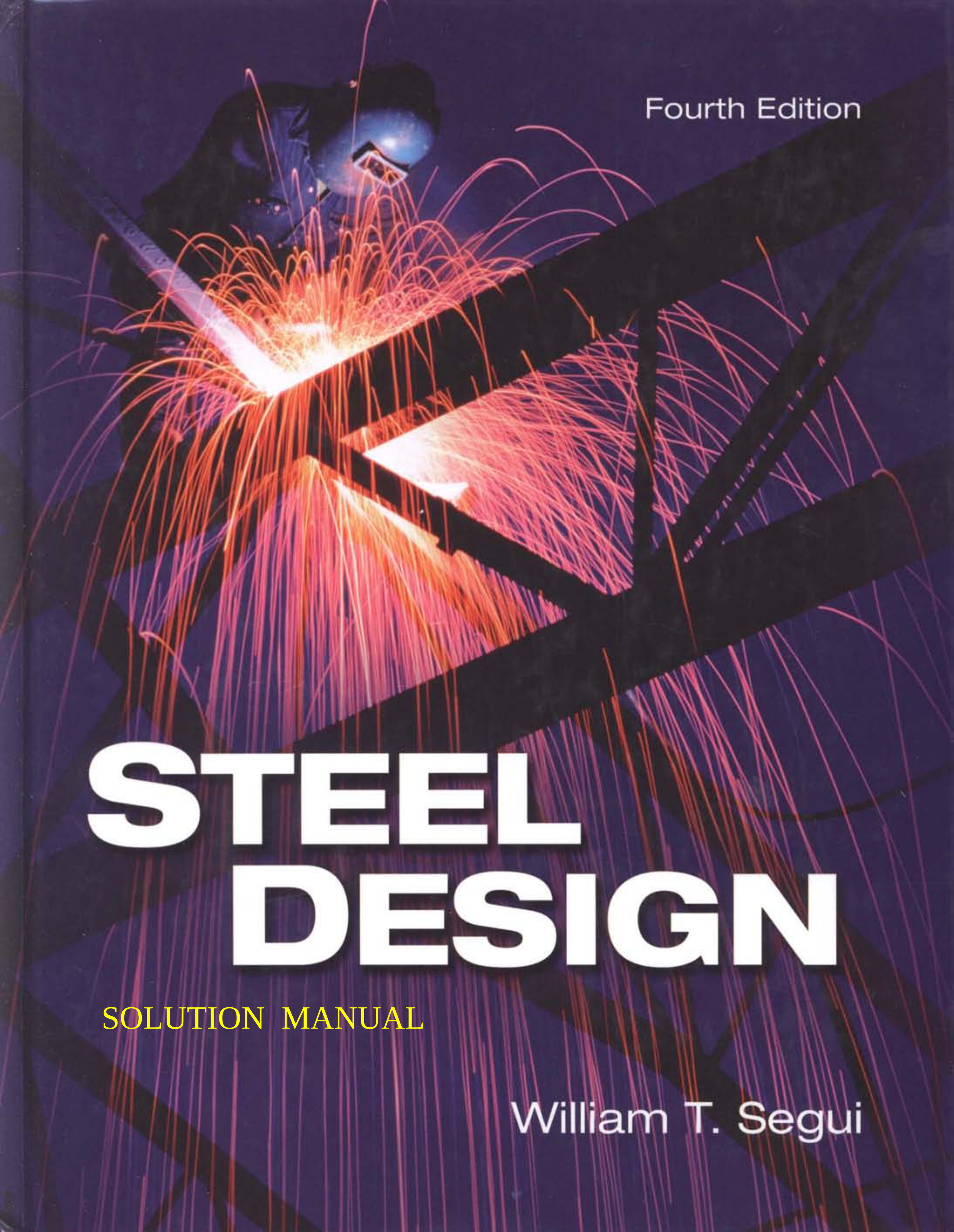


Fourth Edition

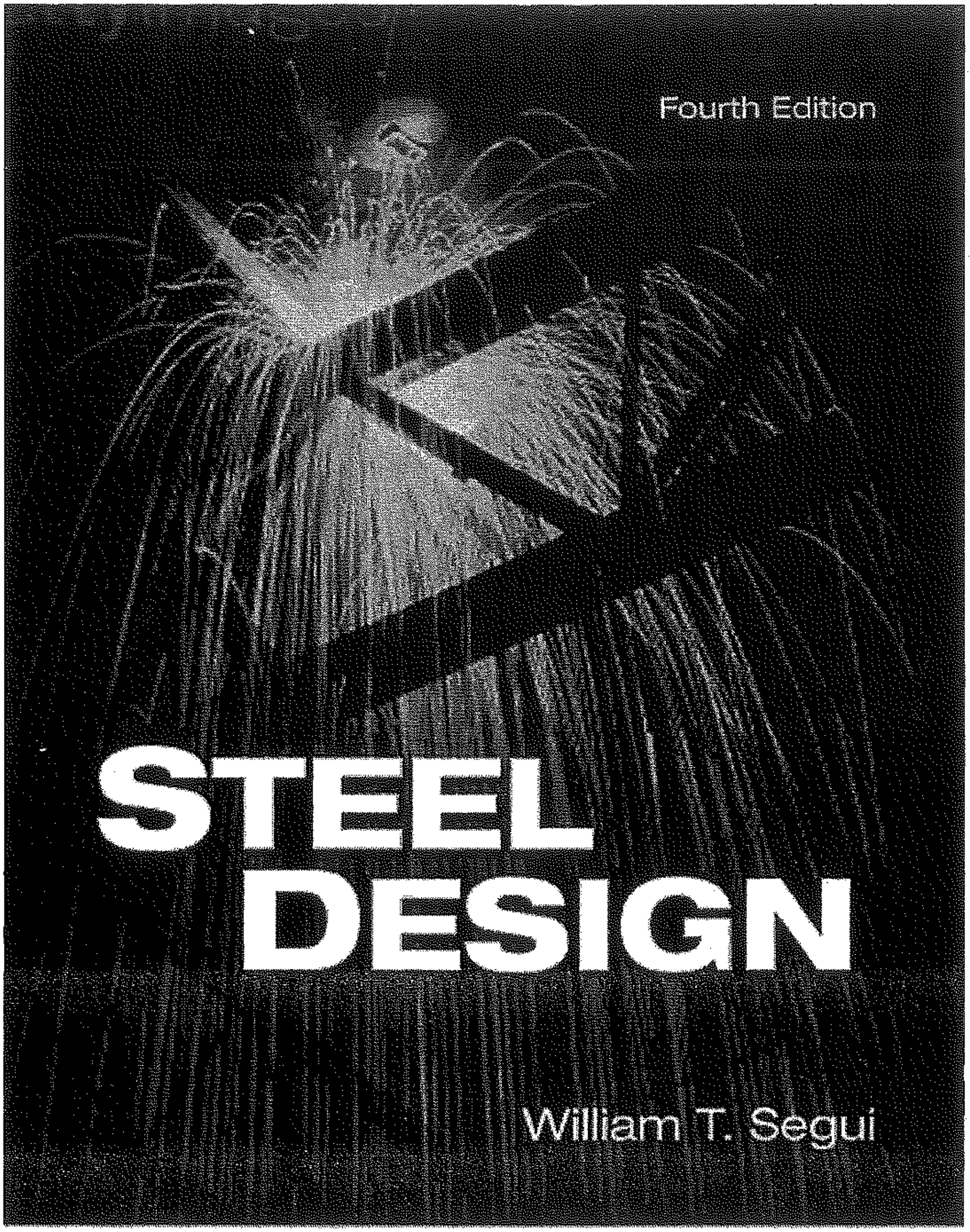
STEEL DESIGN

SOLUTION MANUAL

William T. Segui

INSTRUCTOR'S SOLUTIONS MANUAL

to accompany



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STEEL DESIGN

William T. Segui

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PREFACE

This instructor's manual contains solutions to the problems in Chapters 1-10 of *Steel Design*, 4th edition. Solutions to all problems are given, with the following exceptions: In Chapter 8 some of the ASD solutions are not given. In Chapters 9 and 10, only solutions to the problems whose answers appear in the Answers to Selected Problems section of the textbook are given.

In general, intermediate results to be used in subsequent calculations were recorded to four significant figures, and final results were rounded to three significant figures. Rounding was done according to the following practice: when the first digit to be dropped is less than 5, do not round up; when the first digit to be dropped is equal or greater than 5, add 1 to the last digit retained.

Students following these guidelines should be able to reproduce the numerical results given. However, the precision of the results could depend on the grouping of computations and on whether intermediate values are retained in the calculator between steps.

In many cases, there will be more than one acceptable solution to a design problem; therefore, the solutions given for design problems should be used only as a guide in grading homework.

I would appreciate learning of any errors you may discover. You can contact me at wsegui@memphis.edu. A list of errors and corrections will be maintained at

http://www.ce.memphis.edu/segui/errata.html.

In some instances, there are inconsistencies between dimensions and properties given in the *Manual* and values given in the database on the Companion CD. When I encountered a discrepancy, I used the value given in the *Manual*.

William T. Segui
November 4, 2006

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CHAPTER 1 - INTRODUCTION

1.5-1

(a) $A = \pi(0.550)^2 / 4 = 0.2376 \text{ in.}^2$

$$F_u = \frac{P_u}{A} = \frac{28{,}500}{0.2376} = 120{,}000 \text{ psi} = 120 \text{ ksi}$$

$\underline{F_u = 120 \text{ ksi}}$

(b) $$e = \frac{2.300 - 2.030}{2.030} \times 100 = 13.3\%$$

$\underline{e = 13.3\%}$

(c)

$A_f = \pi(0.430)^2 / 4 = 0.1452 \text{ in.}^2$

$$\text{Change} = \frac{A_f - A_o}{A_o} \times 100 = \frac{0.1452 - 0.2376}{0.2376} \times 100 = -38.9\%$$

$\underline{\text{Reduction} = 38.9\%}$

1.5-2

$A = \pi(0.5)^2 / 4 = 0.1963 \text{ in.}^2$

$$f = \frac{P}{A} = \frac{13.5}{0.1963} = 68.77 \text{ ksi}$$

$$\varepsilon = \frac{\Delta L}{L} = \frac{4.66 \times 10^{-3}}{2} = 2.33 \times 10^{-3}$$

$$E = \frac{f}{\varepsilon} = \frac{68.77}{2.33 \times 10^{-3}} = 29{,}500 \text{ ksi}$$

$\underline{E = 29{,}500 \text{ ksi}}$

1.5-3

$A = \pi(0.510)^2 / 4 = 0.2043 \text{ in.}^2$

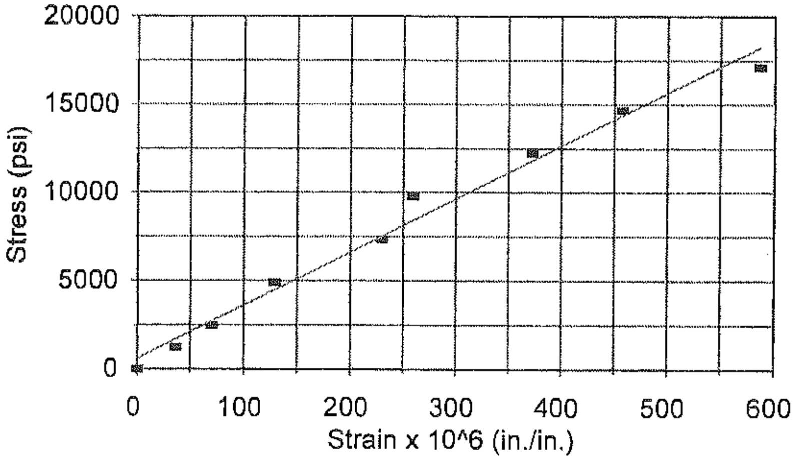
For $P = 250$ lb, $f = \dfrac{P}{A} = \dfrac{250}{0.2043} = 1224$ psi

Spreadsheet results:

(a)

Load (lb)	Stress (psi)	Strain x 10^6 (in./in.)
0	0	0
250	1224	37.1
500	2447	70.3
1000	4895	129.1
1500	7342	230.1
2000	9790	259.4
2500	12237	372.4
3000	14684	457.7
3500	17132	586.5

(b)



(c) E = slope = 30,100 ksi

1.5-4

$$A = \pi(0.5)^2 / 4 = 0.1963 \text{ in.}^2$$

$$E = \frac{f}{\varepsilon} = \frac{P/A}{\Delta L / L} = \frac{P}{\Delta L} \cdot \frac{L}{A} = 1392(4/0.1963) = 28{,}400 \text{ ksi}$$

E = 28,400 ksi

1.5-5

$$A = \pi(3/8)^2/4 = 0.1104 \text{ in.}^2$$

$$\text{For } P = 550 \text{ lb, } f = \frac{P}{A} = \frac{550}{0.1104} = 4982 \text{ psi}$$

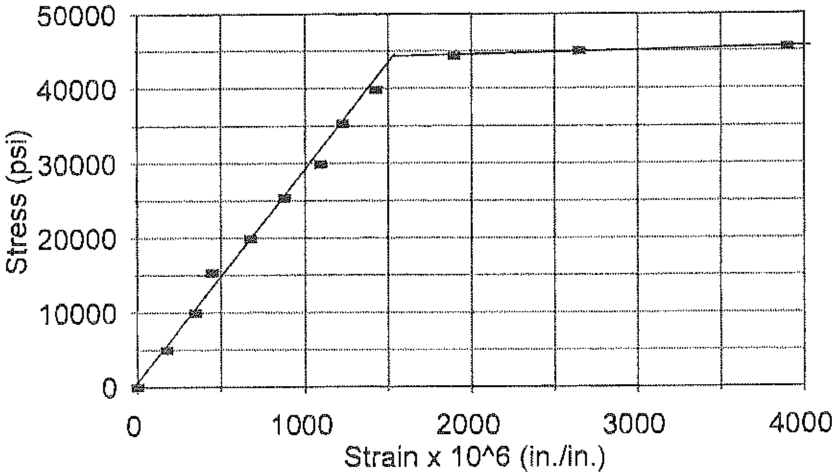
$$\varepsilon = \frac{\Delta L}{L} = \frac{350 \times 10^{-6}}{2} = 175 \times 10^{-6}$$

Spreadsheet results:

(a)

Load (lb)	Elongation x 10^6 (in.)	Stress (psi)	Strain x 10^6 (in./in.)
0	0	0	0
550	350	4982	175
1100	700	9964	350
1700	900	15399	450
2200	1350	19928	675
2800	1760	25362	880
3300	2200	29891	1100
3900	2460	35326	1230
4400	2860	39855	1430
4900	3800	44384	1900
4970	5300	45018	2650
5025	7800	45516	3900

(b)



(c) $E = \text{slope} = \dfrac{15{,}000}{500 \times 10^{-6}} = 30{,}000{,}000 \text{ psi}$

$$\underline{E = 30{,}000{,}000 \text{ psi}}$$

(d) $$\underline{F_y \approx 44{,}000 \text{ psi}}$$

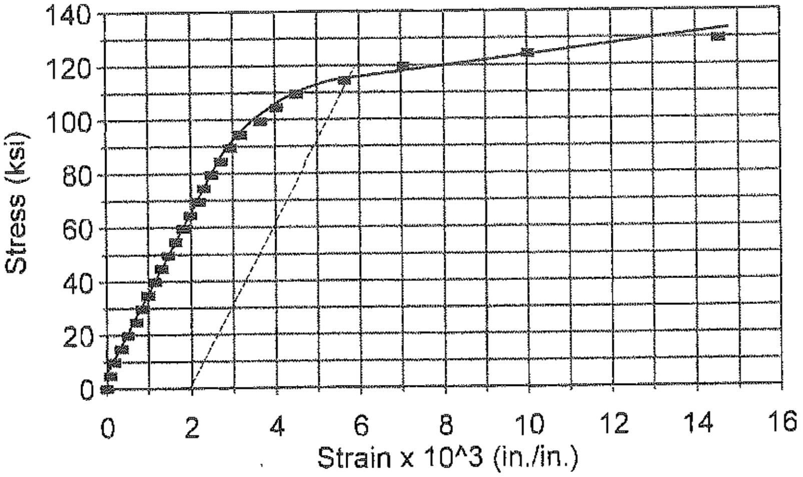
1.5-6

Spreadsheet results:

(a)

Load (kips)	Elongation x 10^3 (in.)	Stress (ksi)	Strain x 10^3 (in./in.)
0	0	0	0
1	0.16	4.973	0.080
2	0.352	9.945	0.176
3	0.706	14.92	0.353
4	1.012	19.89	0.506
5	1.434	24.86	0.717
6	1.712	29.84	0.856
7	1.986	34.81	0.993
8	2.286	39.78	1.143
9	2.612	44.75	1.306
10	2.938	49.73	1.469
11	3.274	54.70	1.637
12	3.632	59.67	1.816
13	3.976	64.64	1.988
14	4.386	69.62	2.193
15	4.64	74.59	2.320
16	4.988	79.56	2.494
17	5.432	84.54	2.716
18	5.862	89.51	2.931
19	6.362	94.48	3.181
20	7.304	99.45	3.652
21	8.072	104.4	4.036
22	9.044	109.4	4.522
23	11.31	114.4	5.655
24	14.12	119.3	7.060
25	20.044	124.3	10.02
26	29.106	129.3	14.55

(b)



(c) $E = \text{slope} = \dfrac{80 - 50}{0.0025 - 0.0015} = 30{,}000 \text{ ksi}$

$\underline{E = 30{,}000 \text{ ksi}}$

(d) $\underline{F_{p\ell} \approx 85 \text{ ksi}}$

(e) $\underline{F_y \approx 116 \text{ ksi}}$

CHAPTER 2 - CONCEPTS IN STRUCTURAL STEEL DESIGN

2-1

$D = 9$ kips, $L_r = 5$ kips, $S = 6$ kips, $R = 7$ kips, $W = 8$ kips

(a)

1: $1.4D = 1.4(9) = 12.6$ kips

2: $1.2D + 1.6L + 0.5R = 1.2(9) + 1.6(0) + 0.5(7) = 14.3$ kips

3: $1.2D + 1.6R + 0.8W = 1.2(9) + 1.6(7) + 0.8(8) = 28.4$ kips

4: $1.2D + 1.6W + 0.5L + 0.5R = 1.2(9) + 1.6(8) + 0.5(0) + 0.5(7) = 27.1$ kips

6: $0.9\text{D} \pm 1.6\text{W} = 0.9(9) + 1.6(8) = 20.9$ kips

or $0.9(9) - 1.6(8) = -4.7$ kips

$\underline{R_u = 28.4 \text{ kips (combination 3)}}$

(but the column must be checked for an uplift of 4.7 kips.)

(b) $\underline{\phi R_n = 28.4 \text{ kips}}$

(c) $R_n = \dfrac{\phi R_n}{\phi} = \dfrac{28.4}{0.90} = 31.6$ kips $\underline{R_n = 31.6 \text{ kips}}$

(d)

3: $D + R = 9 + 7 = 16$ kips

5: $D \pm W = 9 + 8 = 17$ kips

6: $D + 0.75W + 0.75L + 0.75R = 9 + 0.75(8) + 0.75(0) + 0.75(7) = 20.25$ kips

7: $0.6D \pm W = 0.6(9) + 8 = 13.4$ kips

or $0.6(9) - 8 = -2.6$ kips

$\underline{R_a = 20.3 \text{ kips (combination 6)}}$

(but the column must be checked for an uplift of 2.6 kips)

(e) $R_n = \Omega R_a = 1.67(20.3) = 33.9$ kips $\underline{R_n = 33.9 \text{ kips}}$

2-2

1: $1.4D = 1.4(9) = 12.6$ kips

2: $1.2D + 1.6L + 0.5S = 1.2(9) + 1.6(0) + 0.5(6) = 13.8$ kips

3: $1.2D + 1.6S + 0.8W = 1.2(9) + 1.6(6) + 0.8(8) = 26.8$ kips

4: $1.2D + 1.3W + 0.5L + 0.5S = 1.2(9) + 1.3(8) + 0.5(0) + 0.5(6) = 24.2$ kips

(a) 26.8 kips (combination 3)

(b) $\phi R_n = 26.8$ kips

(c) $R_n = \dfrac{\phi R_n}{\phi} = \dfrac{26.8}{0.85} = 31.5\text{ kips}$ $R_n = 31.5$ kips

---

2-3

(a) Combination 1: $1.4D = 1.4(45) = 63$ ft-kips

Combination 2: $1.2D + 1.6L + 0.5L_r = 1.2(45) + 1.6(63) + 0.5(0) = 154.8$ ft-kips

$R_u = 155\text{ ft-kips (combination 2)}$

(b) $R_n = \dfrac{R_u}{\phi} = \dfrac{154.8}{0.9} = 172\text{ ft-kips}$ $R_n = 172\text{ ft-kips}$

(c) Combination 2: $D + L = 45 + 63 = 108$ ft-kips $R_a = 108$ ft-kips (combination 2)

(d) $R_n = \Omega R_a = 1.67(108) = 180\text{ ft-kip}$ $R_n = 180$ ft-kips

---

2-4

$D = 18$ kips, $L = 2$ kips

(a)

1: $1.4D = 1.4(18) = 25.2$ kips

2: $1.2D + 1.6L = 1.2(18) + 1.6(2) = 24.8$ kips

$R_u = 25.2\text{ kips (combination 1)}$

(b)

2: $D + L = 18 + 2 = 20$ kips. $R_a = 20\text{ kips (combination 2)}$

2-5

$D = 21$ psf, $L_r = 12$ psf, $S = 13.5$ psf, $W = 22$ psf upward (*in this particular case, the wind load cannot be reversed, even in those cases where reversal would normally be considered.*)

Treat gravity loads as positive and wind load as negative:

(a)

1: $1.4D = 1.4(21) = 29.4\text{ psf}$

2: $1.2D + 1.6L + 0.5S = 1.2(21) + 1.6(0) + 0.5(13.5) = 32.0$ psf

3: $1.2D + 1.6S + 0.5L = 1.2(21) + 1.6(13.5) + 0.5(0) = 46.8$ psf

4: $1.2D + 1.6W + 0.5L + 0.5S = 1.2(21) + 1.6(-22) + 0.5(0) + 0.5(13.5)$

$= -3.25$ psf (uplift)

5: $1.2D \pm E + 0.5L + 0.2S = 1.2(21) \pm 0 + 0.5(0) + 0.2(13.5) = 27.9$ psf

6: $0.9D \pm (1.6W) = 0.9D + 1.6W = 0.9(21) + 1.6(-22) = -16.3$ psf (uplift)

$\underline{R_u = 46.8 \text{ psf (combination 3)}}$

(Combination 6, with R_u = -16.3 psf, would also need to be considered in the design of the roof in order to prevent uplift.)

(b)

3: $D + S = 21 + 13.5 = 34.5$ psf

5: $D \pm W = 21 + (-22) = -1$ psf

6: $D + 0.75W + 0.75L + 0.75S = 21 + 0.75(-22) + 0 + 0.75(13.5) = 14.6$ psf

7: $0.6D \pm W = 0.6(21) + (-22) = -9.4$ psf

$\underline{R_a = 34.5 \text{ psf (combination 1)}}$

(Combination 7, with R_a = -9.4 psf, would also need to be considered in the design of the roof in order to prevent uplift.)

3.2-1

For yielding of the gross section,

$$A_g = 7(3/8) = 2.625 \text{ in.}^2, \qquad P_n = F_y A_g = 36(2.625) = 94.5 \text{ kips}$$

For fracture of the net section,

$$A_e = (3/8)\left[7 - \left(1 + \frac{1}{8}\right)\right] = 2.203 \text{ in.}^2$$

$$P_n = F_u A_e = 58(2.203) = 127.8 \text{ kips}$$

a) The design strength based on yielding is

$$\phi_t P_n = 0.90(94.5) = 85.05 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(127.8) = 95.85 \text{ kips}$$

The design strength for LRFD is the smaller value: $\underline{\phi_t P_n = 85.1 \text{ kips}}$

b) The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{94.5}{1.67} = 56.59 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{127.8}{2.00} = 63.9 \text{ kips}$$

The allowable service load is the smaller value: $\underline{P_n/\Omega_t = 56.6 \text{ kips}}$

Alternate solution using allowable *stress*: For yielding,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the allowable load is $F_t A_g = 21.6(2.625) = 56.7$ kips

For fracture,

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the allowable load is $F_t A_e = 29.0(2.203) = 63.89 = 63.89$ kips

The allowable service load is the smaller value = 56.7 kips

3.2-2

For yielding of the gross section,

$$A_g = 6(3/8) = 2.25 \text{ in.}^2$$

$$P_n = F_y A_g = 50(2.25) = 112.5 \text{ kips}$$

For fracture of the net section,

$$A_e = A_g = 2.25 \text{ in.}^2$$

$$P_n = F_u A_e = 65(2.25) = 146.3 \text{ kips}$$

a) The design strength based on yielding is

$$\phi_t P_n = 0.90(112.5) = 101 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(146.3) = 110 \text{ kips}$$

The design strength for LRFD is the smaller value: $\underline{\phi_t P_n = 101 \text{ kips}}$

b) The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{112.5}{1.67} = 67.4 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{146.3}{2.00} = 73.2 \text{ kips}$$

The allowable service load is the smaller value: $\underline{P_n/\Omega_t = 67.4 \text{ kips}}$

Alternate solution using allowable *stress*: For yielding,

$$F_t = 0.6F_y = 0.6(50) = 30.0 \text{ ksi}$$

and the allowable load is

$$F_t A_g = 30.0(2.25) = 67.5 \text{ kips}$$

For fracture,

$$F_t = 0.5F_u = 0.5(65) = 32.5 \text{ ksi}$$

and the allowable load is

$$F_t A_e = 32.5(2.25) = 73.1 \text{ kips}$$

he allowable service load is the smaller value = 67.5 kips

3.2-3

For yielding of the gross section,

$$P_n = F_y A_g = 50(3.37) = 168.5 \text{ kips}$$

For fracture of the net section,

$$A_n = A_g - A_{holes} = 3.37 - 0.220\left(\frac{7}{8} + \frac{1}{8}\right) \times 2 \text{ holes} = 2.930 \text{ in.}^2$$

$$A_e = 0.85A_n = 0.85(2.930) = 2.491 \text{ in.}^2$$

$$P_n = F_e A_e = 65(2.491) = 161.9 \text{ kips}$$

a) The design strength based on yielding is

$$\phi_t P_n = 0.90(168.5) = 152 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(161.9) = 121.4 \text{ kips}$$

The design strength is the smaller value: $\phi_t P_n = 121.4$ kips

Let $P_u = \phi_t P_n$

$$1.2D + 1.6(3D) = 121.4, \text{ Solution is: } \{D = 20.23\}$$

$$P = D + L = 20.23 + 3(20.23) = 80.9 \text{ kips}$$

$\underline{P = 80.9 \text{ kips}}$

b) The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{168.5}{1.67} = 100.9 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{161.9}{2.00} = 80.95 \text{ kips}$$

The allowable load is the smaller value = 80.95 kips

$\underline{P = 81.0 \text{ kips}}$

Alternate computation of allowable load using allowable *stress*: For yielding,

$$F_t = 0.6F_y = 0.6(50) = 30.0 \text{ ksi}$$

and the allowable load is

$$F_t A_g = 30.0(3.37) = 101.1 \text{ kips}$$

For fracture,

$$F_t = 0.5F_u = 0.5(65) = 32.5 \text{ ksi}$$

and the allowable load is

$$F_t A_e = 32.5(2.491) = 80.96 \text{ kips}$$

3.2-4

For A242 steel and $t = ½$ in., $F_y = 50$ ksi and $F_u = 70$ ksi. For yielding of the gross section,

$$A_g = 8(1/2) = 4 \text{ in.}^2$$

$$P_n = F_y A_g = 50(4) = 200 \text{ kips}$$

For fracture of the net section,

$$A_n = A_g - A_{holes} = 4 - (1/2)\left(1 + \frac{1}{8}\right) \times 2 \text{ holes} = 2.875 \text{ in.}^2$$

$$A_e = A_n = 2.875 \text{ in.}^2$$

$$P_n = F_u A_e = 70(2.875) = 201.3 \text{ kips}$$

a) The design strength based on yielding is

$$\phi_t P_n = 0.90(200) = 180 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(201.3) = 151 \text{ kips}$$

The design strength for LRFD is the smaller value: $\underline{\phi_t P_n = 151 \text{ kips}}$

b) The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{200}{1.67} = 120 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{201.3}{2.00} = 101 \text{ kips}$$

The allowable service load is the smaller value: $\underline{P_n/\Omega_t = 101 \text{ kips}}$

Alternate solution using allowable *stress*: For yielding,

$$F_t = 0.6F_y = 0.6(50) = 30 \text{ ksi}$$

and the allowable load is

$$F_t A_g = 30(4) = 120 \text{ kips}$$

For fracture,

$$F_t = 0.5F_u = 0.5(70) = 35 \text{ ksi}$$

and the allowable load is

$$F_t A_e = 35(2.875) = 101 \text{ kips}$$

The allowable service load is the smaller value = 101 kips

3.2-5

For a thickness of $t = 3/8$ in., $F_y = 50$ ksi and $F_u = 70$ ksi. First, compute the nominal strengths. For the gross section,

$$A_g = 7.5(3/8) = 2.813 \text{ in.}^2$$

$$P_n = F_y A_g = 50(2.813) = 140.7 \text{ kips}$$

Net section:

$$A_n = 2.813 - \left(\frac{3}{8}\right)\left(1\frac{1}{8} + \frac{1}{8}\right)(2) = 1.876 \text{ in.}^2$$

$$A_e = A_n = 1.876 \text{ in.}^2$$

$$P_n = F_u A_e = 70(1.876) = 131.3 \text{ kips}$$

a) The design strength based on yielding is

$$\phi_t P_n = 0.90(140.7) = 127 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(131.3) = 98.5 \text{ kips}$$

The design strength is the smaller value: $\phi_t P_n = 98.5$ kips

Factored load:

Combination 1: $1.4D = 1.4(25) = 35.0$ kips

Combination 2: $1.2D + 1.6L = 1.2(25) + 1.6(45) = 102$ kips

The second combination controls; $P_u = 102$ kips.

Since $P_u > \phi_t P_n$, (102 kips > 98.5 kips), The member is unsatisfactory.

b) For the gross section, The allowable strength is

$$\frac{P_n}{\Omega_t} = \frac{140.7}{1.67} = 84.3 \text{ kips}$$

Alternately, the allowable stress is

$$F_t = 0.6F_y = 0.6(50) = 30 \text{ ksi}$$

and the allowable strength is $F_t A_g = 30(2.813) = 84.4$ kips

For the net section, the allowable strength is

$$\frac{P_n}{\Omega_t} = \frac{131.3}{2.00} = 65.7 \text{ kips}$$

Alternately, the allowable stress is

$F_t = 0.5F_u = 0.5(70) = 35$ ksi and the allowable strength is

$$F_t A_e = 35(1.876) = 65.7 \text{ kips}$$

The smaller value controls; the allowable strength is 65.7 kips. When the only loads are dead load and live load, ASD load combination 2 will always control:

$$P_a = D + L = 25 + 45 = 70 \text{ kips}$$

Since 70 kips > 65.7 kips, The member is unsatisfactory.

3.2-6

Compute the strength for one angle, then double it. For the gross section,

$$P_n = F_y A_g = 36(1.19) = 42.84 \text{ kips}$$

For two angles, $P_n = 2(42.84) = 85.68$ kips

Net section:

$$A_n = 1.19 - \left(\frac{1}{4}\right)\left(\frac{3}{4} + \frac{1}{8}\right) = 0.9713 \text{ in.}^2$$

$$A_e = 0.85A_n = 0.85(0.9713) = 0.8256 \text{ in.}^2$$

$$P_n = F_u A_e = 58(0.8256) = 47.88 \text{ kips}$$

For two angles, $P_n = 2(47.88) = 95.76$ kips

a) The design strength based on yielding is

$$\phi_t P_n = 0.90(85.68) = 77.1 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(95.76) = 71.8 \text{ kips}$$

The design strength is the smaller value: $\phi_t P_n = 71.8$ kips

$$P_u = 1.2D + 1.6L = 1.2(12) + 1.6(36) = 72.0 \text{ kips} > 71.8 \text{ kips} \quad \text{(N.G.)}$$

The member is unsatisfactory.

b) For the gross section, The allowable strength is

$$\frac{P_n}{\Omega_t} = \frac{85.68}{1.67} = 51.3 \text{ kips}$$

Alternately, the allowable stress is

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the allowable strength is $F_t A_g = 21.6(2 \times 1.19) = 51.4$ kips

For the net section, the allowable strength is

$$\frac{P_n}{\Omega_t} = \frac{95.76}{2.00} = 47.9 \text{ kips}$$

Alternately, the allowable stress is

$$F_t = 0.5F_u = 0.5(58) = 29 \text{ ksi}$$

and the allowable strength is $F_t A_e = 29(2 \times 0.8256) = 47.9$ kips

The net setion strength controls; the allowable strength is 47.9 kips. When the only loads are dead load and live load, ASD load combination 2 will always control:

$$P_a = D + L = 12 + 36 = 48 \text{ kips} > 47.9 \text{ kips} \qquad \text{(N.G.)}$$

The member is unsatisfactory.

3.3-1

(a) $U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.47}{5} = 0.7060$

$A_e = A_g U = 5.86(0.7060) = 4.14 \text{ in.}^2$ $\qquad \underline{A_e = 4.14 \text{ in.}^2}$

(b) Plate with longitudinal welds only:

$\frac{\ell}{w} = \frac{5}{4} = 1.25, \qquad U = 0.75 \text{ in.}^2$

$A_e = A_g U = \left(\frac{3}{8} \times 4\right)(0.75) = 1.125 \text{ in.}^2$ $\qquad \underline{A_e = 1.13 \text{ in.}^2}$

(c) $U = 1.0$

$A_e = A_g U = \left(\frac{5}{8} \times 5\right)(1.0) = 3.13 \text{ in.}^2$ $\qquad \underline{A_e = 3.13 \text{ in.}^2}$

(d) $U = 1.0$

$A_g = 0.5(5.5) = 2.750 \text{ in.}^2$

$A_n = A_g - A_{holes} = 2.750 - \frac{1}{2}\left(\frac{3}{4} + \frac{1}{8}\right) = 2.313 \text{ in.}^2$

$A_e = A_n U = 2.313(1.0) = 2.313 \text{ in.}^2$ $\qquad \underline{A_e = 2.31 \text{ in.}^2}$

(e) $U = 1.0$

$A_g = \frac{5}{8} \times 6 = 3.750 \text{ in.}^2$

$A_n = A_g - A_{holes} = 3.750 - \frac{5}{8}\left(\frac{7}{8} + \frac{1}{8}\right) = 3.125 \text{ in.}^2$

$A_e = A_n U = 3.125(1.0) = 3.125 \text{ in.}^2$ $\qquad \underline{A_e = 3.13 \text{ in.}^2}$

3.3-2

(a) $A_n = A_g - A_{holes} = 3.31 - \frac{7}{16}\left(\frac{7}{8} + \frac{1}{8}\right) = 2.873 \text{ in.}^2$

$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.15}{3} = 0.6167$

$A_e = A_n U = 2.873(0.6167) = 1.772 \text{ in.}^2$

$P_n = F_u A_e = 70(1.772) = 124$ kips $\qquad \underline{P_n = 124 \text{ kips}}$

(b) From AISC Table D3.1, Case 8, $U = 0.60$

$A_e = A_n U = 2.873(0.60) = 1.724$ in.^{2}

$P_n = F_u A_e = 70(1.724) = 121$ kips $\qquad \underline{P_n = 121 \text{ kips}}$

3.3-3

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.775}{8} = 0.9031$$

$A_e = A_g U = 2.48(0.9031) = 2.240$ in.^{2}

$P_n = F_u A_e = 58(2.240) = 129.9$ kips $\qquad \underline{P_n = 130 \text{ kips}}$

3.3-4

For A242 steel, $F_y = 50$ ksi and $F_u = 70$ ksi

For yielding of the gross section,

$P_n = F_y A_g = 50(4.75) = 237.5$ kips

For fracture of the net section,

$$A_n = A_g - A_{holes} = 4.75 - \frac{1}{2}\left(\frac{3}{4} + \frac{1}{8}\right) = 4.313 \text{ in.}^2$$

From AISC Table D3.1, Case 8, $U = 0.80$

$A_e = A_n U = 4.313(0.80) = 3.45$ in.^{2}

$P_n = F_e A_e = 70(3.45) = 241.5$ kips

a) The design strength based on yielding is

$\phi_t P_n = 0.90(237.5) = 213.8$ kips

The design strength based on fracture is

$\phi_t P_n = 0.75(241.5) = 181.1$ kips

The design strength is the smaller value: $\phi_t P_n = 181.1$ kips

Let $P_u = \phi_t P_n$

$1.2D + 1.6(2D) = 181.1$, Solution is: $\{D = 41.16\}$

$P = D + L = 41.16 + 2(41.16) = 124$ kips $\qquad \underline{P = 124 \text{ kips}}$

b) The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{237.5}{1.67} = 142.2 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{241.5}{2.00} = 120.8 \text{ kips}$$

The allowable load is the smaller value = 120.8 kips $\underline{P = 121 \text{ kips}}$

Alternate computation of allowable load using allowable *stress*: For yielding,

$$F_t = 0.6F_y = 0.6(50) = 30.0 \text{ ksi}$$

and the allowable load is

$$F_tA_g = 30.0(4.75) = 142.5 \text{ kips}$$

For fracture,

$$F_t = 0.5F_u = 0.5(70) = 35 \text{ ksi}$$

and the allowable load is

$$F_tA_e = 35(3.45) = 120.8 \text{ kips}$$

3.3-5

Gross section: $P_n = F_yA_g = 36(5.86) = 211.0 \text{ kips}$

Net section: $A_n = 5.86 - \left(\frac{5}{8}\right)\left(1 + \frac{1}{8}\right)(2) = 4.454 \text{ in.}^2$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.03}{(3+3+3)} = 0.8856$$

$$A_e = A_nU = 4.454(0.8856) = 3.944 \text{ in.}^2$$

$$P_n = F_uA_e = 58(3.944) = 228.8 \text{ kips}$$

(a) The design strength based on yielding is

$$\phi_tP_n = 0.90(211.0) = 190 \text{ kips}$$

The design strength based on fracture is

$$\phi_tP_n = 0.75(228.8) = 172 \text{ kips}$$

The design strength is the smaller value: $\phi_tP_n = 172$ kips

Load combination 2 controls:

$$P_u = 1.2D + 1.6L = 1.2(50) + 1.6(100) = 220 \text{ kips}$$

Since $P_u > \phi_tP_n$, (220 kips > 172 kips), The member is not adequate.

(b) For the gross section, The allowable strength is $\frac{P_n}{\Omega_t} = \frac{211.0}{1.67} = 126$ kips

For the net section, the allowable strength is $\frac{P_n}{\Omega_t} = \frac{228.8}{2.00} = 114$ kips

The smaller value controls; the allowable strength is 114 kips.

Load combination 6 controls:

$$P_a = D + 0.75W + 0.75L = 50 + 0.75(45) + 0.75(100) = 159 \text{ kips}$$

Since 159 kips > 114 kips, The member is not adequate.

Alternate ASD solution using allowable stress:

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the allowable strength is $F_tA_g = 21.6(5.86) = 127$ kips

For the net section, $F_t = 0.5F_u = 0.5(58) = 29.0$ ksi

and the allowable strength is $F_tA_e = 29.0(3.944) = 114$ kips

The smaller value controls; the allowable strength is114 kips. From load combination 6,

$$P_a = D + 0.75W + 0.75L = 50 + 0.75(45) + 0.75(100) = 159 \text{ kips}$$

Since 159 kips > 114 kips, the member is not adequate.

3.3-6

For yielding of the gross section,

$$A_g = 5(1/4) = 1.25 \text{ in.}^2$$

$$P_n = F_yA_g = 36(1.25) = 45.0 \text{ kips}$$

For fracture of the net section, from AISC Table D3.1, case 4,

$$L/w = 7/5 = 1.4, \quad \therefore \ U = 0.75$$

$$A_e = A_gU = 1.25(0.75) = 0.9375 \text{ in.}^2$$

$$P_n = F_uA_e = 58(0.9375) = 54.38 \text{ kips}$$

a) The design strength based on yielding is

$$\phi_tP_n = 0.90(45.0) = 40.5 \text{ kips}$$

The design strength based on fracture is

$$\phi_tP_n = 0.75(54.38) = 40.8 \text{ kips}$$

The design strength for LRFD is the smaller value: $\phi_tP_n = 40.5$ kips

b) The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{45.0}{1.67} = 27.0 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{54.38}{2.00} = 27.2 \text{ kips}$$

The allowable service load is the smaller value: $\underline{P_n/\Omega_t = 27.0 \text{ kips}}$

3.3-7

Gross section: $P_n = F_yA_g = 50(10.3) = 515.0 \text{ kips}$

Net section: $A_n = 10.3 - 0.520\left(\frac{7}{8} + \frac{1}{8}\right)(4) = 8.220 \text{ in.}^2$

Connection is through the flanges with four bolts per line.

$$\frac{b_f}{d} = \frac{6.56}{12.5} = 0.525 < \frac{2}{3} \qquad \therefore\ U = 0.85$$

$$A_e = A_nU = 8.220(0.85) = 6.987 \text{ in.}^2$$

$$P_n = F_uA_e = 65(6.987) = 454.2 \text{ kips}$$

(a) The design strength based on yielding is

$$\phi_tP_n = 0.90(515.0) = 464 \text{ kips}$$

The design strength based on fracture is

$$\phi_tP_n = 0.75(454.2) = 341 \text{ kips}$$

The design strength is the smaller value: $\underline{\phi_tP_n = 341 \text{ kips}}$

(b) For the gross section, The allowable strength is $\frac{P_n}{\Omega_t} = \frac{515.0}{1.67} = 308 \text{ kips}$

For the net section, the allowable strength is $\frac{P_n}{\Omega_t} = \frac{454.2}{2.00} = 227 \text{ kips}$

The smaller value controls; $\underline{\frac{P_n}{\Omega_t} = 227 \text{ kips}}$

3.3-8

Gross section: $P_n = F_yA_g = 50(5.17) = 258.5 \text{ kips}$

Net section:

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.30}{10} = 0.87$$

$$A_e = A_gU = 5.17(0.87) = 4.498 \text{ in.}^2$$

$$P_n = F_uA_e = 70(4.498) = 314.9 \text{ kips}$$

(a) The design strength based on yielding is

$$\phi_tP_n = 0.90(258.5) = 233 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(314.9) = 236 \text{ kips}$$

The design strength is the smaller value: $\phi_t P_n = 233$ kips

Load combination 3:

$$P_u = 1.2D + 1.6S + 0.8W = 1.2(75) + 1.6(50) + 0.8(70) = 226 \text{ kips}$$

Load combination 4:

$$P_u = 1.2D + 1.6W + 0.5S = 1.2(75) + 1.6(70) + 0.5(50) = 227 \text{ kips}$$

Load combination 4 controls. Since $P_u < \phi_t P_n$, (227 kips < 233 kips),

The member is adequate.

(b) For the gross section, The allowable strength is $\dfrac{P_n}{\Omega_t} = \dfrac{258.5}{1.67} = 155$ kips

For the net section, the allowable strength is $\dfrac{P_n}{\Omega_t} = \dfrac{314.9}{2.00} = 157$ kips

The smaller value controls; the allowable strength is 155 kips.

Load combination 3: $P_a = D + S = 75 + 50 = 125$ kips

Load combination 6: $P_a = D + 0.75W + 0.75S = 75 + 0.75(70) + 0.75(50) = 165$ kips

Load combination 6 controls. Since 165 kips > 155 kips, The member is not adequate.

3.4-1

Gross section: $A_g = 10(1/2) = 5 \text{ in.}^2$

Net section: Hole diameter $= \dfrac{7}{8} + \dfrac{1}{8} = 1$ in.

Possibilities for net area:

$$A_n = A_g - \sum t \times (d \text{ or } d') = 5 - (1/2)(1)(2) = 4.0 \text{ in.}^2$$

or $A_n = 5 - (1/2)(1) - (1/2)\left[1 - \dfrac{(2)^2}{4(3)}\right] - (1/2)\left[1 - \dfrac{(2)^2}{4(3)}\right] = 3.833 \text{ in.}^2$

or $A_n = 5 - (1/2)(1)(3) = 3.5 \text{ in.}^2$, but because of load transfer,

use $A_n = \dfrac{9}{6}(3.5) = 5.25 \text{ in.}^2$ for this possibility.

The smallest value controls. Use $A_n = 3.833 \text{ in.}^2$

$$A_e = A_n U = A_n(1.0) = 3.833 \text{ in.}^2$$

$$P_n = F_u A_e = 58(3.833) = 222 \text{ kips}$$

The nominal strength based on the net section is $P_n = 222$ kips

3.4-2

Compute the strength of one plate, then double it.

Gross section: $A_g = 10(1/2) = 5.0 \text{ in.}^2$

Net section: Hole diameter $= \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in.

Possibilities for net area:

$$A_n = A_g - \sum t \times (d \text{ or } d') = 5 - (1/2)(7/8)(2) = 4.125 \text{ in.}^2$$

or $A_n = 5 - (1/2)(7/8) - (1/2)\left[\frac{7}{8} - \frac{(5)^2}{4(6)}\right] = 4.646 \text{ in.}^2$

Because of load transfer, use $A_n = \frac{10}{9}(4.646) = 5.162 \text{ in.}^2$ for this possibility.

or $A_n = 5 - (1/2)(7/8) - (1/2)\left[\frac{7}{8} - \frac{(2)^2}{4(3)}\right] - (1/2)\left[\frac{7}{8} - \frac{(2)^2}{4(3)}\right] = 4.021 \text{ in.}^2$

Because of load transfer, use $A_n = \frac{10}{8}(4.021) = 5.026 \text{ in.}^2$ for this possibility.

The smallest value controls. Use $A_n = 4.125 \text{ in.}^2$

$$A_e = A_n U = 4.125(1.0) = 4.125 \text{ in.}^2$$

$$P_n = F_u A_e = 58(4.125) = 239.3 \text{ kips}$$

For two plates, $P_n = 2(239.3) = 478.6$ kips

The nominal strength based on the net section is $\underline{P_n = 479 \text{ kips}}$

3.4-3

Gross section: $A_g = 8(3/8) = 3.0 \text{ in.}^2$, $P_n = F_y A_g = 36(3.0) = 108$ kips

Net section: Hole diameter $= \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$ in.

$$A_n = A_g - \sum t_w \times (d \text{ or } d') = 3 - (3/8)(5/8) = 2.766 \text{ in.}^2$$

or $A_n = 3 - (3/8)(5/8) - (3/8)\left[5/8 - \frac{(3)^2}{4(2)}\right] = 2.954 \text{ in.}^2$

or $A_n = 3 - (3/8)(5/8) - (3/8)\left[5/8 - \frac{(3)^2}{4(2)}\right] \times 2 = 3.141 \text{ in.}^2$

or $A_n = [3 - (3/8)(5/8)(2)] \times \frac{6}{5} = 3.038 \text{ in.}^2$

or $A_n = \left(3 - (3/8)(5/8) - (3/8)\left[5/8 - \frac{(2.5)^2}{4(2)}\right](2)\right) \times \frac{6}{5} = 3.460 \text{ in.}^2$

Use $A_e = A_n = 2.766 \text{ in.}^2$

$P_n = F_u A_e = 58(2.766) = 160.4$ kips

(a) Gross section: $\phi_t P_n = 0.90(108) = 97.2$ kips

Net section: $\phi_t P_n = 0.75(160.4) = 120$ kips $\qquad$ $\underline{\phi_t P_n = 97.2 \text{ kips}}$

(b) Gross section: $\dfrac{P_n}{\Omega_t} = \dfrac{108}{1.67} = 64.7$ kips

Net section: $\dfrac{P_n}{\Omega_t} = \dfrac{160.4}{2.00} = 80.2$ kips $\qquad$ $\underline{P_n/\Omega_t = 64.7 \text{ kips}}$

3.4-4

Gross section: $A_g = 5.87 \text{ in.}^2$, $\quad P_n = F_y A_g = 50(5.87) = 293.5$ kips

Net section: Hole diameter $= 1\frac{1}{8} + \frac{1}{8} = 1.25$ in.

$$A_n = A_g - \sum t_w \times (d \text{ or } d') = 5.87 - 0.448(1.25) = 5.310 \text{ in.}^2$$

or $$A_n = 5.87 - 0.448(1.25) - 0.448\left[1.25 - \frac{(1.5)^2}{4(4)}\right] = 4.813 \text{ in.}^2$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.583}{6(1.5)} = 0.9352$$

$$A_e = A_n U = 4.813(0.9352) = 4.501 \text{ in.}^2$$

$$P_n = F_u A_e = 70(4.501) = 315.1 \text{ kips}$$

(a) Gross section: $\phi_t P_n = 0.90(293.5) = 264$ kips

Net section: $\phi_t P_n = 0.75(315.1) = 236$ kips (controls)

$P_u = 1.2D + 1.6L = 1.2(36) + 1.6(110) = 219 \text{ kips} < 236 \text{ kips}$ (OK)

Since $P_u < \phi_t P_n$ (219 kips < 236 kips), $\qquad$ The member has enough strength.

(b) Gross section: $\dfrac{P_n}{\Omega_t} = \dfrac{293.5}{1.67} = 176$ kips

Net section: $\dfrac{P_n}{\Omega_t} = \dfrac{315.1}{2.00} = 158$ kips (controls)

$P_a = D + L = 36 + 110 = 146 \text{ kips} < 158 \text{ kips}$ (OK)

Since $P_a < \dfrac{P_n}{\Omega_t}$ (146 kips < 158 kips), $\qquad$ The member has enough strength.

3.4-5

For A572 Grade 50 steel, $F_y = 50$ ksi and $F_u = 65$ ksi.

Compute the strength for one angle, then multiply by 2.

Gross section: $A_g = 3.98 \text{ in.}^2, \quad P_n = F_y A_g = 50(3.98) = 199.0 \text{ kips}$

For two angles, $P_n = 2(199.0) = 398.0$ kips

Net section: Hole diameter $= \frac{7}{8} + \frac{1}{8} = 1$ in.

$$A_n = A_g - \sum t \times (d \text{ or } d') = 3.98 - (3/8)(1) = 3.605 \text{ in.}^2$$

or $A_n = 3.98 - (3/8)(1) - (3/8)\left[1 - \frac{(3)^2}{4(1.5)}\right] = 3.793 \text{ in.}^2$

or $A_n = 3.98 - (3/8)(1) - (3/8)\left[1 - \frac{(3)^2}{4(1.5)}\right] \times 2 = 3.980 \text{ in.}^2$

or $A_n = 3.98 - (3/8)(1) \times 2 = 3.230 \text{ in.}^2$, but because of load transfer,

use $A_n = \frac{7}{6}(3.230) = 3.768 \text{ in.}^2$ for this possibility.

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.861}{3+3+3} = 0.9043$$

$$A_e = A_n U = 3.605(0.9043) = 3.260 \text{ in.}^2$$

$$P_n = F_u A_e = 65(3.260) = 211.9 \text{ kips}$$

For two angles, $P_n = 2(211.9) = 423.8$ kips

(a) LRFD Solution

Gross section: $\phi_t P_n = 0.90(398.0) = 358$ kips

Net section: $\phi_t P_n = 0.75(423.8) = 318$ kips (controls)

$\underline{\phi_t P_n = 318 \text{ kips}}$

(b) ASD Solution

Gross section: $\frac{P_n}{\Omega_t} = \frac{398.0}{1.67} = 238$ kips

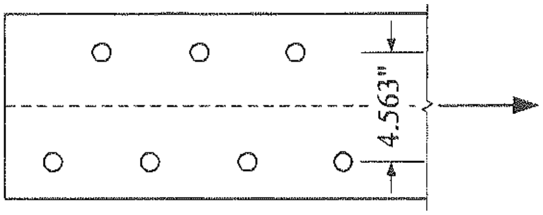
Net section: $\frac{P_n}{\Omega_t} = \frac{423.8}{2.00} = 212$ kips

$\underline{\frac{P_n}{\Omega_t} = 212 \text{ kips}}$

3.4-6

Gross section: $P_n = F_y A_g = 36(3.31) = 119.2$ kips

Net section: Use a gage distance of $2.5 + 2.5 - \frac{7}{16} = 4.563$ in.



Hole diameter = $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in.

$$A_n = A_g - \sum t \times (d \text{ or } d')$$

$$= 3.31 - (7/16)(7/8) = 2.927 \text{ in.}^2$$

or $A_n = 3.31 - (7/16)(7/8) - (7/16)\left[\frac{7}{8} - \frac{(2)^2}{4(4.563)}\right] = 2.640 \text{ in.}^2$

Use $A_e = A_n = 2.640 \text{ in.}^2$, and $P_n = F_u A_e = 58(2.640) = 153.1$ kips

(a) Gross section: $\phi_t P_n = 0.90(119.2) = 107$ kips

Net section: $\phi_t P_n = 0.75(153.1) = 115$ kips

Gross section controls. $\underline{\phi_t P_n = 107 \text{ kips}}$

(b) Gross section: $\frac{P_n}{\Omega_t} = \frac{119.2}{1.67} = 71.4$ kips

Net section: $\frac{P_n}{\Omega_t} = \frac{153.1}{2.00} = 76.6$ kips

Gross section controls. $\underline{P_n/\Omega_t = 71.4 \text{ kips}}$

3.5-1

Shear areas:

$$A_{gv} = \frac{7}{16}(4.5) = 1.969 \text{ in.}^2$$

$$A_{nv} = \frac{7}{16}[4.5 - 1.5(1.0)] = 1.313 \text{ in.}^2$$

Tension area $= A_{nt} = \frac{7}{16}[1.75 - 0.5(1.0)] = 0.5469 \text{ in.}^2$

For this type of connection, $U_{bs} = 1.0$, and from AISC Equation J4-5,

$$R_n = 0.6F_u A_{nv} + U_{bs} F_u A_{nt}$$

$$= 0.6(65)(1.313) + 1.0(65)(0.5469) = 86.8 \text{ kips}$$

with an upper limit of

$$0.6F_y A_{gv} + U_{bs} F_u A_{nt} = 0.6(50)(1.969) + 1.0(65)(0.5469) = 94.6 \text{ kips}$$

$\underline{R_n = 86.8 \text{ kips}}$

3.5-2

Shear areas:

$$A_{gv} = \frac{1}{2}(2+4) \times 2 = 6 \text{ in.}^2$$

$$A_{nv} = \frac{1}{2}[2+4-1.5(1.125)] \times 2 = 4.313 \text{ in.}^2$$

Tension area = $A_{nt} = \frac{1}{2}[7.5-2-2-(0.5+0.5)(1.125)] = 1.188 \text{ in.}^2$

For this type of connection, $U_{bs} = 1.0$, and from AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(4.313) + 1.0(58)(1.188) = 219 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(6) + 1.0(58)(1.188) = 199 \text{ kips}$$

$\underline{R_n = 199 \text{ kips}}$

3.5-3

Tension member:

The shear areas are $A_{gv} = \frac{7}{16}(3.5+1.5) \times 2 = 4.375 \text{ in.}^2$

$$A_{nv} = \frac{7}{16}\left[3.5+1.5-1.5\left(\frac{3}{4}+\frac{1}{8}\right)\right] \times 2 = 3.227 \text{ in.}^2$$

The tension area is $A_{nt} = \frac{7}{16}\left[3.0-(0.5+0.5)\left(\frac{3}{4}+\frac{1}{8}\right)\right] = 0.9297 \text{ in.}^2$

For this type of connection, $U_{bs} = 1.0$, and from AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(3.227) + 1.0(58)(0.9297) = 166.2 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(4.375) + 1.0(58)(0.9297) = 148.4 \text{ kips}$$

The nominal block shear strength of the tension member is therefore 148.4 kips.

Gusset Plate:

$$A_{gv} = \frac{3}{8}(3.5+2.5) \times 2 = 4.5 \text{ in.}^2$$

$$A_{nv} = \frac{3}{8}[3.5+2.5-1.5(7/8)] \times 2 = 3.516 \text{ in.}^2$$

$$A_{nt} = \frac{3}{8}[3.0 - (0.5 + 0.5)(7/8)] = 0.7969 \text{ in.}^2$$

From AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(3.516) + 1.0(58)(0.7969) = 168.6 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(4.5) + 1.0(58)(0.7969) = 143.4 \text{ kips}$$

The nominal block shear strength of the gusset plate is therefore 143.4 kips

The gusset plate controls, and the nominal block shear strength of the connection is 143.4 kips

(a) The design strength is $\phi R_n = 0.75(143.4) = 108$ kips $\underline{\phi R_n = 108 \text{ kips}}$

(b) The allowable strength is $\frac{R_n}{\Omega} = \frac{143.4}{2.00} = 71.7$ kips $\underline{R_n/\Omega = 71.7 \text{ kips}}$

3.5-4

Gross section nominal strength:

$$P_n = F_yA_g = 50(3.60) = 180.0 \text{ kips}$$

Net section nominal strength:

$$A_n = 3.60 - 0.314(7/8)(2) = 3.051 \text{ in.}^2$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.525}{(3+3)} = 0.9125$$

$$A_e = A_nU = 3.051(0.9125) = 2.784 \text{ in.}^2$$

$$P_n = F_uA_e = 65(2.784) = 181.0 \text{ kips}$$

Block shear strength of tension member:

The shear areas are $A_{gv} = 0.314(1.5 + 3 + 3) \times 2 = 4.710 \text{ in.}^2$

$$A_{nv} = 0.314[1.5 + 3 + 3 - 2.5(7/8)] \times 2 = 3.336 \text{ in.}^2$$

The tension area is

$$A_{nt} = 0.314[3.0 - (0.5 + 0.5)(7/8)] = 0.6673 \text{ in.}^2$$

For this type of connection, $U_{bs} = 1.0$, and from AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(65)(3.336) + 1.0(65)(0.6673) = 173.5 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(50)(4.710) + 1.0(65)(0.6673) = 184.7 \text{ kips}$$

The nominal block shear strength of the tension member is therefore 173.5 kips.

Block shear strength of gusset plate:

$$A_{gv} = \frac{3}{8}(1.5+3+3)\times 2 = 5.625 \text{ in.}^2$$

$$A_{nv} = \frac{3}{8}[1.5+3+3-2.5(7/8)]\times 2 = 3.984 \text{ in.}^2$$

$$A_{nt} = \frac{3}{8}[3-(0.5+0.5)(7/8)] = 0.7969 \text{ in.}^2$$

From AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(3.984) + 1.0(58)(0.7969) = 184.9 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(5.625) + 1.0(58)(0.7969) = 167.7 \text{ kips}$$

The nominal block shear strength of the gusset plate is therefore 167.7 kips. The gusset plate controls, and the nominal block shear strength of the connection is 167.7 kips

(a) Design strength for LRFD:

For tension on the gross area, $\phi_tP_n = 0.90(180.0) = 162$ kips

For tension on the net area, $\phi_tP_n = 0.75(181.0) = 136$ kips

For block shear, $\phi R_n = 0.75(167.7) = 126$ kips

Block shear controls. Maximum factored load = design strength = 126 kips

(b) Allowable strength for ASD:

For tension on the gross area, $\dfrac{P_n}{\Omega_t} = \dfrac{180.0}{1.67} = 108$ kips

For tension on the net area, $\dfrac{P_n}{\Omega_t} = \dfrac{181.0}{2.00} = 90.5$ kips

For block shear, $\dfrac{R_n}{\Omega} = \dfrac{167.7}{2.00} = 83.9$ kips

Block shear controls. Maximum service load = allowable strength = 83.9 kips

---

3.6-1

(a) $P_u = 1.2D + 1.6L = 1.2(28) + 1.6(84) = 168$ kips

$$\text{Required } A_g = \frac{P_u}{0.9F_y} = \frac{168}{0.9(36)} = 5.19 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{0.75F_u} = \frac{168}{0.75(58)} = 3.86 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{18 \times 12}{300} = 0.72 \text{ in.}$$

Try L5 × 3½ ×¾

$A_g = 5.81 \text{ in.}^2 > 5.19 \text{ in.}^2$ (OK)

$r_{\min} = r_z = 0.744 \text{ in.} > 0.72 \text{ in.}$ (OK)

$A_n = 5.81 - 0.75(1.125) = 4.966 \text{ in.}^2$

$A_e = A_n U = 4.966(0.80) = 3.97 \text{ in.}^2 > 3.86 \text{ in.}^2$ (OK)

Use an L5 × 3½ × ¾

(b) $P_a = D + L = 28 + 84 = 112 \text{ kips}$

$$\text{Required } A_g = \frac{P_a}{F_t} = \frac{P_a}{0.6F_y} = \frac{112}{0.6(36)} = 5.19 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_a}{0.5F_u} = \frac{112}{0.5(58)} = 3.86 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{18(12)}{300} = 0.72 \text{ in.}$$

Try L5 × 3½ ×¾

$A_g = 5.81 \text{ in.}^2 > 5.19 \text{ in.}^2$ (OK)

$r_{\min} = r_z = 0.744 \text{ in.} > 0.72 \text{ in.}$ (OK)

$A_n = 5.81 - 0.75(1.125) = 4.966 \text{ in.}^2$

$A_e = A_n U = 4.966(0.80) = 3.97 \text{ in.}^2 > 3.86 \text{ in.}^2$ (OK)

Use an L5 × 3½ × ¾

3.6-2

(a) $P_u = 1.2D + 1.6L = 1.2(100) + 1.6(50) = 200.0 \text{ kips}$

$$\text{Required } A_g = \frac{P_u}{0.9F_y} = \frac{200}{0.9(36)} = 6.17 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{0.75F_u} = \frac{200}{0.75(58)} = 4.60 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{20 \times 12}{300} = 0.8 \text{ in.}$$

Try C12 × 25

$A_g = 7.34 \text{ in.}^2 > 6.17 \text{ in.}^2$ (OK)

$r_{\min} = r_y = 0.779 \text{ in.} < 0.8 \text{ in.}$ (N.G.)

(Although this value for the radius of gyration does not quite satisfy the AISC recommendation for maximum slenderness, tensile strength is not affected by slenderness, so some leeway is permitted. Therefore, we will consider this value acceptable.)

$$A_n = 7.34 - 0.387(1.125)(2) = 6.469 \text{ in.}^2$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.674}{6} = 0.8877$$

$$A_e = A_n U = 6.469(0.8877) = 5.74 \text{ in.}^2 > 4.60 \text{ in.}^2 \quad \text{(OK)}$$

Use a C12 × 25

Alternate solution using a larger radius of gyration:

Try C15 × 33.9

$$A_g = 10.0 \text{ in.}^2 > 6.17 \text{ in.}^2 \quad \text{(OK)}$$

$$r_{\min} = r_y = 0.901 \text{ in.} > 0.8 \text{ in.} \quad \text{(OK)}$$

$$A_n = 10 - 0.400(1.125)(2) = 9.100 \text{ in.}^2$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.788}{6} = 0.8687$$

$$A_e = A_n U = 9.100(0.8687) = 7.91 \text{ in.}^2 > 4.60 \text{ in.}^2 \quad \text{(OK)}$$

Alternate: Use a C15 × 40

(b) $P_a = D + L = 100 + 50 = 150$ kips

$$\text{Required } A_g = \frac{P_a}{F_t} = \frac{P_a}{0.6F_y} = \frac{150}{0.6(36)} = 6.94 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_a}{0.5F_u} = \frac{150}{0.5(58)} = 5.17 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{20 \times 12}{300} = 0.8 \text{ in.}$$

Try C12 × 25

$$A_g = 7.34 \text{ in.}^2 > 6.94 \text{ in.}^2 \quad \text{(OK)}$$

$$r_{\min} = r_y = 0.779 \text{ in.} < 0.8 \text{ in.} \quad \text{(N.G.)}$$

(Although this value for the radius of gyration does not quite satisfy the AISC recommendation for maximum slenderness, tensile strength is not affected by slenderness, so some leeway is permitted. Therefore, we will consider this value acceptable.)

$$A_n = 7.34 - 0.387(1.125)(2) = 6.469 \text{ in.}^2$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.674}{6} = 0.8877$$

$$A_e = A_n U = 6.469(0.8877) = 5.74 \text{ in.}^2 > 5.17 \text{ in.}^2 \quad \text{(OK)}$$

Use a C12 × 25

Alternate solution using a larger radius of gyration:

Try C15 × 33.9

$A_g = 10.0 \text{ in.}^2 > 6.94 \text{ in.}^2$ (OK)

$r_{\min} = r_y = 0.901 \text{ in.} > 0.8 \text{ in.}$ (OK)

$A_n = 10 - 0.400(1.125)(2) = 9.100 \text{ in.}^2$

$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.788}{6} = 0.8687$

$A_e = A_n U = 9.100(0.8687) = 7.91 \text{ in.}^2 > 5.17 \text{ in.}^2$ (OK)

Alternate: Use a C15 × 40

3.6-3

(a) $P_u = 1.2D + 1.6L = 1.2(30) + 1.6(90) = 180.0 \text{ kips}$

Required $A_g = \frac{P_u}{0.9F_y} = \frac{180}{0.90(50)} = 4.00 \text{ in.}^2$

Required $A_e = \frac{P_u}{0.75F_u} = \frac{180}{0.75(65)} = 3.69 \text{ in.}^2$

Required $r_{\min} = \frac{L}{300} = \frac{25 \times 12}{300} = 1.0 \text{ in.}$

The angle leg must be at least 5 in. long to accommodate two lines of bolts (See usual gages for angles, Fig. 3.24. Also see the last table in the single-angle section of the Dimensions and Properties tables in the *Manual.*)

Try 2L5 × 5 × $^5/_{16}$

$A_g = 6.13 \text{ in.}^2 > 4.00 \text{ in.}^2$ (OK) $r_{\min} = r_x = 1.56 \text{ in.} > 1.0 \text{ in.}$ (OK)

$A_n = 6.13 - 4(7/8 + 1/8)(5/16) = 4.880 \text{ in.}^2$

From AISC Table D4.1, for 4 or more bolts per line, $U = 0.80$

$A_e = A_n U = 4.880(0.80) = 3.90 \text{ in.}^2 > 3.69 \text{ in.}^2$ (OK)

Use 2L5 × 5 × $^5/_{16}$

(b) $P_a = D + L = 30 + 90 = 120 \text{ kips}$

Required $A_g = \frac{P_a}{0.6F_y} = \frac{120}{0.6(50)} = 4.00 \text{ in.}^2$

Required $A_e = \frac{P_a}{0.5F_u} = \frac{120}{0.5(65)} = 3.69 \text{ in.}^2$

Required $r_{\min} = \frac{L}{300} = \frac{25 \times 12}{300} = 1.0 \text{ in.}$

The angle leg must be at least 5 in. long to accommodate two lines of bolts (See usual gages for angles, Fig. 3.24. Also see the last table in the single-angle section of the Dimensions and Properties tables in the *Manual.*)

Try 2L5 × 5 × $^5/_{16}$

$A_g = 6.13 \text{ in.}^2 > 4.00 \text{ in.}^2$ (OK) $r_{\min} = r_x = 1.56 \text{ in.} > 1.0 \text{ in.}$ (OK)

$A_n = 6.13 - 4(7/8 + 1/8)(5/16) = 4.880 \text{ in.}^2$

From AISC Table D4.1, for 4 or more bolts per line, $U = 0.80$

$A_e = A_n U = 4.880(0.80) = 3.90 \text{ in.}^2 > 3.69 \text{ in.}^2$ (OK)

Use 2L5 × 5 × $^5/_{16}$

3.6-4

(a) Load combination 4 controls:

$$P_u = 1.2D + 1.6W + 0.5L = 1.2(54) + 1.6(75) + 0.5(80) = 224.8 \text{ kips}$$

$$\text{Required } A_g = \frac{P_u}{0.90F_y} = \frac{224.8}{0.90(50)} = 5.00 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{0.75F_u} = \frac{224.8}{0.75(65)} = 4.61 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{17.5 \times 12}{300} = 0.7 \text{ in.}$$

Try C10 × 20:

$A_g = 5.87 \text{ in.}^2 > 5.00 \text{ in.}^2$ (OK)

$r_{\min} = r_y = 0.690 \text{ in.} \approx 0.7 \text{ in.}$ (say OK)

(Since the slenderness ratio limit is a recommendation rather than a requirement, this is close enough.)

$A_e = A_g U = 5.87(0.85) = 4.99 \text{ in.}^2 \approx 5.00 \text{ in.}^2$ (say OK) Use a C10 × 20

(b) Load combination 6 controls:

$$P_a = D + 0.75W + 0.75L = 54 + 0.75(75) + 0.75(80) = 170.3 \text{ kips}$$

$$\text{Required } A_g = \frac{P_a}{0.6F_y} = \frac{170.3}{0.6(50)} = 5.68 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_a}{0.5F_u} = \frac{170.3}{0.5(65)} = 5.24 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{17.5 \times 12}{300} = 0.7 \text{ in.}$$

Try C12 × 25:

$A_g = 7.34 \text{ in.}^2 > 5.68 \text{ in.}^2$ (OK)

$r_{\min} = r_y = 0.779 \text{ in.} > 0.7 \text{ in.}$ (OK)

$A_e = A_g U = 7.34(0.85) = 6.24 \text{ in.}^2 > 5.24 \text{ in.}^2$ (OK) Use a C12 × 25

3.6-5

$$\text{Required } A_g = \frac{P_u}{0.9F_y} = \frac{180}{0.9(36)} = 5.56 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{0.75F_u} = \frac{180}{0.75(58)} = 4.14 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{15 \times 12}{300} = 0.6 \text{ in.}$$

Try C10 × 20

$A_g = 5.87 \text{ in.}^2 > 5.56 \text{ in.}^2$ (OK)

$r_{\min} = r_y = 0.690 \text{ in.} > 0.6 \text{ in.}$ (OK)

$A_n = 5.87 - 0.379(1.0)(2) = 5.112 \text{ in.}^2$

$A_e = A_nU = 5.112(0.85) = 4.35 \text{ in.}^2 > 4.15 \text{ in.}^2$ (OK)

Use a C10 × 20

3.6-6

From Part 1 of the *Manual*, all W10 shapes have a flange thickness ≤ 1.25 in. Therefore, from Table 2-3 in Part 2 of the *Manual*, $F_y = 50$ ksi and $F_u = 70$ ksi.

$P_u = 1.2D + 1.6L = 1.2(175) + 1.6(175) = 490.0 \text{ kips}$

$$\text{Required } A_g = \frac{P_u}{0.9F_y} = \frac{490}{0.9(50)} = 10.9 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{0.75F_u} = \frac{490}{0.75(70)} = 9.33 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{30 \times 12}{300} = 1.2 \text{ in.}$$

Try W10 × 49

$A_g = 14.4 \text{ in.}^2 > 10.9 \text{ in.}^2$ (OK)

$r_{\min} = r_y = 2.54 \text{ in.} > 1.2 \text{ in.}$ (OK)

$A_n = 14.4 - 0.560(1.25 + 0.125)(4) = 11.32 \text{ in.}^2$

$\frac{b_f}{d} = \frac{10.0}{10.0} > \frac{2}{3} \quad \Rightarrow$ From AISC Table D3.1, Case 7, $U = 0.90$

$A_e = A_nU = 11.32(0.90) = 10.2 \text{ in.}^2 > 9.33 \text{ in.}^2$ (OK)

Use a W10 × 49

3.7-1

(a) LRFD: Load combination 1 controls: $P_u = 1.4(45) = 63.00$ kips

$$\text{Required } A_b = \frac{P_u}{0.75(0.75F_u)} = \frac{63.00}{0.75(0.75)(58)} = 1.931 \text{ in.}^2$$

Let $\frac{\pi d^2}{4} = 1.931, \qquad d = 1.568$ in.

Required $d = 1.57$ in. Use $1^5/8$ in.

(b) ASD: Load combination 2 controls: $P_a = D + L = 45 + 5 = 50$ kips

$F_t = 0.375F_u = 0.375(58) = 21.75$ ksi

$$\text{Required } A_b = \frac{P_a}{F_t} = \frac{50}{21.75} = 2.299 \text{ in.}^2$$

Let $\frac{\pi d^2}{4} = 2.299, \qquad d = 1.71$ in.

Required $d = 1.71$ in. Use $d = 1\frac{3}{4}$ in.

3.7-2

(a) Dead load = beam weight = 0.048 kips/ft

$w_u = 1.2w_D + 1.6w_L = 1.2(0.048) = 0.0576$ kips/ft

$P_u = 1.2P_D + 1.6P_L = 1.6(20) = 32.0$ kips

Because of symmetry, the tension is the same in both rods.

$$T_u = \frac{1}{2}[0.0576(30) + 32] = 16.86 \text{ kips}$$

$$\text{Required Area} = A_b = \frac{T_u}{0.75(0.75F_u)} = \frac{16.86}{0.75(0.75)(58)} = 0.5168 \text{ in.}^2$$

From $A_b = \frac{\pi d^2}{4}$, required $d = \sqrt{\frac{4(0.5168)}{\pi}} = 0.811$ in.

Required $d = 0.811$ in., use $d = 7/8$ in.

(b) Maximum force in rod occurs when live load is a A or D. Entire live load is taken by one rod.

$$T_u = \frac{0.0576(30)}{2} + 32 = 32.86 \text{ kips}$$

$$\text{Required } A_b = \frac{T_u}{0.75(0.75F_u)} = \frac{32.86}{0.75(0.75)(58)} = 1.007 \text{ in.}^2$$

Let $\frac{\pi d^2}{4} = 1.007, \qquad d = 1.13$ in. Required $d = 1.13$ in., use $d = 1\frac{1}{4}$ in.

3.7-3

(a) Dead load = beam weight = 0.048 kips/ft

Because of symmetry, the tension is the same in both rods.

$$T_a = \frac{1}{2}[0.048(30) + 20] = 10.72 \text{ kips}$$

$$F_t = 0.375F_u = 0.375(58) = 21.75 \text{ ksi}$$

$$\text{Required } A_b = \frac{T_a}{F_t} = \frac{10.72}{21.75} = 0.4929 \text{ in.}^2$$

Let $\frac{\pi d^2}{4} = 0.4929, \quad d = 0.792$ in. Required $d = 0.792$ in., use $d = 13/16$ in.

(b) Maximum force in rod occurs when live load is a *A* or *D*. Entire live load is taken by one rod.

$$T_a = \frac{0.048(30)}{2} + 20 = 20.72 \text{ kips}$$

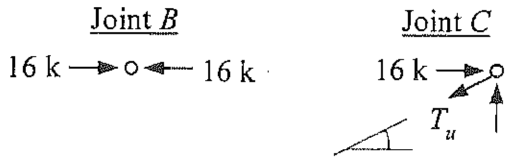
$$\text{Required } A_b = \frac{T_a}{F_t} = \frac{20.72}{21.75} = 0.9526 \text{ in.}^2$$

Let $\frac{\pi d^2}{4} = 0.9526, \quad d = 1.10$ in. Required $d = 1.10$ in., use $d = 1^1/8$ in.

3.7-4

All members are pin-connected, and all loads are applied at the joints; therefore, all members are two-force members (either tension members or compression members). Load combination 4 controls.

$$1.6W = 1.6(10) = 16 \text{ kips}$$



tan^{-1} (20/40) = 26.57°

At joint *C*,

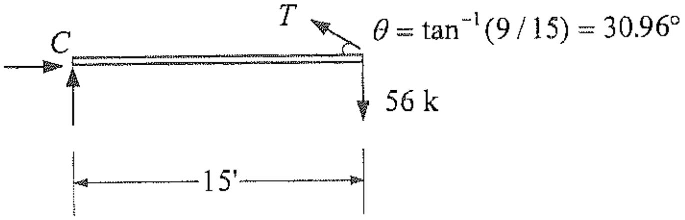
$$\sum F_x = 16 - T_u \cos 26.57° = 0 \quad \Rightarrow \quad T_u = 17.89 \text{ kips}$$

$$\text{Required } A_b = \frac{T_u}{0.75(0.75F_u)} = \frac{17.89}{0.75(0.75)(58)} = 0.5484 \text{ in.}^2$$

Let $\frac{\pi d^2}{4} = 0.5484, \quad d = 0.836$ in. Required $d = 0.836$ in., use $d = 7/8$ in.

3.7-5

(a) LRFD: $P_u = 1.2D + 1.6L = 1.6(35) = 56$ kips



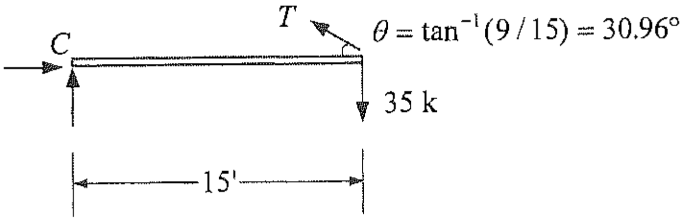
$$\sum M_C = 56(15) - [T\sin(30.96°)](15) = 0, \qquad T = 108.9 \text{ kips}$$

$$\text{Required } A_b = \frac{T}{0.75(0.75F_u)} = \frac{108.9}{0.75(0.75)(58)} = 3.338 \text{ in.}^2$$

$$\text{Let } \frac{\pi d^2}{4} = 3.338, \qquad d = 2.062 \text{ in.}$$

Required $d = 2.06$ in. Use 2 $^1/_8$ in.

(b) ASD: $P_a = D + L = 35$ kips



$$\sum M_C = 35(15) - [T\sin(30.96°)](15) = 0, \qquad T = 68.04 \text{ kips}$$

$$F_t = 0.375F_u = 0.375(58) = 21.75 \text{ ksi}$$

$$\text{Required } A_b = \frac{T}{F_t} = \frac{68.04}{21.75} = 3.128 \text{ in.}^2$$

$$\text{Let } \frac{\pi d^2}{4} = 3.128, \qquad d = 2.00 \text{ in.}$$

$d = 2$ in.

3.7-6

From Part 1 of the *Manual*, the inside diameter is $d = 10.0$ in.

$$\text{Volume of water per foot of length} = \frac{\pi d^2}{4} \times 12 = \frac{\pi(10.0)^2}{4} \times 12 = 942.5 \text{ in.}^3$$

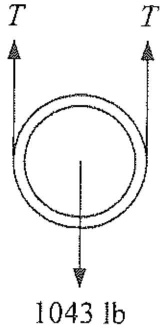
The total weight per foot is

$$\text{weight of water} + \text{weight of pipe} = \frac{942.5}{(12)^3}(62.4) + 40.5 = 74.53 \text{ lb/ft}$$

where the density of water has been taken as 62.4 lb/ft³

(a) Treat the load as 100% dead load: $w_u = 1.4(74.53) = 104.3$ lb/ft

The load at each support is 104.3 lb/ft × 10 ft = 1043 lb



$$\Sigma F_y = 2T - 1043 = 0$$

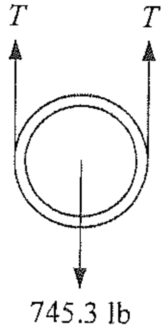
$$T = 521.5 \text{ lb}$$

$$\text{Required } A_b = \frac{T}{0.75(0.75F_u)} = \frac{0.5215}{0.75(0.75)(58)} = 1.598 \times 10^{-2} \text{ in.}^2$$

Let $\frac{\pi d^2}{4} = 0.01598$, $d = 0.143$ in.

Required $d = 0.143$ in. Use $^5/_8$ in.minimum

(b) The load at each support is 74.53 lb/ft × 10 ft = 745.3 lb.



$$\Sigma F_y = 2T - 745.3 = 0$$

$$T = 372.7 \text{ lb}$$

$$F_t = 0.375F_u = 0.375(58) = 21.75 \text{ ksi}$$

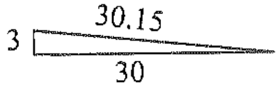
$$\text{Required } A_b = \frac{T}{F_t} = \frac{0.3727}{21.75} = 1.714 \times 10^{-2} \text{ in.}^2$$

Let $\frac{\pi d^2}{4} = 0.01714$, $d = 0.148$ in.

Required $d = 0.148$ in. Use $^5/_8$ in.minimum

3.8-1

Interior joint load:



Snow: $20(10)(12.5) = 2500$ lb

Roofing: $12(10)(30.15/30)(12.5) = 1508$ lb

Purlins: $8.5(12.5) = 106.3$ lb

Truss weight: $1000/3 = 333.3$ lb

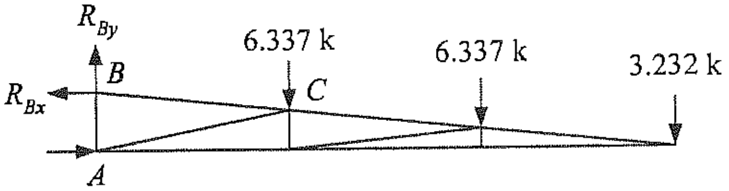
(The assumption that the truss weight is distributed equally to the joints is approximate but is consistent with the approximate nature of the estimate of total truss weight.)

(a) Load combination 3 controls:

$$1.2D + 1.6S = 1.2(1.508 + 0.1063 + 0.3333) + 1.6(2.5) = 6.337 \text{ kips}$$

Exterior joint load. Use half of the above loads except for the purlin weight, which is the same:

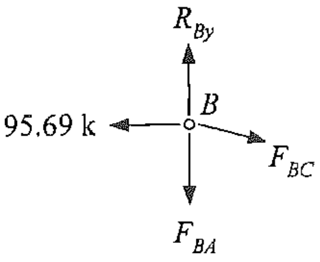
$$1.2D + 1.6S = 1.2\left(\frac{1.508}{2} + 0.1063 + \frac{0.3333}{2}\right) + 1.6\left(\frac{2.5}{2}\right) = 3.232 \text{ kips}$$



$$\sum M_A = 6.337(10) + 6.337(20) + 3.232(30) - R_{Bx}(3) = 0$$

$$R_{Bx} = 95.69 \text{ kips} \leftarrow$$

Joint B:



$$\sum F_x = -95.69 + \frac{30}{30.15} F_{BC} = 0, \qquad F_{BC} = 96.17 \text{ kips}$$

$$\text{Required } A_g = \frac{F_{BC}}{0.9F_y} = \frac{96.17}{0.9(36)} = 2.97 \text{ in.}^2$$

$$\text{Required } A_e = \frac{F_{BC}}{0.75F_u} = \frac{96.17}{0.75(58)} = 2.21 \text{ in.}^2$$

$$L = 10\left(\frac{30.15}{30}\right) = 10.05 \text{ ft}$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{10.05 \times 12}{300} = 0.402 \text{ in.}$$

Try WT5 × 11

$A_g = 3.24 \text{ in.}^2 > 2.79 \text{ in.}^2$ (OK) $r_{\min} = 1.33 \text{ in.} > 0.402 \text{ in.}$ (OK)

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.07}{11} = 0.9027$$

$A_e = A_gU = 3.24(0.9027) = 2.93 \text{ in.}^2 > 2.21 \text{in.}^2$ (OK) Use WT5 × 9.5

(b) Load combination 3 controls:

$$D + L = 1.508 + 0.1063 + 0.3333 + 2.5 = 4.448 \text{ kips}$$

Exterior joint load: use half of the above loads except for the purlin weight, which is the same:

$$D + L = \frac{1.508}{2} + 0.1063 + \frac{0.3333}{2} + \frac{2.5}{2} = 2.277 \text{ kips}$$

For a free-body diagram of the entire truss,

$$\sum M_A = 4.448(10) + 4.448(20) + 2.277(30) - R_{Bx}(3) = 0$$

$$R_{Bx} = 67.25 \text{ kips} \leftarrow$$

For a free body of joint B:

$$\sum F_x = -67.25 + \frac{30}{30.15}F_{BC} = 0, \qquad F_{BC} = 67.59 \text{ kips}$$

$$\text{Required } A_g = \frac{F_{BC}}{0.6F_y} = \frac{67.59}{0.6(36)} = 3.13 \text{ in.}^2$$

$$\text{Required } A_e = \frac{F_{BC}}{0.5F_u} = \frac{67.59}{0.5(58)} = 2.33 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{10.05 \times 12}{300} = 0.402 \text{ in.}$$

Try WT5 × 11

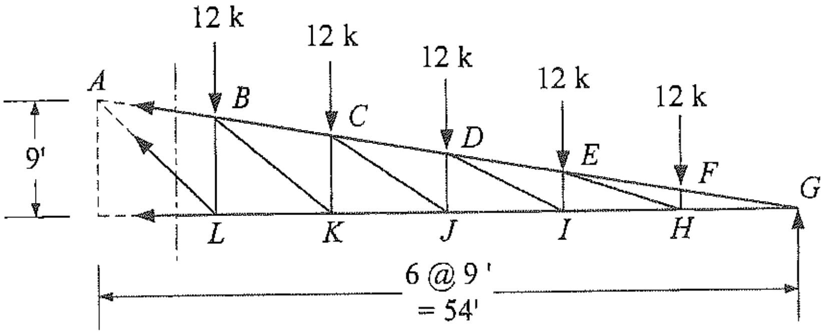
$A_g = 3.24 \text{ in.}^2 > 3.13 \text{ in.}^2$ (OK) $r_{\min} = 1.33 \text{ in.} > 0.402 \text{ in.}$ (OK)

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.07}{11} = 0.9027$$

$A_e = A_gU = 3.24(0.9027) = 2.93 \text{ in.}^2 > 2.33 \text{ in.}^2$ (OK) Use WT5 × 9.5

3.8-2

The diagonal web members are the tension members, and member AL has the largest force.



Using the method of sections and considering the force in member AL to act at L,

$$\sum M_G = 45(F_{AL}\sin 45°) - 12(45+36+27+18+9) = 0$$

$$F_{AL} = 50.91 \text{ kips}$$

$$\text{Required } A_g = \frac{F_{AL}}{0.9F_y} = \frac{50.91}{0.9(50)} = 1.13 \text{ in.}^2$$

$$\text{Required } A_e = \frac{F_{AL}}{0.75F_u} = \frac{50.91}{0.75(65)} = 1.04 \text{ in.}^2$$

$$L = \sqrt{(9)^2+(9)^2} = 12.73 \text{ ft}$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{12.73 \times 12}{300} = 0.509 \text{ in.}$$

Try L$3\frac{1}{2} \times 3 \times \frac{1}{4}$

$A_g = 1.56 \text{ in.}^2 > 1.13 \text{ in.}^2$ (OK) $r_{\min} = 0.628 \text{ in.} > 0.509 \text{ in.}$ (OK)

$$A_n = 1.56 - \left(\frac{3}{4}+\frac{1}{8}\right)\left(\frac{1}{4}\right) = 1.341 \text{ in.}^2$$

From AISC Table D3.1, Case 8, use a value of $U = 0.80$

$A_e = A_nU = 1.341(0.80) = 1.07 \text{ in.}^2 > 1.04 \text{ in.}^2$ (OK)

Use L$3\frac{1}{2} \times 3 \times \frac{1}{4}$ for member AL

This shape can be used for all of the web tension members. Although each member could be a different size, this would not usually be practical. The following table shows the relatively small difference in requirements for all the web tension members.

Member	Force (kips)	Req'd A_g (in.^{2})
AL	50.91	1.13
BK	46.86	1.04
CJ	43.27	0.962
DI	40.25	0.894
EH	37.95	0.843

3.8-3

Use load combination 3: $1.2D + 1.6S$.

Tributary surface area per joint $= 15\sqrt{(9)^2 + (9/6)^2} = 136.9\text{ ft}^2$

Roofing:	$1.2D = 1.2(12)(136.9) = 1971$ lb
Snow:	$1.6S = 1.6(18)(9 \times 15) = 3888$ lb
Truss weight:	$1.2D = 1.2(5000)/12 = 500$ lb
Purlin weight:	$1.2D = 1.2(33 \times 15) = 594.0$ lb
Interior joint:	$1971 + 3888 + 500 + 594 = 6953$ lb = 6.95 kips
At peak:	$1971 + 3888 + 500 + 2(594) = 7547$ lb = 7.55 kips

Load = 7.55 kips at peak, 6.95 kips elsewhere

3.8-4

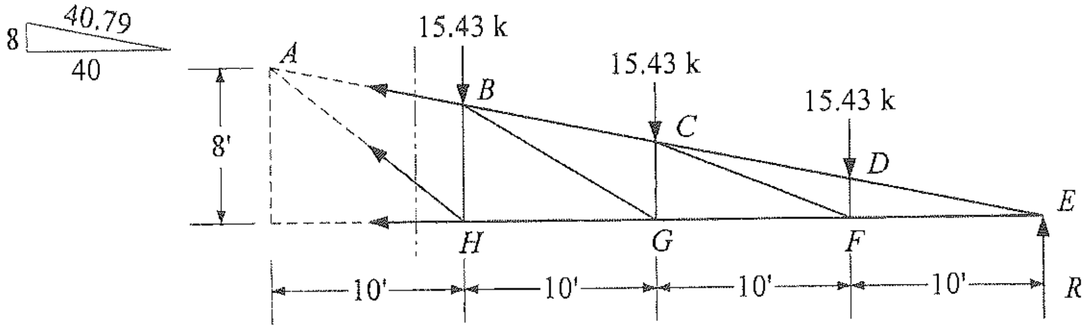
Dead load per truss $= (4 + 12 + 6)(40.79 \times 2)(25) + 5(80)(25) = 54{,}870$ lb

Snow load per truss $= 18(80)(25) = 36{,}000$ lb

$D = 54870/8 = 6859$ lb/joint, $\quad S = 36000/8 = 4500$ lb/joint

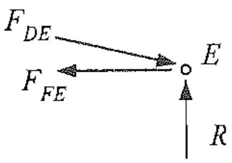
(a) Load combination 3 controls:

Factored joint load $= 1.2D + 1.6S = 1.2(6.859) + 1.6(4.500) = 15.43$ kips



Bottom chord: Member *FE* has the largest tension force.

Use a free body of joint *E*:



$$R = \text{Reaction} = 7(15.43)/2 = 54.01 \text{ kips}$$

$$\sum F_y = 54.01 - \frac{8}{40.79} F_{DE} = 0, \qquad F_{DE} = 275.4 \text{ kips}$$

$$\sum F_x = 275.4\left(\frac{40}{40.79}\right) - F_{FE} = 0, \qquad F_{FE} = 270.1 \text{ kips}$$

$$\text{Required } A_g = \frac{F_{FE}}{0.9F_y} = \frac{270.1}{0.9(50)} = 6.002 \text{ in.}^2$$

$$\text{Required } A_e = \frac{F_{FE}}{0.75F_u} = \frac{270.1}{0.75(65)} = 5.541 \text{ in.}^2$$

From $A_e = A_g U$, $\quad$ Required $A_g = \dfrac{\text{Required } A_e}{U} = \dfrac{5.541}{0.85} = 6.52 \text{ in.}^2$

(This controls the gross area requirement.)

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{10 \times 12}{300} = 0.4 \text{ in.}$$

Try 2L $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2}$

$A_g = 6.53 \text{ in.}^2 > 6.52 \text{ in.}^2$ (OK)

$r_x = 1.05$ in., $r_y = 1.63$ in., $\therefore r_{\min} = 1.05 \text{ in.} > 0.4$ in. (OK)

Use 2L $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2}$ for bottom chord

Web members: Design for the maximum tensile force, which occurs in member *AH*, and use one shape for all tension web members (the diagonal web members). Using the method of sections (see figure), consider the force in member *AH* to act at *H*.

$$\text{Length} = \sqrt{(8)^2 + (10)^2} = 12.81 \text{ ft.}$$

$$\sum M_E = \frac{8}{12.81}F_{AH}(30) - 15.43(10+20+30) = 0, \qquad F_{AH} = 49.41 \text{ kips}$$

$$\text{Required } A_g = \frac{F_{AH}}{0.9F_y} = \frac{49.41}{0.9(50)} = 1.098 \text{ in.}^2$$

$$\text{Required } A_e = \frac{F_{AH}}{0.75F_u} = \frac{49.41}{0.75(65)} = 1.014 \text{ in.}^2$$

From $A_e = A_g U$, $\quad \text{Required } A_g = \dfrac{\text{Required } A_e}{U} = \dfrac{1.014}{0.85} = 1.19 \text{ in.}^2$ (controls)

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{12.81 \times 12}{300} = 0.512 \text{ in.}$$

Try 2L $2 \times 2 \times \frac{3}{16}$

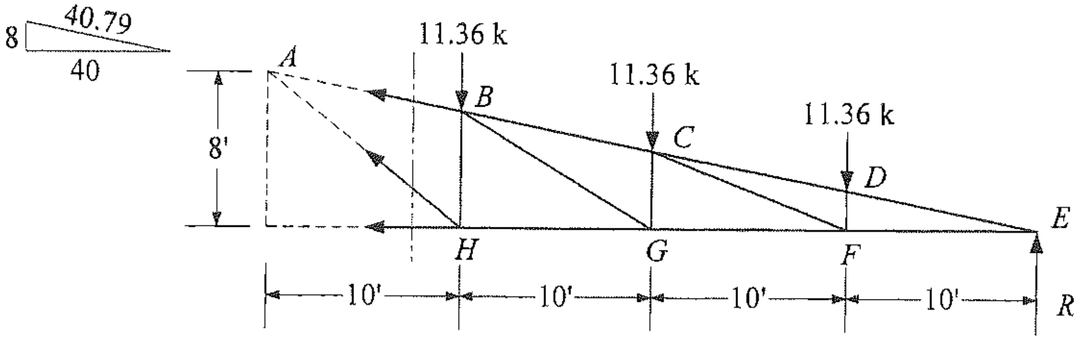
$A_g = 1.44 \text{ in.}^2 > 1.19 \text{ in.}^2$ (OK)

$r_x = 0.612$ in., $r_y = 0.967$ in., $\therefore\ r_{\min} = 0.612 \text{ in.} > 0.512$ in. (OK)

Use 2L $2 \times 2 \times \frac{3}{16}$ for diagonal web members

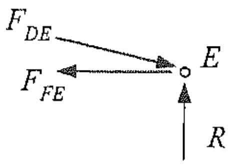
(b) Load combination 3 controls:

Joint load $= D + S = 6859 + 4500 = 11{,}360$ lb



Bottom chord: Member *FE* has the largest tension force.

Use a free body of joint *E*:



R = Reaction = 7(11.36)/2 = 39.76 kips

$$\sum F_y = 39.76 - \frac{8}{40.79}F_{DE} = 0, \qquad F_{DE} = 202.7 \text{ kips}$$

$$\sum F_x = 202.7\left(\frac{40}{40.79}\right) - F_{FE} = 0, \qquad F_{FE} = 198.8 \text{ kips}$$

$$\text{Required } A_g = \frac{F_{FE}}{0.6F_y} = \frac{198.8}{0.6(50)} = 6.627 \text{ in.}^2$$

$$\text{Required } A_e = \frac{F_{FE}}{0.5F_u} = \frac{198.8}{0.5(65)} = 6.117 \text{ in.}^2$$

From $A_e = A_g U$, $\quad \text{Required } A_g = \frac{\text{Required } A_e}{U} = \frac{6.117}{0.85} = 7.20 \text{ in.}^2$

(This controls the gross area requirement.)

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{10 \times 12}{300} = 0.4 \text{ in.}$$

Try 2L $5 \times 5 \times \frac{3}{8}$

$A_g = 7.30 \text{ in.}^2 > 7.20 \text{ in.}^2$ (OK)

$r_x = 1.55$ in., $r_y = 2.20$ in., $\therefore\ r_{\min} = 1.55 \text{ in.} > 0.4 \text{ in.}$ (OK)

Use 2L $5 \times 5 \times \frac{3}{8}$ for bottom chord

Web members: Design for the maximum tensile force, which occurs in member *AH*, and use one shape for all tension web members (the diagonal web members). Using the method of sections (see figure), consider the force in member *AH* to act at *H*.

$$\text{Length} = \sqrt{(8)^2 + (10)^2} = 12.81 \text{ ft.}$$

$$\sum M_E = \frac{8}{12.81} F_{AH}(30) - 11.36(10 + 20 + 30) = 0, \qquad F_{AH} = 36.38 \text{ kips}$$

$$\text{Required } A_g = \frac{F_{AH}}{0.6F_y} = \frac{36.38}{0.6(50)} = 1.213 \text{ in.}^2$$

$$\text{Required } A_e = \frac{F_{AH}}{0.5F_u} = \frac{36.38}{0.5(65)} = 1.119 \text{ in.}^2$$

From $A_e = A_g U$, $\quad \text{Required } A_g = \frac{\text{Required } A_e}{U} = \frac{1.119}{0.85} = 1.32 \text{ in.}^2$ (controls)

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{12.81 \times 12}{300} = 0.512 \text{ in.}$$

Try 2L $2 \times 2 \times \frac{3}{16}$

$A_g = 1.44 \text{ in.}^2 > 1.32 \text{ in.}^2$ (OK)

$r_x = 0.612$ in., $r_y = 0.967$ in., $\therefore\ r_{\min} = 0.612 \text{ in.} > 0.512 \text{ in.}$ (OK)

Use 2L $2 \times 2 \times \frac{3}{16}$ for diagonal web members

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3.8-5

Use sag rods at midspan of purlins.

$$\text{Top Chord length} = \sqrt{(40)^2 + (8)^2} = 40.79 \text{ ft}$$

$$\text{Tributary area} = 40.79(25/2) = 509.9 \text{ ft}^2$$

(a) Total vertical load $= 6(509.9) = 3059$ lb

Component parallel to roof $= 3059\left(\frac{8}{40.79}\right) = 600.0$ lb

Since the design is for dead load only, use load combination 1:

$P_u = 1.4D = 1.4(600) = 840$ lb

Required $A_g = \frac{P_u}{\phi_t(0.75F_u)} = \frac{0.840}{0.75(0.75)(58)} = 0.02575 \text{ in.}^2$

Let $\frac{\pi d^2}{4} = 0.02575$: $\quad d = 0.181$ in. $\quad$ Required $d = 0.181$ in., Use $\frac{5}{8}$ in. minimum

(b) $P_a = 600.0$ lb

$F_t = 0.375F_u = 0.375(58) = 21.75$ ksi

Required $A_b = \frac{T}{F_t} = \frac{0.6000}{21.75} = 0.02759 \text{ in.}^2$

Let $\frac{\pi d^2}{4} = 0.02759$, $\quad d = 0.187$ in. $\quad$ Required $d = 0.187$ in., Use $\frac{5}{8}$ in. minimum

CHAPTER 4 - COMPRESSION MEMBERS

4.3-1

(a) $\frac{KL}{r} = \frac{1.0(10 \times 12)}{2.65} = 45.28$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(45.28)^2} = 139.6 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113.4$$

Since $KL/r = 45.28 < 113.4$, use AISC Eq. E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/139.6)}(50) = 43.04 \text{ ksi}$$

$$P_n = F_{cr}A_g = 43.04(29.4) = 1265 \text{ kips}$$

$\underline{P_n = 1270 \text{ kips}}$

(b) $\frac{KL}{r} = \frac{1.0(30 \times 12)}{2.65} = 135.8$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(135.8)^2} = 15.52 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113.4$$

Since $KL/r = 135.8 > 113.4$, use AISC Eq. E3-3:

$$F_{cr} = 0.877F_e = 0.877(15.52) = 13.61 \text{ ksi}$$

$$P_n = F_{cr}A_g = 13.61(29.4) = 400.1 \text{ kips}$$

$\underline{P_n = 400 \text{ kips}}$

4.3-2

$$\frac{KL}{r} = \frac{1.0(25 \times 12)}{3.68} = 81.52$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(81.52)^2} = 43.07 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{35}} = 135.6$$

Since $KL/r = 81.52 < 135.6$, use AISC Eq. E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(35/43.07)}(35) = 24.91 \text{ ksi}$$

$$P_n = F_{cr}A_g = 24.91(11.1) = 276.5 \text{ kips}$$

$\underline{P_n = 277 \text{ kips}}$

4.3-3

$$\frac{KL}{r} = \frac{2.1(15 \times 12)}{2.45} = 154.3$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(154.3)^2} = 12.02 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113.4$$

Since $KL/r = 154.3 > 113.4$, use AISC Eq. E3-3:

$$F_{cr} = 0.877F_e = 0.877(12.02) = 10.54 \text{ ksi}$$

$$P_n = F_{cr}A_g = 10.54(16.8) = 177.1 \text{ kips}$$ $\underline{P_n = 177 \text{ kips}}$

4.3-4

(a) $$\frac{KL}{r} = \frac{0.65(15 \times 12)}{2.39} = 48.95$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(48.95)^2} = 119.5 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{46}} = 118.3$$

Since $KL/r = 48.95 < 118.3$, use AISC Eq. E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(46/119.5)}(46) = 39.15 \text{ ksi}$$

$$P_n = F_{cr}A_g = 39.15(13.5) = 528.5 \text{ kips}$$

Design strength = $\phi_c P_n = 0.90(528.5) = 475.7$ kips $\underline{\phi_c P_n = 476 \text{ kips}}$

Allowable strength = $\dfrac{P_n}{\Omega_c} = \dfrac{528.5}{1.67} = 316.5$ kips $\underline{\dfrac{P_n}{\Omega_c} = 316 \text{ kips}}$

(b) From *Manual* Table 4-22, for $KL/r = 48.95$ and $F_y = 46$ ksi,

$\phi_c F_{cr} = 35.21$ ksi (by interpolation)

$\phi_c P_n = \phi_c F_{cr} A_g = 35.21(13.5) = 475.3$ kips $\underline{\phi_c P_n = 475 \text{ kips}}$

$\dfrac{F_{cr}}{\Omega_c} = 23.41$ ksi (by interpolation)

$\dfrac{P_n}{\Omega_c} = \dfrac{F_{cr}A_g}{\Omega_c} = 23.41(13.5) = 316.0$ kips $\underline{\dfrac{P_n}{\Omega_c} = 316 \text{ kips}}$

4.3-5

(a) $$\frac{KL}{r} = \frac{0.8(13 \times 12)}{2.48} = 50.32$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(50.32)^2} = 113.0 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4$$

Since $KL/r = 50.32 < 113.4$, use AISC Eq. E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/113.0)}(50) = 41.55 \text{ ksi}$$

$$P_n = F_{cr}A_g = 41.55(24.0) = 997.2 \text{ kips}$$

Design strength $= \phi_c P_n = 0.90(997.2) = 897.5$ kips $\qquad \underline{\phi_c P_n = 897 \text{ kips}}$

Allowable strength $= \dfrac{P_n}{\Omega_c} = \dfrac{997.2}{1.67} = 597.1$ kips $\qquad \underline{\dfrac{P_n}{\Omega_c} = 597 \text{ kips}}$

(b) From *Manual* Table 4-22, for $KL/r = 50.32$ and $F_y = 50$ ksi,

$\phi_c F_{cr} = 37.40$ ksi (by interpolation)

$\phi_c P_n = \phi_c F_{cr} A_g = 37.40(24.0) = 897.6$ kips $\qquad \underline{\phi_c P_n = 898 \text{ kips}}$

$\dfrac{F_{cr}}{\Omega_c} = 24.87$ ksi (by interpolation)

$\dfrac{P_n}{\Omega_c} = \dfrac{F_{cr}A_g}{\Omega_c} = 24.87(24.0) = 596.9$ kips $\qquad \underline{\dfrac{P_n}{\Omega_c} = 597 \text{ kips}}$

4.3-6

(a) $\dfrac{KL}{r} = \dfrac{0.8(12 \times 12)}{2.69} = 42.83$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(42.83)^2} = 156.0 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4$$

Since $KL/r = 42.83 < 113.4$, use AISC Eq. E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/156.0)}(50) = 43.72 \text{ ksi}$$

$$P_n = F_{cr}A_g = 43.72(35.1) = 1535 \text{ kips}$$

Design strength $= \phi_c P_n = 0.90(1535) = 1382$ kips $\qquad \underline{\phi_c P_n = 1380 \text{ kips}}$

Allowable strength $= \dfrac{P_n}{\Omega_c} = \dfrac{1535}{1.67} = 919.2$ kips $\qquad \underline{\dfrac{P_n}{\Omega_c} = 919 \text{ kips}}$

(b) From *Manual* Table 4-22, for $KL/r = 42.83$ and $F_y = 50$ ksi,

$\phi_c F_{cr} = 39.33$ ksi (by interpolation)

$\phi_c P_n = \phi_c F_{cr} A_g = 39.33(35.1) = 1380$ kips $\qquad \underline{\phi_c P_n = 1380 \text{ kips}}$

$$\frac{F_{cr}}{\Omega_c} = 26.22 \text{ ksi (by interpolation)}$$

$$\frac{P_n}{\Omega_c} = \frac{F_{cr}A_g}{\Omega_c} = 26.22(35.1) = 920.3 \text{ kips} \qquad \underline{\frac{P_n}{\Omega_c} = 920 \text{ kips}}$$

4.3-7

Compute the overall, or flexural, buckling strength.

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{2.0(12 \times 12)}{3.28} = 87.80 < 200 \quad \text{(OK)}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{46}} = 118$$

Since 87.80 < 118, use AISC Equation E3-2.

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(87.80)^2} = 37.13 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(46/37.13)}(46) = 27.39 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 27.39(6.06) = 166.0 \text{ kips}$$

Check width-thickness ratios. From the dimensions and properties table in the *Manual*, the width-thickness ratio for the larger overall dimension is

$$\frac{h}{t} = 54.5$$

The ratio for the smaller dimension is

$$\frac{b}{t} = 43.0$$

From AISC Table B4.1, case 12 (and Figure 4.9 in this book), the upper limit for nonslender elements is

$$1.40\sqrt{\frac{E}{F_y}} = 1.40\sqrt{\frac{29,000}{46}} = 35.15$$

Since both $\frac{h}{t}$ and $\frac{b}{t}$ are > $1.40\sqrt{\frac{E}{F_y}}$, both elements are slender and the local buckling strength must be computed. (Although the limiting width-thickness ratio is labeled *b/t* in the table, that is a generic notation, and it applies to *h/t* as well.)

Because these cross-sectional elements are stiffened elements, $Q_s = 1.0$, and Q_a must be computed from AISC Section E7.2. The shape is a rectangular section of uniform thickness, so AISC E7.2(b) applies, provided that

$$\frac{b}{t} \geq 1.40\sqrt{\frac{E}{f}}$$

where

$$f = \frac{P_n}{A_{eff}}$$

and A_{eff} is the reduced effective area. The Specification User Note for square and rectangular sections permits a value of $f = F_y$ to be used in lieu of determining f by iteration. From AISC Equation E7-18, the effective width of the slender element is

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.38}{(b/t)}\sqrt{\frac{E}{f}}\right] \leq b \qquad \text{(AISC Equation E7-18)}$$

For the 10-inch side, using $f = F_y$ and the *design* thickness from the dimensions and properties table,

$$b_e = 1.92(0.174)\sqrt{\frac{29,000}{46}}\left[1 - \frac{0.38}{(54.5)}\sqrt{\frac{29,000}{46}}\right] = 6.920 \text{ in.}$$

From AISC B4.2(d) and the discussion in Part 1 of the Manual, the unreduced length of the 10-inch side between the corner radii can be taken as

$$b = 10 - 2(1.5t) = 10 - 2(1.5)(0.174) = 9.478 \text{ in.}$$

where the corner radius is taken as 1.5 times the design thickness. The loss in area for the two10-inch sides is therefore

$$2(b - b_e)t = 2(9.478 - 6.920)(0.174) = 0.8902 \text{ in.}^2$$

For the 8-inch sides, the unreduced length between the corner radii can be taken as

$$b = 8 - 2(1.5t) = 8 - 2(1.5)(0.174) = 7.478 \text{ in.}$$

$$b_e = 1.92(0.174)\sqrt{\frac{29,000}{46}}\left[1 - \frac{0.38}{(43.0)}\sqrt{\frac{29,000}{46}}\right] = 6.527 \text{ in.}$$

The loss in area for the two 8-inch sides is

$$2(b - b_e)t = 2(7.478 - 6.527)(0.174) = 0.3309 \text{ in.}^2$$

The reduced area is

$$A_{eff} = 6.06 - 0.8902 - 0.3309 = 4.839 \text{ in.}^2$$

The reduction factor is

$$Q_a = \frac{A_{eff}}{A} = \frac{4.893}{6.06} = 0.8074$$

$$Q = Q_s Q_a = 1.0(0.8074) = 0.8074$$

Compute the local buckling strength.

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29,000}{0.8074(46)}} = 131.6$$

$$\frac{KL}{r} = 87.80 < 131.6 \qquad \therefore \text{ use AISC Equation E7-2.}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y = 0.8074\left(0.658^{\frac{0.8074(46)}{37.13}}\right)46 = 24.44 \text{ ksi}$$

$P_n = F_{cr}A_g = 24.44(6.06) = 148$ kips $\underline{P_n = 148 \text{ kips}}$

Iterative solution for f:

As an initial trial value use

$f = F_{cr} = 24.44$ ksi (the value obtained above after using an initial value of $f = F_y$)

For the 10-inch side, $b = 9.478$ in., and

$$b_e = 1.92(0.174)\sqrt{\frac{29{,}000}{24.44}}\left[1 - \frac{0.38}{(54.5)}\sqrt{\frac{29{,}000}{24.44}}\right] = 8.744 \text{ in..}$$

The loss in area for the two10-inch sides is therefore

$2(b - b_e)t = 2(9.478 - 8.744)(0.174) = 0.2554 \text{ in.}^2$

For the 8-inch sides, $b = 7.478$ in., and

$$b_e = 1.92(0.174)\sqrt{\frac{29{,}000}{24.44}}\left[1 - \frac{0.38}{(43.0)}\sqrt{\frac{29{,}000}{24.44}}\right] = 8.005 \text{ in.} > b$$

Therefore, there is no reduction for the 8-inch sides, and the reduced area is

$A_{eff} = 6.06 - 0.2554 = 5.805 \text{ in.}^2$

The reduction factor is

$$Q_a = \frac{A_{eff}}{A} = \frac{5.805}{6.06} = 0.9579$$

$Q = Q_sQ_a = 1.0(0.9579) = 0.9579$

Compute the local buckling strength.

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000}{0.9579(46)}} = 120.8$$

$\frac{KL}{r} = 87.80 < 120.8$ $\quad\therefore$ use AISC Equation E7-2.

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y$$

$$= 0.9579\left(0.658^{\frac{0.9579(46)}{37.13}}\right)46 = 26.81 \text{ ksi} \neq 24.44 \text{ ksi (the assumed value)}$$

Try $f = 26.81$ ksi:

$$b_e = 1.92(0.174)\sqrt{\frac{29,000}{26.81}}\left[1 - \frac{0.38}{(54.5)}\sqrt{\frac{29,000}{26.81}}\right] = 8.468 \text{ in.}$$

The total loss in area is

$$2(b - b_e)t = 2(9.478 - 8.468)(0.174) = 0.3515 \text{ in.}^2$$

and the reduced area is

$$A_{eff} = 6.06 - 0.3515 = 5.709 \text{ in.}^2$$

The reduction factor is

$$Q_a = \frac{A_{eff}}{A} = \frac{5.709}{6.06} = 0.9421$$

$$Q = Q_s Q_a = 1.0(0.9421) = 0.9421$$

Compute the local buckling strength.

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29,000}{0.9421(46)}} = 121.8$$

$$\frac{KL}{r} = 87.80 < 121.8 \qquad \therefore \text{ use AISC Equation E7-2.}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y$$

$$= 0.9421\left(0.658^{\frac{0.9421(46)}{37.13}}\right)46 = 26.59 \text{ ksi} \neq 26.81 \text{ ksi}$$

Try $f = 26.59$ ksi

$$b_e = 1.92(0.174)\sqrt{\frac{29,000}{26.59}}\left[1 - \frac{0.38}{(54.5)}\sqrt{\frac{29,000}{26.59}}\right] = 8.492 \text{ in..}$$

Loss in area $= 2(b - b_e)t = 2(9.478 - 8.492)(0.174) = 0.3431 \text{ in.}^2$

Reduced area $= A_{eff} = 6.06 - 0.3431 = 5.717 \text{ in.}^2$

$$Q_a = \frac{A_{eff}}{A} = \frac{5.717}{6.06} = 0.9434$$

$$Q = Q_s Q_a = 1.0(0.9434) = 0.9434$$

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29,000}{0.9434(46)}} = 121.8$$

$$\frac{KL}{r} = 87.80 < 121.8 \qquad \therefore \text{ use AISC Equation E7-2.}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y$$

$$= 0.9434\left(0.658^{\frac{0.9434(46)}{37.13}}\right)46 = 26.61 \text{ ksi} \approx 26.59 \text{ ksi}$$

Recall that AISC Equation E7-18 for b_e applies when $b/t \geq 1.40\sqrt{E/f}$. In the present case,

$$1.40\sqrt{\frac{E}{f}} = 1.40\sqrt{\frac{29{,}000}{26.61}} = 46.2$$

Since $54.5 > 46.2$, AISC Equation E7-18 does apply.

$$P_n = F_{cr}A_g = 26.61(6.06) = 161.3 \text{ kips} \qquad \therefore \text{ local buckling controls.}$$

$\underline{P_n = 161 \text{ kips}}$

4.3-8

Compute the overall, or flexural, buckling strength.

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{2.10(10 \times 12)}{2.89} = 87.20 < 200 \quad \text{(OK)}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4$$

Since 87.20 < 113.4, use AISC Equation E3-2.

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(87.20)^2} = 37.64 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/37.64)}(50) = 28.68 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 28.68(29.8) = 855 \text{ kips}$$

Check width-thickness ratios. From the dimensions and properties table in the *Manual*, the width-thickness ratio of the web is

$$\frac{h}{t_w} = 37.5$$

From AISC Table B4.1, case 10 (and Figure 4.9 in this book), the upper limit for nonslender elements is

$$1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29{,}000}{50}} = 35.88$$

Since $\frac{h}{t_w} > 1.49\sqrt{\frac{E}{F_y}}$, the web is slender.

For the flange,

$$\frac{b_f}{2t_f} = 7.68 < 0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000}{50}} = 13.49 \therefore \text{ flange is not slender}$$

Because the web is a stiffened element, $Q_s = 1.0$, and Q_a must be computed from AISC Section E7.2. AISC E7.2(a) applies, provided that

$$\frac{b}{t} \geq 1.49\sqrt{\frac{E}{f}}$$

where b/t is the generic notation for the width-thickness ratio and $f = F_{cr}$ computed with $Q = 1.0$. From the flexural buckling strength computations above, $F_{cr} = 28.68$ ksi.

$$1.49\sqrt{\frac{E}{f}} = 1.49\sqrt{\frac{29{,}000}{28.68}} = 47.38$$

Since $\frac{h}{t_w} < 1.49\sqrt{\frac{E}{f}}$, local buckling does not control. $\underline{P_n = 855 \text{ kips}}$

4.3-9

$$\frac{KL}{r} = \frac{0.8(10 \times 12)}{1.66} = 57.83$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(57.83)^2} = 85.58 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{46}} = 118.3 > 57.83$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(46/85.58)}(46) = 36.73 \text{ ksi}$$

$$P_n = F_{cr}A_g = 36.73(5.24) = 192.5 \text{ kips}$$

(a) Let $P_u = \phi_c P_n$

$1.2D + 1.6(2D) = 0.90(192.5)$, Solution is: $\{D = 39.38\}$

$P = D + L = 39.38 + 2(39.38) = 118$ kips $\underline{P = 118 \text{ kips}}$

(b) Let $P_a = P_n/\Omega_c$

$D + L = 192.5/1.67 = 115$ kips $\underline{P = 115 \text{ kips}}$

4.3-10

$$\frac{KL}{r} = \frac{0.8(30 \times 12)}{3.07} = 93.81$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(93.81)^2} = 32.52 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4 > 93.81$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/32.52)}(50) = 26.27 \text{ ksi}$$

$$P_n = F_{cr}A_g = 26.27(25.6) = 672.5 \text{ kips}$$

(a) $\phi_c P_n = 0.90(672.5) = 605$ kips

$P_u = 1.2D + 1.6L = 1.2(110) + 1.6(280) = 580 \text{ kips} < 605 \text{ kips}$ (OK)

A W12 × 87 is adequate

(b) $\dfrac{P_n}{\Omega_c} = \dfrac{672.5}{1.67} = 403$ kips

$P_a = D + L = 110 + 280 = 390 \text{ kips} < 403 \text{ kips}$ (OK)

A W12 × 87 is adequate

4.6-1

$KL = 1.0(18) = 18$ ft

(a-1) $P_u = 1.2D + 1.6L = 1.2(265) + 1.6(130) = 526.0$ kips

From the column load tables for $KL = 18$ ft, a W14 × 74 has a design strength of 563 kips.

Use a W14 × 74

(a-2) $P_a = D + L = 265 + 130 = 395$ kips

From the column load tables for $KL = 18$ ft, a W14 × 82 has an allowable strength of 413 kips.

Use a W14 × 82

(b-1) Assume $F_{cr} = 25$ ksi

$$A_g > \frac{P_u}{\phi_c F_{cr}} = \frac{526.0}{0.90(25)} = 23.4 \text{ in.}^2$$

Try W16 × 77 (a nonslender shape), $A_g = 22.6 \text{ in.}^2$, $r_y = 2.47$ in.

$$\frac{KL}{r_y} = \frac{(18 \times 12)}{2.47} = 87.45 < 200 \quad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(87.45)^2} = 37.43 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4 > 87.45$$

$$\therefore F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/37.43)}(50) = 28.59 \text{ ksi}$$

$$P_n = F_{cr}A_g = 28.59(22.6) = 646.1 \text{ kips}$$

$\phi_c P_n = 0.90(646.1) = 582 \text{ kips} > P_u = 526 \text{ kips}$ (OK) Use a W16 ×77

(b-2) Assume $F_{cr} = 25$ ksi

$$A_g > \frac{P_a}{0.6F_{cr}} = \frac{395}{0.6(25)} = 26.3 \text{ in.}^2$$

Try W16 × 89 (a nonslender shape), $A_g = 26.2 \text{ in.}^2$, $r_y = 2.49$ in.

$$\frac{KL}{r_y} = \frac{(18 \times 12)}{2.49} = 86.75 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(86.75)^2} = 38.03 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4 > 86.75$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/38.03)}(50) = 28.84 \text{ ksi}$$

$$P_n = F_{cr}A_g = 28.84(26.2) = 755.6 \text{ kips}$$

$$\frac{P_n}{\Omega_c} = \frac{755.6}{1.67} = 452 > P_a = 395 \text{ kips} \qquad \text{(OK)}$$

Use a W16 ×89

4.6-2

$$KL = 2.0(20) = 40 \text{ ft}$$

(a-1) $P_u = 1.2D + 1.6L = 1.2(110) + 1.6(110) = 308$ kips

From the column load tables for $KL = 40$ ft, a W12 × 120 has a design strength of 339 kips.

Use a W12 × 120

(a-2) $P_a = D + L = 110 + 110 = 220$ kips

From the column load tables for $KL = 40$ ft, a W12 × 120 has an allowable strength of 225 kips.

Use a W12 × 120

(b-1) $P_u = 1.2D + 1.6L = 1.2(110) + 1.6(110) = 308$ kips

Assume $F_{cr} = 25$ ksi

$$A_g > \frac{P_u}{\phi_c F_{cr}} = \frac{308}{0.90(25)} = 13.69 \text{ in.}^2$$

Try W18 × 65 (a nonslender shape), $A_g = 19.1 \text{ in.}^2$, $r_y = 1.69$ in.

$$\frac{KL}{r_y} = \frac{2.0(20 \times 12)}{1.69} = 284.0 > 200$$

Try W18 × 86 (a nonslender shape), $A_g = 25.3 \text{ in.}^2$, $r_y = 2.63$ in.

$$\frac{KL}{r_y} = \frac{2.0(20 \times 12)}{2.63} = 182.5 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(182.5)^2} = 8.594 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4 < 182.5$$

$\therefore\ F_{cr} = 0.877F_e = 0.877(8.594) = 7.537$ ksi

$P_n = F_{cr}A_g = 7.537(25.3) = 190.7$ kips

$\phi_c P_n = 0.90(190.7) = 171.6 \text{ kips} < P_u = 308 \text{ kips}$ (N.G.)

Assume $F_{cr} = 8$ ksi

$$A_g > \frac{P_u}{\phi_c F_{cr}} = \frac{308}{0.90(8)} = 42.8 \text{ in.}^2$$

Try W18 × 158 (a nonslender shape), $A_g = 46.3$ in.^{2}, $r_y = 2.74$ in.

$$\frac{KL}{r_y} = \frac{2.0(20 \times 12)}{2.74} = 175.2 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(175.2)^2} = 9.325 \text{ ksi}$$

$$\frac{KL}{r_y} > 4.71\sqrt{\frac{E}{F_y}} \qquad \therefore\ F_{cr} = 0.877F_e = 0.877(9.325) = 8.178 \text{ ksi}$$

$P_n = F_{cr}A_g = 8.178(46.3) = 378.6$ kips

$\phi_c P_n = 0.90(378.6) = 341 \text{ kips} > P_u = 308 \text{ kips}$ (OK)

Try the next lightest shape; try a W18 × 143 (a nonslender shape), $A_g = 42.1$ in.^{2}, $r_y =$ 2.72 in.

$$\frac{KL}{r_y} = \frac{2.0(20 \times 12)}{2.72} = 176.5 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(176.5)^2} = 9.188 \text{ ksi}$$

$$\frac{KL}{r_y} > 4.71\sqrt{\frac{E}{F_y}} \qquad \therefore\ F_{cr} = 0.877F_e = 0.877(9.188) = 8.058 \text{ ksi}$$

$P_n = F_{cr}A_g = 8.058(42.1) = 339.2$ kips

$\phi_c P_n = 0.90(339.2) = 305 \text{ kips} < P_u = 308 \text{ kips}$ (N.G.) Use a W18 × 158

(b-2) $P_a = D + L = 110 + 110 = 220$ kips

Assume $F_{cr} = 8$ ksi

$$A_g > \frac{P_a}{0.6F_{cr}} = \frac{220}{0.6(8)} = 45.8 \text{ in.}^2$$

Try W18 × 158 (a nonslender shape), $A_g = 46.3$ in.^{2}, $r_y = 2.74$ in.

$$\frac{KL}{r_y} = \frac{2.0(20 \times 12)}{2.74} = 175.2 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(175.2)^2} = 9.325 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4 < 175.2$$

$$\therefore\ F_{cr} = 0.877F_e = 0.877(9.325) = 8.178 \text{ ksi}$$

$$P_n = F_{cr}A_g = 8.178(46.3) = 378.6 \text{ kips}$$

$$\frac{P_n}{\Omega_c} = \frac{378.6}{1.67} = 227 > P_a = 220 \text{ kips} \qquad \text{(OK)}$$

Use a W18 × 158

4.6-3

$KL = 2.1(12) = 25.2$ ft

(a) $P_u = 1.2D + 1.6L = 1.2(100) + 1.6(300) = 600.0$ kips

HSS 14 × 10 × $\frac{5}{8}$: $\phi_c P_n = 721 \text{ kips} > 600 \text{ kips}, w = 93.1$ lb/ft

HSS 20 × 12 × $\frac{3}{8}$: $\phi_c P_n = 630 \text{ kips} > 600 \text{ kips}, w = 78.4$ lb/ft

Use HSS 20 × 12 × $\frac{3}{8}$

(b) $P_a = D + L = 100 + 300 = 400$ kips

HSS 20 × 12 × $\frac{3}{8}$: $P_n/\Omega_c = 419 \text{ kips} > 400 \text{ kips}, w = 78.4$ lb/ft

Use HSS 20 × 12 × $\frac{3}{8}$

4.6-4

$KL = 0.8(15) = 12$ ft

(a) $P_u = 1.2D + 1.6L = 1.2(42) + 1.6(42) = 118$ kips

Pipe 6 Std: $\phi_c P_n = 133 \text{ kips} > 118 \text{ kips};\ w = 19.0$ lb/ft Use a Pipe 6 Std.

(b) $P_a = D + L = 42 + 42 = 84$ kips

Pipe 6 Std: $P_n/\Omega_c = 88.7 \text{ kips} > 84 \text{ kips};\ w = 19.0$ lb/ft Use a Pipe 6 Std.

4.6-5

$KL = 2.1(12) = 25.2$ ft

(a) $P_u = 1.2D + 1.6L = 1.2(100) + 1.6(300) = 600.0$ kips

HP14 × 89: $\phi_c P_n = 688$ kips Use HP14 × 89

(b) $P_a = D + L = 100 + 300 = 400$ kips

HP14 × 89: $P_n/\Omega_c = 458$ kips Use HP14 × 89

4.6-6

$KL = 0.8(15) = 12$ ft

(a) $P_u = 1.2D + 1.6L = 1.2(42) + 1.6(42) = 118$ kips

HSS 5 × 4 × 3/8: $\phi_c P_n = 124$ kips > 118 kips, $w = 19.7$ lb/ft

HSS 6 × 4 × 5/16: $\phi_c P_n = 125$ kips > 118 kips, $w = 19.1$ lb/ft

HSS 6 × 5 × 1/4: $\phi_c P_n = 125$ kips > 118 kips, $w = 17.3$ lb/ft

HSS 7 × 5 × 3/16: $\phi_c P_n = 118$ kips, $w = 14.5$ lb/ft Use HSS 7 × 5 × 3/16

(b) $P_a = D + L = 42 + 42 = 84$ kips

HSS 6 × 5 × 1/4: $P_n/\Omega_c = 92.0$ kips > 84 kips, $w = 17.3$ lb/ft

HSS 8 × 6 × 3/16: $P_n/\Omega_c = 99.6$ kips > 84 kips, $w = 17.1$ lb/ft

Use HSS 8 × 6 × 3/16

4.6-7

$P_u = 1.2D + 1.6L = 1.2(90) + 1.6(260) = 524$ kips

$KL = 0.65(15.33) = 9.965$ say 10 ft

(a) Use a W12 × 53 ($\phi_c P_n = 590$ kips)

(b) Use a Pipe 12 XS ($w = 65.5$ lb/ft, $\phi_c P_n = 544$ kips)

(c) Use an HSS 9 × 9 × $\frac{1}{2}$ ($w = 55.5$ lb/ft, $\phi_c P_n = 585$ kips)

(d) Use an HSS 12 ×10 × $\frac{3}{8}$ ($w = 52.9$ lb/ft, $\phi_c P_n = 567$ kips)

4.6-8

$P_a = D + L = 90 + 260 = 350$ kips

$KL = 0.65(15.33) = 9.965$ say 10 ft

(a) Use a W12 × 53 ($P_n/\Omega_c = 393$ kips)

(b) Use a Pipe 12 XS ($w = 65.5$ lb/ft, $P_n/\Omega_c = 362$ kips)

(c) Use an HSS 9 × 9 × $\frac{1}{2}$ ($w = 55.5$ lb/ft, $P_n/\Omega_c = 389$ kips)

(d) Use an HSS 12 ×10 × $\frac{3}{8}$ ($w = 52.9$ lb/ft, $P_n/\Omega_c = 378$ kips)

4.6-9

(a) $P_u = 1.2D + 1.6L = 1.2(90) + 1.6(260) = 524$ kips

Assume $F_{cr} = 25$ ksi

$$A_g > \frac{P_u}{\phi_c F_{cr}} = \frac{524}{0.90(25)} = 23.29 \text{ in.}^2$$

Try W21 × 93 (a nonslender shape), $A_g = 27.3$ in.^{2}, $r_y = 1.84$ in.

$$\frac{KL}{r_y} = \frac{0.65(15.33 \times 12)}{1.84} = 64.99 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(64.99)^2} = 67.76 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113.4 > 64.99$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/67.76)}(50) = 36.71 \text{ ksi}$$

$$P_n = F_{cr}A_g = 36.71(27.3) = 1002 \text{ kips}$$

$\phi_c P_n = 0.90(1002) = 902$ kips >524 kips (OK, but too conservative)

Assume $F_{cr} = 35$ ksi

$$A_g > \frac{P_u}{\phi_c F_{cr}} = \frac{524}{0.90(35)} = 16.63 \text{ in.}^2$$

Try W21 × 68 (a slender shape), $A_g = 20.0$ in.^{2}, $r_y = 1.80$ in.

$$\frac{KL}{r_y} = \frac{0.65(15.33 \times 12)}{1.80} = 66.43 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(66.43)^2} = 64.86 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113.4 > 64.43$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/64.86)}(50) = 36.21 \text{ ksi}$$

$$P_n = F_{cr}A_g = 36.21(20.0) = 724.2 \text{ kips}$$

$\phi_c P_n = 0.90(724.2) = 652$ kips >524 kips (OK)

Check web local buckling. From the dimensions and properties table in the *Manual*, the width-thickness ratio of the web is

$$\frac{h}{t_w} = 43.6$$

Because this cross-sectional element is a stiffened element, $Q_s = 1.0$, and Q_a must be computed from AISC Section E7.2. AISC E7.2(a) applies, provided that

$$\frac{b}{t} \geq 1.49\sqrt{\frac{E}{f}}$$

where b/t is the generic notation for the width-thickness ratio and $f = F_{cr}$ computed with $Q = 1.0$. From the flexural buckling strength computations above, $F_{cr} = 36.21$ ksi.

$$1.49\sqrt{\frac{E}{f}} = 1.49\sqrt{\frac{29,000}{36.21}} = 42.17$$

Since $\frac{h}{t_w} > 1.49\sqrt{\frac{E}{f}}$, local buckling strength must be checked. The unreduced width of the web is $b = d - 2k_{des} = 21.1 - 2(1.19) = 18.72$ in. From AISC Eq. E7-17, The reduced effective width is

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{(b/t)}\sqrt{\frac{E}{f}}\right] \leq b$$

$$= 1.92(0.430)\sqrt{\frac{29,000}{36.21}}\left[1 - \frac{0.34}{(43.6)}\sqrt{\frac{29,000}{36.21}}\right] = 18.21 \text{ in.} < 18.72 \text{ in.}$$

The reduced area is

$$A_{eff} = A - t_w(b - b_e) = 20.0 - 0.430(18.72 - 18.21) = 19.78 \text{ in.}^2$$

$$Q_a = Q = \frac{A_{eff}}{A} = \frac{19.78}{20.0} = 0.989$$

Determine which critical stress equation to use:

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29,000}{0.989(50)}} = 114.1 > \frac{KL}{r} = 66.43$$

$$\therefore F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y = 0.989\left[0.658^{\frac{0.989(50)}{64.86}}\right](50) = 35.94 \text{ ksi}$$

$$P_n = F_{cr}A_g = 35.94(20.0) = 718.8 \text{ kips}$$

$$\phi_c P_n = 0.90(718.8) = 647 \text{ kips} > 524 \text{ kips} \qquad \text{(OK, but still too conservative)}$$

Try W21 × 62 (a slender shape), $A_g = 18.3$ in.^{2}, $r_y = 1.77$ in.

$$\frac{KL}{r_y} = \frac{0.65(15.33 \times 12)}{1.77} = 67.56 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(67.56)^2} = 62.71 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113.4 > 67.56$$

$$\therefore F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/62.71)}(50) = 35.81 \text{ ksi}$$

$$P_n = F_{cr}A_g = 35.81(18.3) = 655.3 \text{ kips}$$

$$\phi_c P_n = 0.90(655.3) = 590 \text{ kips} > 524 \text{ kips} \qquad \text{(OK)}$$

From the dimensions and properties table in the *Manual*, the width-thickness ratio of the web is

$$\frac{h}{t_w} = 46.9$$

From the flexural buckling strength computations above, $F_{cr} = f = 35.81$ ksi.

$$1.49\sqrt{\frac{E}{f}} = 1.49\sqrt{\frac{29{,}000}{35.81}} = 42.4$$

Since $\frac{h}{t_w} > 1.49\sqrt{\frac{E}{f}}$, local buckling strength must be checked. The unreduced width of the web is $b = d - 2k_{des} = 21.0 - 2(1.12) = 18.76$ in. From AISC Eq. E7-17, The reduced effective width is

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{(b/t)}\sqrt{\frac{E}{f}}\right] \le b$$

$$= 1.92(0.400)\sqrt{\frac{29{,}000}{35.81}}\left[1 - \frac{0.34}{(46.9)}\sqrt{\frac{29{,}000}{35.81}}\right] = 17.35 \text{ in.} < b = 18.76 \text{ in.}$$

The reduced area is

$$A_{eff} = A - t_w(b - b_e) = 18.3 - 0.400(18.76 - 17.35) = 17.74 \text{ in.}^2$$

$$Q_a = Q = \frac{A_{eff}}{A} = \frac{17.74}{18.3} = 0.9694$$

Determine which critical stress equation to use:

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000}{0.9694(50)}} = 115.2 > \frac{KL}{r} = 67.56$$

$$\therefore\ F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y = 0.9694\left[0.658^{\frac{0.9694(50)}{62.71}}\right](50) = 35.07 \text{ ksi}$$

$$P_n = F_{cr}A_g = 35.07(18.3) = 641.8 \text{ kips}$$

$$\phi_c P_n = 0.90(641.8) = 578 \text{ kips} > 524 \text{ kips} \quad \text{(OK)}$$

Use W21 × 62

(b) $P_a = D + L = 90 + 260 = 350$ kips

Assume $F_{cr} = 35$ ksi

$$A_g > \frac{P_u}{\phi_c F_{cr}} = \frac{524}{0.90(35)} = 16.63 \text{ in.}^2$$

Try W21 × 62 (a slender shape), $A_g = 18.3 \text{ in.}^2$, $r_y = 1.77$ in.

$$\frac{KL}{r_y} = \frac{0.65(15.33 \times 12)}{1.77} = 67.56 < 200$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(67.56)^2} = 62.71 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113.4 > 67.56$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/62.71)}(50) = 35.81 \text{ ksi}$$

$$P_n = F_{cr}A_g = 35.81(18.3) = 655.3 \text{ kips}$$

$$\frac{P_n}{\Omega_c} = \frac{655.3}{1.67} = 392 \text{ kips} > 350 \text{ kips} \qquad \text{(OK)}$$

Check web local buckling. From the dimensions and properties table in the *Manual*, the width-thickness ratio of the web is

$$\frac{h}{t_w} = 46.9$$

Because this cross-sectional element is a stiffened element, $Q_s = 1.0$, and Q_a must be computed from AISC Section E7.2. AISC E7.2(a) applies, provided that

$$\frac{b}{t} \geq 1.49\sqrt{\frac{E}{f}}$$

where b/t is the generic notation for the width-thickness ratio and $f = F_{cr}$ computed with $Q = 1.0$. From the strength calculations above, $F_{cr} = f = 35.81$ ksi.

$$1.49\sqrt{\frac{E}{f}} = 1.49\sqrt{\frac{29,000}{35.81}} = 42.4$$

Since $\frac{h}{t_w} > 1.49\sqrt{\frac{E}{f}}$, local buckling strength must be checked. The unreduced width of the web is $b = d - 2k_{des} = 21.0 - 2(1.12) = 18.76$ in. From AISC Eq. E7-17, The reduced effective width is

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{(b/t)}\sqrt{\frac{E}{f}}\right] \leq b$$

$$= 1.92(0.400)\sqrt{\frac{29,000}{35.81}}\left[1 - \frac{0.34}{(46.9)}\sqrt{\frac{29,000}{35.81}}\right] = 17.35 \text{ in.} < b = 18.76 \text{ in.}$$

The reduced area is

$$A_{eff} = A - t_w(b - b_e) = 18.3 - 0.400(18.76 - 17.35) = 17.74 \text{ in.}^2$$

$$Q_a = Q = \frac{A_{eff}}{A} = \frac{17.74}{18.3} = 0.9694$$

Determine which critical stress equation to use:

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29,000}{0.9694(50)}} = 115.2 > \frac{KL}{r} = 67.56$$

$$\therefore\ F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y = 0.9694\left[0.658^{\frac{0.9694(50)}{62.71}}\right](50) = 35.07 \text{ ksi}$$

$$P_n = F_{cr}A_g = 35.07(18.3) = 641.8 \text{ kips}$$

$$\frac{P_n}{\Omega_c} = \frac{641.8}{1.67} = 384 \text{ kips} > 350 \text{ kips} \qquad \text{(OK)}$$

Use W21 × 62

4.7-1

$$\frac{K_xL}{r_x} = \frac{2.1(13 \times 12)}{7.10} = 46.14, \qquad \frac{K_yL}{r_y} = \frac{1.0(13 \times 12)}{2.51} = 62.15 \text{ (controls)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(62.15)^2} = 74.10 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{60}} = 103.5 > 62.15$$

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(60/74.10)}(60) = 42.75 \text{ ksi}$$

$$P_n = F_{cr}A_g = 42.75(29.5) = 1260 \text{ kips}$$

Check for slender compression elements for $F_y = 60$ ksi.

Flange: $\lambda = \frac{b_f}{2t_f} = 5.29, \qquad \lambda_r = 0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000}{60}} = 12.3$

Since $\lambda < \lambda_r$, flange is nonslender.

Web: $\lambda = \frac{h}{t_w} = 24.3, \qquad \lambda_r = 1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29{,}000}{60}} = 32.8$

Since $\lambda < \lambda_r$, web is nonslender. Therefore, the nominal compressive strength is

$$P_n = 1260 \text{ kips.}$$

$P_n = 1260$ kips

4.7-2

For an HSS 10 × 6 × 5/16, $A_g = 8.76$ in.^{2}, $r_x = 3.66$ in., $r_y = 2.47$ in., and there are no slender elements (see table that follows *Manual* Table 1-12).

$$\frac{K_xL}{r_x} = \frac{15 \times 12}{3.66} = 49.18, \qquad \frac{K_yL}{r_y} = \frac{9 \times 12}{2.47} = 43.72$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(49.18)^2} = 118.3 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{46}} = 118.3$$

Since $KL/r = 49.18 < 118.3$, use AISC Eq. E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(46/118.3)}(46) = 39.09 \text{ ksi}$$

$$P_n = F_{cr}A_g = 39.09(8.76) = 342.4 \text{ kips}$$

(a) $\phi_c P_n = 0.90(342.4) = 308.2$ kips $\underline{\phi_c P_n = 308 \text{ kips}}$

(b) $\dfrac{P_n}{\Omega_c} = \dfrac{342.4}{1.67} = 205.0$ kips $\underline{\dfrac{P_n}{\Omega_c} = 205 \text{ kips}}$

4.7-3

For a W12 × 79, $A_g = 23.2 \text{ in.}^2$, $r_x = 5.34$, in., $r_y = 3.05$ in.

$$\frac{K_x L}{r_x} = \frac{28 \times 12}{5.34} = 62.92, \qquad \frac{K_y L}{r_y} = \frac{16 \times 12}{3.05} = 62.95 \text{ (controls)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 (29{,}000)}{(62.95)^2} = 72.23 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{60}} = 103.5$$

Since $KL/r = 62.95 < 103.5$, use AISC Eq. E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(60/72.23)}(60) = 42.38 \text{ ksi}$$

$$P_n = F_{cr} A_g = 42.38(23.2) = 983.2 \text{ kips}$$

Check for slender compression elements for $F_y = 60$ ksi.

Flange: $\lambda = \dfrac{b}{2t_f} = 8.22, \qquad \lambda_r = 0.56\sqrt{\dfrac{E}{F_y}} = 0.56\sqrt{\dfrac{29{,}000}{60}} = 12.3$

Since $\lambda < \lambda_r$, flange is nonslender.

Web: $\lambda = \dfrac{h}{t_w} = 20.7, \qquad \lambda_r = 1.49\sqrt{\dfrac{E}{F_y}} = 1.49\sqrt{\dfrac{29{,}000}{60}} = 32.8$

Since $\lambda < \lambda_r$, web is nonslender. Therefore, the nominal compressive strength is

$P_n = 983.2$ kips.

(a) LRFD Solution:

$$\phi_c P_n = 0.90(983.2) = 885 \text{ kips}$$

$$P_u = 1.2D + 1.6L = 1.2(180) + 1.6(320) = 728 \text{ kips} < 885 \text{ kips} \qquad \text{(OK)}$$

Yes; member is satisfactory.

(b) ASD Solution:

$$\frac{P_n}{\Omega_c} = \frac{983.2}{1.67} = 589 \text{ kips}$$

$$P_a = D + L = 180 + 320 = 500 \text{ kips} < 589 \text{ kips} \qquad \text{(OK)}$$

Yes; member is satisfactory.

4.7-4

$K_xL = 22 \text{ ft}, \qquad K_yL = 22 - 10 = 12 \text{ ft}$

(a) $P_u = 1.2D + 1.6L = 1.2(142) + 1.6(356) = 740.0 \text{ kips}$

From the column load tables for $KL = 12$ ft, try a W14 × 74

$\phi_c P_n = 766 \text{ kips for } KL = 12 \text{ ft.}$

$\frac{K_xL}{r_x/r_y} = \frac{22}{2.44} = 9.02 \text{ ft} < 12 \text{ ft}$ Use a W14 × 74

(b) $P_a = D + L = 142 + 356 = 498 \text{ kips}$

From the column load tables for $KL = 12$ ft, try a W14 × 74

$\frac{P_n}{\Omega_c} = 510 \text{ kips for } KL = 12 \text{ ft.}$

$\frac{K_xL}{r_x/r_y} = \frac{22}{2.44} = 9.02 \text{ ft} < 12 \text{ ft}$ Use a W14 × 74

4.7-5

$K_xL = 35 \text{ ft}, \qquad K_yL = 15 \text{ ft}$

(a) $P_u = 1.2D + 1.6L = 1.2(380) + 1.6(1140) = 2280 \text{ kips}$

From the column load tables for $KL = 15$ ft, there are no W8 or W10 shapes with enough strength. Try a W12 × 230 :

$\phi_c P_n = 2450 \text{ kips for } KL = 15 \text{ ft}$

$\frac{K_xL}{r_x/r_y} = \frac{35}{1.80} = 19.44 \text{ ft} > 15 \text{ ft}$

For $KL = 19$ ft, $\phi_c P_n = 2150 \text{ kips} < 2280 \text{ kips}$ (N.G.)

Try a W12 × 252:

$\frac{K_xL}{r_x/r_y} = \frac{35}{1.81} = 19.34 \text{ ft} > 15 \text{ ft}$

For $KL = 20$ ft, $\phi_c P_n = 2280 \text{ kips} = P_u$ (OK)

Investigate W14 shapes: Try a W14 × 211. $\phi_c P_n = 2420$ kips for $KL = 15$ ft

$\frac{K_xL}{r_x/r_y} = \frac{35}{1.61} = 21.74 \text{ ft.} > 15 \text{ ft}$

For $KL = 20$ ft, $\phi_c P_n = 2160 \text{ kips} < 2280 \text{ kips}$ (N.G.)

Try a W14 × 233:

$\frac{K_xL}{r_x/r_y} = \frac{35}{1.62} = 21.60 \text{ ft} > 15 \text{ ft}$

For $KL = 21.60$ ft, $\phi_c P_n = 2300$ kips > 2280 kips (OK)

The W14 × 233 is the lightest W shape (in the column load tables) that will work.

Use a W14 × 233

(b) $P_a = D + L = 380 + 1140 = 1520$ kips

From the column load tables for $KL = 15$ ft, there are no W8 or W10 shapes with enough strength.

Try a W12 × 230 :

$$\frac{P_n}{\Omega_c} = 1630 \text{ kips for } KL = 15 \text{ ft}$$

$$\frac{K_x L}{r_x/r_y} = \frac{35}{1.80} = 19.44 \text{ ft} > 15 \text{ ft}$$

For $KL = 19$ ft, $\frac{P_n}{\Omega_c} = 1430$ kips < 1520 kips (N.G.)

Try a W12 × 252:

$$\frac{K_x L}{r_x/r_y} = \frac{35}{1.81} = 19.34 \text{ ft} > 15 \text{ ft}$$

For $KL = 20$ ft, $\frac{P_n}{\Omega_c} = 1520 \text{ kips} = P_a$ (OK)

Investigate W14 shapes: Try a W14 × 211. $\frac{P_n}{\Omega_c} = 1610$ kips for $KL = 15$ ft

$$\frac{K_x L}{r_x/r_y} = \frac{35}{1.61} = 21.74 \text{ ft.} > 15 \text{ ft}$$

For $KL = 20$ ft, $\frac{P_n}{\Omega_c} = 1440$ kips < 1520 kips (N.G.)

Try a W14 × 233:

$$\frac{K_x L}{r_x/r_y} = \frac{35}{1.62} = 21.60 \text{ ft} > 15 \text{ ft}$$

For $KL = 21.60$ ft, $\frac{P_n}{\Omega_c} = 1530$ kips > 1520 kips (OK)

The W14 × 233 is the lightest W shape (in the column load tables) that will work.

Use a W14 × 233

4.7-6

$K_x L = 15$ ft, $K_y L = 7.5$ ft

The x-axis strength will control when $r_x/r_y < \frac{K_x L}{K_y L} = 2.0$. This is true for all rectangular HSS.

(a) $P_u = 1.2D + 1.6L = 1.2(35) + 1.6(80) = 170.0$ kips

Assume $r_x/r_y = 1.2$, $\quad \dfrac{K_xL}{r_x/r_y} = \dfrac{15}{1.2} = 12.5$ ft

Try an HSS 9 × 7 × 3/16 ($w = 19.7$ lb/ft):

$$\frac{K_xL}{r_x/r_y} = \frac{15}{1.21} = 12.40 \text{ ft}, \qquad \phi_c P_n = 170 \text{ kips for } KL = 13 \text{ ft} \qquad \text{(OK)}$$

Use an HSS 9 × 7 × 3/16

(b) $P_a = D + L = 35 + 80 = 115$ kips

Assume $r_x/r_y = 1.2$, $\quad \dfrac{K_xL}{r_x/r_y} = \dfrac{15}{1.2} = 12.5$ ft

Try an HSS 9 × 7 × 3/16 ($w = 19.7$ lb/ft):

$$\frac{K_xL}{r_x/r_y} = \frac{15}{1.21} = 12.40 \text{ ft}, \qquad \frac{P_n}{\Omega_c} = 115 \text{ kips} = P_a \qquad \text{(OK)}$$

Use an HSS 9 × 7 × 3/16

4.7-7

$K_xL = 27$ ft, $\quad K_yL = 15$ ft

K_xL will control when $\dfrac{K_xL}{r_x/r_y} > K_yL$, or $\dfrac{K_xL}{K_yL} > r_x/r_y$

For this column, $\dfrac{K_xL}{K_yL} = \dfrac{27}{15} = 1.8$

r_x/r_y is < 1.8 for all HSS, so $\dfrac{K_xL}{r_x/r_y}$ will control for this column.

(a) $P_u = 1.2D + 1.6L = 1.2(33) + 1.6(82) = 171$ kips

Check within each range of r_x/r_y for possible choices.

For $r_x/r_y \approx 1.2$, $\dfrac{K_xL}{r_x/r_y} = \dfrac{27}{1.2} = 22.5$ ft. $\quad$ Try an HSS 10 × 8 × $\frac{1}{4}$, $w = 29.2$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{27}{1.19} = 22.7 \text{ ft}, \ \phi_c P_n > 205 \text{ kips} \qquad \text{(OK)}$$

For $r_x/r_y \approx 1.3$, $\dfrac{K_xL}{r_x/r_y} = \dfrac{27}{1.3} = 21$ ft. $\quad$ Try an HSS 8 × 6 × $\frac{1}{2}$, $w = 41.9$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{27}{1.25} = 21.6 \text{ ft}, \ \phi_c P_n > 201 \text{ kips} \qquad \text{(OK)}$$

For $r_x/r_y \approx 1.4$, $\dfrac{K_xL}{r_x/r_y} = \dfrac{27}{1.4} = 19$ ft. $\quad$ Try an HSS 12 × 8 × $\frac{3}{16}$, $w = 24.7$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{27}{1.36} = 19.85 \text{ ft}, \ \phi_c P_n > 171 \text{ kips} \qquad \text{(OK)}$$

For $r_x/r_y \approx 1.6$, $\dfrac{K_xL}{r_x/r_y} = \dfrac{27}{1.6} = 17$ ft. Try an HSS $9 \times 5 \times \frac{3}{8}$, $w = 32.5$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{27}{1.58} = 17.09 \text{ ft}, \ \phi_c P_n = 188 \text{ kips} > 171 \text{ kips} \quad \text{(OK)}$$

For $r_x/r_y \approx 1.7$, $\dfrac{K_xL}{r_x/r_y} = \dfrac{27}{1.7} = 16$ ft. Try an HSS $12 \times 6 \times \frac{1}{4}$, $w = 29.2$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{27}{1.71} = 15.79 \text{ ft}, \ \phi_c P_n > 218 \text{ kips} \quad \text{(OK)}$$

Use an HSS $12 \times 8 \times \frac{3}{16}$, $w = 24.7$ lb/ft

(b) $P_a = D + L = 33 + 82 = 115$ kips

Check within each range of r_x/r_y for possible choices.

For $r_x/r_y \approx 1.2$, $\dfrac{K_xL}{r_x/r_y} = \dfrac{27}{1.2} = 22.5$ ft. Try an HSS $10 \times 8 \times \frac{1}{4}$, $w = 29.2$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{27}{1.19} = 22.7 \text{ ft}, \ \frac{P_n}{\Omega_c} > 142 \text{ kips} \quad \text{(OK)}$$

For $r_x/r_y \approx 1.3$, $\dfrac{K_xL}{r_x/r_y} = \dfrac{27}{1.3} = 21$ ft. Try an HSS $8 \times 6 \times \frac{1}{2}$, $w = 41.9$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{27}{1.25} = 21.6 \text{ ft}, \ \frac{P_n}{\Omega_c} > 134 \text{ kips} \quad \text{(OK)}$$

For $r_x/r_y \approx 1.4$, $\dfrac{K_xL}{r_x/r_y} = \dfrac{27}{1.4} = 19$ ft. Try an HSS $10 \times 6 \times \frac{1}{4}$, $w = 25.8$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{27}{1.48} = 18.24 \text{ ft}, \ \frac{P_n}{\Omega_c} = 116 \text{ kips} > 115 \text{ kips} \quad \text{(OK)}$$

For $r_x/r_y \approx 1.6$, $\dfrac{K_xL}{r_x/r_y} = \dfrac{27}{1.6} = 17$ ft. Try an HSS $9 \times 5 \times \frac{3}{8}$, $w = 32.5$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{27}{1.58} = 17.09 \text{ ft}, \ \frac{P_n}{\Omega_c} = 125 \text{ kips} > 115 \text{ kips} \quad \text{(OK)}$$

For $r_x/r_y \approx 1.7$, $\dfrac{K_xL}{r_x/r_y} = \dfrac{27}{1.7} = 16$ ft. Try an HSS $12 \times 6 \times \frac{1}{4}$, $w = 29.2$ lb/ft

$$\frac{K_xL}{r_x/r_y} = \frac{27}{1.71} = 15.79 \text{ ft}, \ \frac{P_n}{\Omega_c} > 145 \text{ kips} \quad \text{(OK)}$$

Use an HSS $10 \times 6 \times \frac{1}{4}$, $w = 25.8$ lb/ft

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4.7-8

(a) Column AB: $G_A = 10$, $\quad G_B = \dfrac{\sum I_c/L_c}{\sum I_g/L_g} = \dfrac{2(303)/12}{2(510)/20} = 0.991$

From the alignment chart, $K_x \approx 1.90$ $K_x = 1.90$

(b) Column BC: $G_C = G_B = 0.991$ and $\underline{K_x = 1.31}$

(c) Column AB:

$$\frac{K_x L}{r_x} = \frac{1.90(12 \times 12)}{4.37} = 62.61$$

$$4.71\sqrt{\frac{E}{F_y}} = 113.4$$

Since $\frac{K_x L}{r_x} < 4.71\sqrt{\frac{E}{F_y}}$, the column is inelastic. Since K_x for column BC is smaller, K_xL/r_x is smaller, so column BC is also inelastic.

$\underline{\tau_a \text{ is applicable to both columns.}}$

4.7-9

$$G_A = 1.0, \qquad G_B = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{2460/13 + 1750/13}{800/25} = 10.1$$

From the alignment chart, $K_x \approx 1.9$

$$\frac{K_x L}{r_x} = \frac{1.9(13 \times 12)}{8.03} = 36.91$$

$$4.71\sqrt{\frac{E}{F_y}} = 113.4$$

Since $\frac{K_x L}{r_x} < 4.71\sqrt{\frac{E}{F_y}}$, the column is inelastic.

(a) $P_u = 1.2D + 1.6L = 1.2(204) + 1.6(408) = 897.6$ kips

$$\frac{P_u}{A_g} = \frac{897.6}{38.2} = 23.50 \text{ ksi}$$

From Table 4-21 in the *Manual*, $\tau_a = 0.924$ by interpolation.

Use $G_B = 0.924(10.1) = 9.3$

From the alignment chart, $K_x \approx 1.86$ $\underline{K_x = 1.86}$

(b) $P_a = D + L = 204 + 408 = 612$ kips

$$\frac{P_a}{A_g} = \frac{612}{38.2} = 16.02 \text{ ksi}$$

From Table 4-21 in the *Manual*, $\tau_a = 0.912$

Use $G_B = 0.912(10.1) = 9.2$

From the alignment chart, $K_x \approx 1.86$ $\underline{K_x = 1.86}$

4.7-10

$$G_A = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{597/15}{800/18 + 800/20} = 0.471, \qquad G_B = 10$$

From the alignment chart, $K_x \approx 1.78$

$$\frac{K_xL}{r_x} = \frac{1.78(15 \times 12)}{5.31} = 60.34$$

$$4.71\sqrt{\frac{E}{F_y}} = 113.4$$

Since $\frac{K_xL}{r_x} < 4.71\sqrt{\frac{E}{F_y}}$, the column is inelastic.

(a) $P_u = 1.2D + 1.6L = 1.2(50) + 1.6(150) = 300$ kips

$$\frac{P_u}{A_g} = \frac{300}{21.1} = 14.22 \text{ ksi}$$

From Table 4-21 in the *Manual*, $\tau_a = 1.00$ $\therefore$ no modification is necessary.

$$\frac{K_yL}{r_y} = \frac{1.0(15 \times 12)}{3.04} = 59.21 < 60.34 \therefore \frac{K_xL}{r_x} \text{ controls}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(60.34)^2} = 78.61 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/78.61)}(50) = 38.31 \text{ ksi}$$

$$P_n = F_{cr}A_g = 38.31(21.1) = 808.3 \text{ kips}$$

$$\phi_c P_n = 0.90(808.3) = 728 \text{ kips}$$

$\underline{\phi_c P_n = 728 \text{ kips}}$

(b) $P_a = D + L = 50 + 150 = 200$ kips

$$\frac{P_a}{A_g} = \frac{200}{21.1} = 9.479 \text{ ksi}$$

From Table 4-21 in the *Manual*, $\tau_a = 1.00$ $\therefore$ no modification is necessary. From part (a), $P_n = 808.3$ kips. The allowable strength is

$$\frac{P_n}{\Omega_c} = \frac{808.3}{1.67} = 484 \text{ kips}$$

$\underline{\frac{P_n}{\Omega_c} = 484 \text{ kips}}$

4.7-11

(a) Member *AB*: $G_A = 10, \qquad G_B = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{2I/13}{2I/30} = 2.3$

From the alignment chart, $K_x \approx 2.15$

$\underline{K_x = 2.15}$

(b) Member *BC*: From part (a), $G_B = 2.3$

$$G_C = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{I/13}{1.5I/30} = 1.54$$ $\underline{K_x = 1.57}$

(c) Member DE: $$G_D = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{I/13}{2(1.5I/30)} = 0.77$$

$$G_E = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{2I/13}{2(2I/30)} = 1.15$$ $\underline{K_x = 1.3}$

(d) Member EF: From part (c), $G_E = 1.15$. Use $G_F = 1.0$ $\underline{K_x = 1.35}$

4.7-12

$$G_G = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{2 \times 640/15}{1330/30} = 1.92$$

$$G_F = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{2 \times 640/15}{2 \times 1330/30} = 0.962$$

From the alignment chart, $K_x \approx 1.45$

$$\frac{K_xL}{r_x} = \frac{1.45(15 \times 12)}{5.98} = 43.65$$

$$4.71\sqrt{\frac{E}{F_y}} = 113.4$$

Since $\frac{K_xL}{r_x} < 4.71\sqrt{\frac{E}{F_y}}$, the column is inelastic.

(a) Use LRFD to determine the stiffness reduction factor (Alternatively, ASD could be used).

$$P_u = 1.2D + 1.6L = 1.2(80) + 1.6(159) = 350.4 \text{ kips}$$

$$\frac{P_u}{A_g} = \frac{350.4}{17.9} = 19.58 \text{ ksi}$$

From Table 4-21 in the *Manual*, $\tau_a = 0.986$ by interpolation.

Use $G_G = 0.986(1.92) = 1.9$, $\quad G_F = 0.986(0.962) = 0.95$

From the alignment chart, $K_x \approx 1.42$ $\underline{K_x = 1.42}$

(b) $$\frac{K_xL}{r_x} = \frac{1.42(15 \times 12)}{5.98} = 42.74, \quad \frac{K_yL}{r_y} = \frac{1.0(15 \times 12)}{2.45} = 73.47 \text{ (controls)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(73.47)^2} = 53.02 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4 > 73.47$$

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/53.02)}(50) = 33.69 \text{ ksi}$$

$$P_n = F_{cr}A_g = 33.69(17.9) = 603 \text{ kips} \qquad \underline{P_n = 603 \text{ kips}}$$

4.7-13

(a) $G_A = 10.0, \quad G_B = \dfrac{\sum I_c/L_c}{\sum I_g/L_g} = \dfrac{55.2/13}{156/20 + 156/20} = 0.27$

From the alignment chart, $K_x \approx 1.7$

$$\frac{K_xL}{r_x} = \frac{1.7(13 \times 12)}{2.17} = 122.2$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{46}} = 118.3$$

Since $\dfrac{K_xL}{r_x} > 4.71\sqrt{\dfrac{E}{F_y}}$, the column is elastic, and the stiffness reduction factor cannot be used.

$\underline{K_x = 1.7}$

(b) $\dfrac{K_yL}{r_y} = \dfrac{1.0(13 \times 12)}{2.17} = 71.89 < \dfrac{K_xL}{r_x}$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(122.2)^2} = 19.17 \text{ ksi}$$

$$F_{cr} = 0.877F_e = 0.877(19.17) = 16.81 \text{ ksi}$$

$$P_n = F_{cr}A_g = 16.81(11.7) = 196.7 \text{ kips} \qquad \underline{P_n = 197 \text{ kips}}$$

4.7-14

(a) $P_u = 1.2D + 1.6L = 1.2(25) + 1.6(75) = 150.0$ kips

For purposes of determining G, assume that y-axis buckling controls and select a shape for AB. For $KL = 1.0(14) = 14$ ft, select a W8 × 31 with $\phi_c P_n = 248$ kips.

$$G_A = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{110/14}{2(291/20)} = 0.27$$

$$G_B = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{110/14 + 248/14}{2(448/20)} = 0.57$$

From the alignment chart, $K_x \approx 1.14$

$$\frac{K_xL}{r_x} = \frac{1.14(14 \times 12)}{3.47} = 55.19$$

$$4.71\sqrt{\frac{E}{F_y}} = 113.4$$

Since $\frac{K_xL}{r_x} < 4.71\sqrt{\frac{E}{F_y}}$, the column is inelastic, and the stiffness reduction factor can be used. But

$$\frac{K_yL}{r_y} = \frac{1.0(14 \times 12)}{2.02} = 83.17 > \frac{K_xL}{r_x}$$

so y-axis buckling controls, and the stiffness reduction factor is not needed.

Use a W8 × 31

(b) $P_a = D + L = 25 + 75 = 100$ kips

For purposes of determining G, assume that y-axis buckling controls and select a shape for AB. For $KL = 1.0(14) = 14$ ft, select a W8 × 31 with $\frac{P_n}{\Omega_c} = 165$ kips.

$$G_A = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{110/14}{2(291/20)} = 0.27$$

$$G_B = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{110/14 + 248/14}{2(448/20)} = 0.57$$

From the alignment chart, $K_x \approx 1.14$

$$\frac{K_xL}{r_x} = \frac{1.14(14 \times 12)}{3.47} = 55.19$$

$$4.71\sqrt{\frac{E}{F_y}} = 113.4$$

Since $\frac{K_xL}{r_x} < 4.71\sqrt{\frac{E}{F_y}}$, the column is inelastic, and the stiffness reduction factor can be used. But

$$\frac{K_yL}{r_y} = \frac{1.0(14 \times 12)}{2.02} = 83.17 > \frac{K_xL}{r_x}$$

so y-axis buckling controls, and the stiffness reduction factor is not needed.

Use a W8 × 31

4.8-1

Compute the flexural buckling strength for the x-axis:

$$\frac{K_xL}{r_x} = \frac{18 \times 12}{3.07} = 70.36$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(70.36)^2} = 57.82 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113$$

Since $\frac{KL}{r} < 4.71\sqrt{\frac{E}{F_y}}$, AISC Equation E3-2 applies.

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/57.82)}(50) = 34.82 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 34.82(26.8) = 933 \text{ kips}$$

Compute the flexural-torsional buckling strength about the y-axis (the axis of symmetry):

$$\frac{K_yL}{r_y} = \frac{18 \times 12}{3.00} = 72.00$$

From the *Manual* Companion CD, $\bar{r}_o = 4.64$ in. and $H = 0.859$

Compute F_{cry} using AISC E3. From AISC Equation E3-4,

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 E}{(K_yL/r_y)^2} = \frac{\pi^2(29,000)}{(72.00)^2} = 55.21 \text{ ksi}$$

Since $K_yL/r_y < 4.71\sqrt{\frac{E}{F_y}} = 113$

$$F_{cry} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/55.21)}(50) = 34.23 \text{ ksi}$$

$$F_{crz} = \frac{GJ}{A_g\bar{r}_o^2} = \frac{11,200(15.3)}{26.8(4.64)^2} = 297.0 \text{ ksi}$$

$$F_{cry} + F_{crz} = 34.23 + 297.0 = 331.2 \text{ ksi}$$

$$F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}}\right]$$

$$= \frac{331.2}{2(0.859)}\left[1 - \sqrt{1 - \frac{4(34.23)(297.0)(0.859)}{(331.2)^2}}\right] = 33.63 \text{ ksi}$$

$$P_n = F_{cr}A_g = 33.63(26.8) = 901 \text{ kips}$$

The flexural-torsional buckling strength controls. $\underline{P_n = 901 \text{ kips}}$

4.8-2

AISC E4(b) must be used, because this shape is nonslender and is neither a double-angle shape nor a tee shape. Check flexural buckling strength about the *y* axis (this is the axis of no symmetry for a channel):

$$\frac{K_yL}{r_y} = \frac{0.65(12 \times 12)}{0.762} = 122.8 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(122.8)^2} = 18.98 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113.4 < 122.8$$

$$\therefore\ F_{cr} = 0.877F_e = 0.877(18.98) = 16.65 \text{ ksi}$$

$$P_n = F_{cr}A_g = 16.65(8.81) = 147 \text{ kips}$$

Flexural-torsional buckling strength about the x axis (this is the axis of symmetry for a channel):

$$\frac{K_xL}{r_x} = \frac{0.65(12 \times 12)}{4.29} = 21.82$$

$$F_{ey} = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(21.82)^2} = 601.2 \text{ ksi}$$

$$F_{ez} = \left[\frac{\pi^2 EC_W}{(K_zL)^2} + GJ\right]\frac{1}{A\bar{r}_o^2}$$

$$= \left[\frac{\pi^2(29,000)(151)}{(0.65 \times 12 \times 12)^2} + 11,200(0.861)\right]\frac{1}{8.81(4.54)^2} = 80.27 \text{ ksi}$$

$$F_{ey} + F_{ez} = 601.2 + 80.27 = 681.5 \text{ ksi}$$

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right]$$

$$= \frac{681.5}{2(0.919)}\left[1 - \sqrt{1 - \frac{4(601.2)(80.27)(0.919)}{(681.5)^2}}\right] = 79.29 \text{ ksi}$$

To determine which compressive strength equation to use, compare this value of F_e with

$$0.44F_y = 0.44(50) = 22.0 \text{ ksi}$$

Since 79.29 ksi > 22.0 ksi, use AISC Equation E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/79.29)}(50) = 38.40 \text{ ksi}$$

$$P_n = F_{cr}A_g = 38.40(8.81) = 338 \text{ kips}$$

The flexural buckling strength controls. $\underline{P_n = 147 \text{ kips}}$

4.8-3

$$KL = 0.8(21) = 16.8 \text{ ft}$$

For a live load-to-dead load ratio of 2.5,

$$D + 2.5D = 175, \qquad D = 50 \text{ kips},\ L = 2.5(50) = 125 \text{ kips}$$

(a) $P_u = 1.2D + 1.6L = 1.2(50) + 1.6(125) = 260.0 \text{ kips}$

From the column load tables, a WT9 × 38 has a design strength of 277 kips based on buckling about the y axis.

Use a WT9 × 38

(b) $P_a = 175$ kips

From the column load tables, a WT9 × 38 has an allowable strength of 184 kips based on buckling about the y axis.

Use a WT9 × 38

4.8-4

(a) $P_u = 1.2D + 1.6L = 1.2(30) + 1.6(70) = 148$ kips

Try a C15 × 33.9

AISC E4(b) must be used, because this shape is nonslender and is neither a double-angle shape nor a tee shape. Check the flexural buckling strength about the y axis (this is the axis of no symmetry for a channel):

$$\frac{K_yL}{r_y} = \frac{0.65(12 \times 12)}{0.901} = 103.9 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(103.9)^2} = 26.51 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{36}} = 133.7$$

Since $\frac{KL}{r} < 4.71\sqrt{\frac{E}{F_y}}$, AISC Equation E3-2 applies.

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(36/26.51)}(36) = 20.39 \text{ ksi}$$

$$P_n = F_{cr}A_g = 20.39(10.0) = 203.9 \text{ kips}$$

$$\phi_c P_n = 0.90(203.9) = 184 \text{ kips} > 148 \text{ kips} \qquad \text{(OK)}$$

Flexural-torsional buckling strength about the x axis (this is the axis of symmetry for a channel):

$$\frac{K_xL}{r_x} = \frac{0.65(12 \times 12)}{5.62} = 16.65$$

$$F_{ey} = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(16.65)^2} = 1032 \text{ ksi}$$

$$F_{ez} = \left[\frac{\pi^2 E C_W}{(K_zL)^2} + GJ\right]\frac{1}{A\bar{r}_o^2}$$

$$= \left[\frac{\pi^2(29{,}000)(358)}{(0.65 \times 12 \times 12)^2} + 11{,}200(1.01)\right]\frac{1}{10.0(5.94)^2} = 65.21 \text{ ksi}$$

$$F_{ey} + F_{ez} = 1032 + 65.21 = 1097 \text{ ksi}$$

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right]$$

$$= \frac{1097}{2(0.920)}\left[1 - \sqrt{1 - \frac{4(1032)(65.21)(0.920)}{(1097)^2}}\right] = 64.88 \text{ ksi}$$

To determine which compressive strength equation to use, compare this value of F_e with

$$0.44F_y = 0.44(36) = 15.8 \text{ ksi}$$

Since 64.88 ksi > 15.8 ksi, use AISC Equation E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(36/64.88)}(36) = 28.54 \text{ ksi}$$

$$P_n = F_{cr}A_g = 28.54(10.0) = 285.4 \text{ kips}$$

$\phi_c P_n = 0.90(285.4) = 257 \text{ kips} > 148 \text{ kips}$ (OK)   Use a C15 × 33.9

(b) $P_a = D + L = 30 + 70 = 100 \text{ kips}$

Try a C15 × 33.9

AISC E4(b) must be used, because this shape is nonslender and is neither a double-angle shape nor a tee shape. First, check the flexural buckling strength about the *y* axis (this is the axis of no symmetry for a channel). From the LRFD solution in Part (a),

$$P_n = 203.9 \text{ kips}$$

$\dfrac{P_n}{\Omega_c} = \dfrac{203.9}{1.67} = 122 \text{ kips} > 100 \text{ kips}$ (OK)

Next, check the flexural-torsional buckling strength about the *x* axis (this is the axis of symmetry for a channel). From the LRFD solution in Part (a),

$$P_n = 285.4 \text{ kips}$$

$\dfrac{P_n}{\Omega_c} = \dfrac{285.4}{1.67} = 171 \text{ kips} > 100 \text{ kips}$ (OK)   Use a C15 × 33.9

4.9-1

With the long leg vertical, the needed properties of a single angle are

$I_y = 4.02 \text{ in.}^4, \quad \bar{x} = 0.901 \text{ in.}, \quad A = 4.00 \text{ in.}^2$

For the two angles,

$$I_y = \left[4.02 + 4.00\left(0.901 + \frac{3}{16}\right)^2\right] \times 2 = 17.52 \text{ in.}^4$$

$r_y = \sqrt{\dfrac{I_y}{A}} = \sqrt{\dfrac{17.52}{2(4.00)}} = 1.48 \text{ in.}$ $r_y = 1.48$ in.

4.9-2

x-axis:

Component	A	y	Ay	$\bar{I}$	d	$\bar{I}+Ad^2$
C10 × 15.3	4.48	0.6340	2.840	2.27	3.598	60.27
W12 × 26	7.65	6.340	48.50	204	2.108	238.0
Sum	12.13		51.34			298.3

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{51.34}{12.13} = 4.232 \text{ in.} \qquad I_x = 298.3 \text{ in.}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{298.3}{12.13}} = 4.959 \text{ in.}$$

y-axis: $I_y = 67.3 + 17.3 = 84.60 \text{ in.}^4, \qquad r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{84.60}{12.13}} = 2.641 \text{ in}$

$\underline{y_2 = 4.23 \text{ in.}, r_x = 4.96 \text{ in.}, r_y = 2.64 \text{ in.}}$

4.9-3

For one angle, $A = 7.13 \text{ in.}^2$, $I_x = I_y = 24.1 \text{ in.}^4$, $\bar{x} = \bar{y} = 1.72$ in.

$$I = [24.1 + 7.13(1.72 + 6/2)^2] \times 4 = 731.8 \text{ in.}^4, \qquad A = 7.13 \times 4 = 28.52 \text{ in.}^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{731.8}{28.52}} = 5.07 \text{ in.}$$

$\underline{r_x = r_y = 5.07 \text{ in.}}$

4.9-4

x-axis:

Segment	A	y	Ay	$\bar{I}$	d	$\bar{I}+Ad^2$
Top fl.	6	0.25	1.5	0.125	6.918	287.3
Web	6	8.5	51.0	128	1.332	138.6
Bot. fl.	3.5	16.75	58.63	7.292×10^{-2}	9.582	321.4
$\sum$	15.5		111.1			747.3

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{111.1}{15.5} = 7.168 \text{ in.} \qquad I = 747.3 \text{ in.}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{747.3}{15.5}} = 6.94 \text{ in.}$$

y-axis:

$$I_y = \frac{1}{12}(0.5)(12)^3 + \frac{1}{12}(16)(3/8)^3 + \frac{1}{12}(0.5)(7)^3 = 86.36 \text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{86.36}{15.5}} = 2.36 \text{ in.} \qquad \underline{r_x = 6.94 \text{ in.}, \ r_y = 2.36 \text{ in.}}$$

4.9-5

$$F_y = 42 \text{ ksi}$$

$$A = 32(27) - 20(15) = 564 \text{ in.}^2$$

$$I_{\min} = I_x = \frac{1}{12}(32)(27)^3 - \frac{1}{12}(20)(15)^3 = 4.686 \times 10^4 \text{ in.}^3$$

$$r_{\min} = r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{46860}{564}} = 9.115 \text{ in.}$$

$$\frac{KL}{r} = \frac{0.8(12 \times 12)}{9.115} = 12.64$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(12.64)^2} = 1791 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{42}} = 123.8 > 12.64$$

$$\therefore \ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(42/1791)}(42) = 41.59 \text{ ksi}$$

$$P_n = F_{cr}A_g = 41.59(564) = 2.346 \times 10^4 \text{ kips} \qquad \underline{P_n = 23{,}500 \text{ kips}}$$

4.9-6

From Table 2-4 in Part 2 of the *Manual*, for A242 steel and 1-in.-thick plate material, $F_y = 46$ ksi.

$$A = 2(18) + 16 = 52 \text{ in.}^2$$

$$I_x = \sum \bar{I} + Ad^2 = 2\left[\frac{1}{12}(18)(1)^3 + 18(8.5)^2\right] + \frac{1}{12}(1)(16)^3 = 2944 \text{ in.}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{2944}{52}} = 7.524 \text{ in.}$$

$$I_y = \frac{1}{12}[16(1)^3 + 1(18)^3(2)] = 973.3 \text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{973.3}{52}} = 4.326 \text{ in.}$$

$$\frac{K_xL}{r_x} = \frac{26 \times 12}{7.524} = 41.47$$

$$\frac{K_yL}{r_y} = \frac{13 \times 12}{4.326} = 36.06 < 41.47 \ \therefore \ x\text{-axis buckling controls.}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(41.47)^2} = 166.4 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{46}} = 118.3$$

Since $KL/r = 41.47 < 118.3$, use AISC Eq. E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(46/166.4)}(46) = 40.97 \text{ ksi}$$

$$P_n = F_{cr}A_g = 40.97(52) = 2130 \text{ kips} \qquad \underline{P_n = 2130 \text{ kips}}$$

4.9-7

(a) $A = 14.4 + 2(9/16)(10) = 25.65 \text{ in.}^2$

$$I_x = 272 + 2\left(\frac{1}{12}\right)\left(\frac{9}{16}\right)(10)^3 = 365.8 \text{ in.}^4$$

$$I_y = \sum \bar{I} + Ad^2 = 93.4 + 2\left[\frac{1}{12}(10)(9/16)^3 + 10(9/16)\left(\frac{10 + 9/16}{2}\right)^2\right]$$

$$= 407.5 \text{ in.}^4 > 365.8 \text{ in.}^4 \quad \therefore x \text{ axis controls.}$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{365.8}{25.65}} = 3.776 \text{ in.}$$

$$\frac{KL}{r} = \frac{25 \times 12}{3.776} = 79.45$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(79.45)^2} = 45.34 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4 > 79.45$$

$$\therefore F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/45.34)}(50) = 31.51 \text{ ksi}$$

$$P_n = F_{cr}A_g = 31.51(25.65) = 808.2 \text{ kips} \qquad \underline{P_n = 808 \text{ kips}}$$

(b) $\dfrac{KL}{r} = \dfrac{K_yL}{r_y} = \dfrac{25 \times 12}{2.54} = 118.1$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(118.1)^2} = 20.52 \text{ ksi}$$

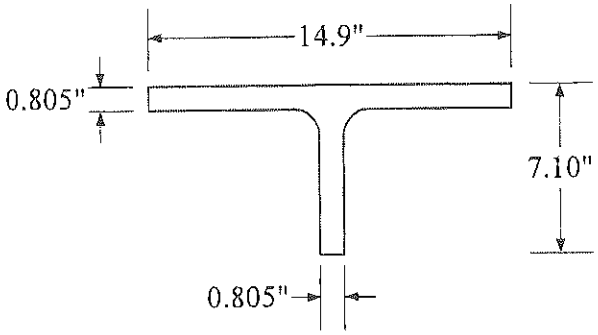
$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4 < 118.1$$

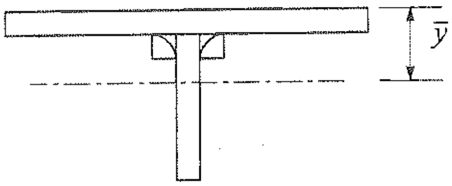
$$\therefore F_{cr} = 0.877F_e = 0.877(20.52) = 18.00 \text{ ksi}$$

$$P_n = F_{cr}A_g = 18.00(14.4) = 259.2 \text{ kips}$$

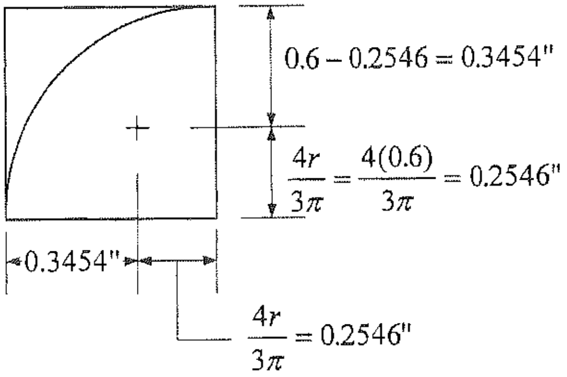
$$\% \text{ difference} = \frac{808.2 - 259.2}{259.2} \times 100 = 211.8 \qquad \underline{\text{Increase} = 212\%}$$

4.9-8





Component shapes


0.6 − 0.2546 = 0.3454"
$\frac{4r}{3\pi} = \frac{4(0.6)}{3\pi} = 0.2546"$
0.3454"
$\frac{4r}{3\pi} = 0.2546"$

Components for fillet

Location of centroid and moment of inertia about the x axis:

Component	A	y	Ay	$\bar{I}$	d	$\bar{I} + Ad^2$
flange	11.99	0.4025	4.826	0.6477	1.051	13.89
square	0.3600	1.105	0.3978	0.01080	0.3480	0.05440
¼ circle	-0.2827	1.150	-0.3251	-0.007112	0.3030	-0.03307
square	0.3600	1.105	0.3978	0.01080	0.3480	0.05440
¼ circle	-0.2827	1.150	-0.3251	-0.007112	0.3030	-0.03307
web	5.067	3.953	20.03	16.73	-2.500	48.40
Sum	17.21		25.00			62.33

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{25.00}{17.21} = 1.453 \text{ in.} \qquad I_x = 62.33 \text{ in.}^4$$

Moment of inertia about the y axis:

Component	A	$\bar{I}$	d	$\bar{I}+Ad^2$
flange	11.99	221.9	0	221.9
square	0.3600	0.01080	0.7025	0.18846
¼ circle	-0.2827	-0.007112	0.7479	-0.16524
square	0.3600	0.01080	0.7025	0.18846
¼ circle	-0.2827	-0.007112	0.7479	-0.16524
web	5.067	0.2736	0	0.2736
Sum	17.21			222.2

$I_y = 222.2 \text{ in.}^4 > 62.33 \text{ in.}^4$ $\therefore$ x axis controls.

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{62.33}{17.21}} = 1.903 \text{ in.}$$

$$\frac{KL}{r} = \frac{10 \times 12}{1.903} = 63.06$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(63.06)^2} = 71.98 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4 > 63.06$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/71.98)}(50) = 37.39 \text{ ksi}$$

$$P_n = F_{cr}A_g = 37.39(17.21) = 643 \text{ kips}$$

$\underline{P_n = 643 \text{ kips}}$

4.9-9

$$A = 4(6.94) = 27.76 \text{ in.}^2$$

$$I_x = I_y = \sum \bar{I} + Ad^2 = 4\left[15.7 + 6.94\left(\frac{18}{2} - 1.52\right)^2\right] = 1616 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1616}{27.76}} = 7.630 \text{ in.}$$

$$\frac{KL}{r} = \frac{30 \times 12}{7.630} = 47.18$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(47.18)^2} = 128.6 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4 > 47.18$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/128.6)}(50) = 42.49 \text{ ksi}$$

$$P_n = F_{cr}A_g = 42.49(27.76) = 1180 \text{ kips}$$

(a) $\phi_c P_n = 0.90(1180) = 1060$ kips $\underline{\phi_c P_n = 1060 \text{ kips}}$

(b) $P_n/\Omega_c = 1180/1.67 = 707$ kips $\underline{P_n/\Omega_c = 707 \text{ kips}}$

4.9-10

Check width-thickness ratio. From AISC B4 (see also Figure 4.9 in textbook),

$$0.45\sqrt{\frac{E}{F_y}} = 0.45\sqrt{\frac{29{,}000}{50}} = 10.84$$

$$\frac{b}{t} = \frac{6}{5/8} = 9.6 < 10.84 \therefore \text{ this is not a slender shape.}$$

Flexural buckling strength about the x axis (this is the axis of no symmetry):

$$\frac{KL}{r_x} = \frac{18 \times 12}{1.89} = 114.3$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(114.3)^2} = 21.91 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4 < 114.3$$

$$\therefore\ F_{cr} = 0.877F_e = 0.877(21.91) = 19.22 \text{ ksi}$$

$$P_n = F_{cr}A_g = 19.22(11.7) = 224.9 \text{ kips}$$

For flexural-torsional buckling strength about the y axis (the axis of symmetry), assume fully-tensioned bolts:

$$\left(\frac{KL}{r}\right)_o = \frac{KL}{r_y} = \frac{18 \times 12}{1.66} = 130.1$$

$$a = \frac{18 \times 12}{3 \text{ spaces}} = 72 \text{ in.}$$

$$\frac{Ka}{r_z} = \frac{72}{0.859} = 83.82 < 0.75(130.1) = 97.58 \text{ (OK)}$$

$$r_{ib} = r_y = 1.13 \text{ in.}$$

$$h = 2(1.03) + \frac{3}{8} = 2.435 \text{ in.}$$

$$\alpha = \frac{h}{2r_{ib}} = \frac{2.435}{2(1.13)} = 1.077$$

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + 0.82\frac{\alpha^2}{(1+\alpha^2)}\left(\frac{a}{r_{ib}}\right)^2}$$

$$= \sqrt{(130.1)^2 + 0.82\left[\frac{(1.077)^2}{1+(1.077)^2}\right]\left(\frac{72}{1.13}\right)^2} = 136.8$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(136.8)^2} = 15.29 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4 < 136.8$$

$$\therefore\ F_{cry} = 0.877F_e = 0.877(15.29) = 13.41 \text{ ksi}$$

$$F_{crz} = \frac{GJ}{A\bar{r}_o^2} = \frac{11{,}200(2 \times 0.775)}{11.7(3.05)^2} = 159.5 \text{ ksi}$$

$$F_{cry} + F_{crz} = 13.41 + 159.5 = 172.9 \text{ ksi}$$

$$F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}}\right]$$

$$= \frac{172.9}{2(0.684)}\left[1 - \sqrt{1 - \frac{4(13.41)(159.5)(0.684)}{(172.9)^2}}\right] = 13.04 \text{ ksi}$$

$$P_n = F_{cr}A_g = 13.04(11.7) = 152.6 \text{ kips}$$

Since 152.6 kips < 224.9 kips, flexural-torsional buckling controls and $P_n = 152.6$ kips

Available strength for LRFD = $\phi_c P_n = 0.90(152.6) = 137.3$ kips. $\underline{\phi_c P_n = 137 \text{ kips}}$

Available strength for ASD = $P_n/\Omega_c = 152.6/1.67 = 91.38$ kips. $\underline{P_n/\Omega_c = 91.4 \text{ kips}}$

4.9-11

$$K_xL = K_yL = 0.65(16) = 10.4 \text{ ft}$$

(a) $P_u = 1.2D + 1.6L = 1.2(33) + 1.6(100) = 200$ kips

From the column load tables,

Try 2L6 × 4 × ½ LLBB: $\phi_c P_n > 206$ kips (y axis controls), $w = 32.3$ lb/ft

Determine the number of intermediate connectors. To obtain the tabulated strength for the y axis, 2 connectors must be used. Also, from AISC E6.2,

$$\frac{Ka}{r_i} \le \frac{3}{4}\frac{KL}{r}$$

$$a = \text{spacing} = \frac{16 \times 12}{n + 1}$$

$$r_i = r_z = 0.864 \text{ in.,} \qquad \frac{Ka}{r_i} = \frac{16 \times 12}{(n+1)(0.864)}$$

For the member, use the maximum slenderness ratio:

$$\frac{KL}{r} = \frac{K_yL}{r_y} = \frac{10.4 \times 12}{1.64} = 76.10$$

For $\frac{Ka}{r_i} \le \frac{3}{4}\frac{KL}{r}$,

$$\frac{16 \times 12}{(n+1)(0.864)} \le \frac{3}{4}(76.10) \qquad \Rightarrow \qquad n \ge 2.9, \qquad \text{use 3.}$$

Use 2L6 × 4 × ½ LLBB with 3 intermediate connectors

(b) $P_a = D + L = 33 + 100 = 133$ kips

From the column load tables, Try 2L6 × 4 × ½ LLBB:

$$\frac{P_n}{\Omega_c} > 137 \text{ kips } (y \text{ axis controls}),\quad w = 32.6 \text{ lb/ft}$$

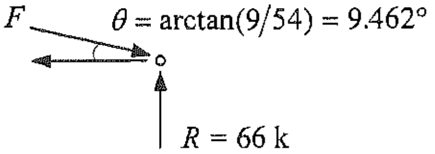
From Part (a), 3 intermediate connectors are required.

Use 2L6 × 4 × ½ LLBB with 3 intermediate connectors

4.9-12

$$\text{Reaction} = \frac{\sum P}{2} = \frac{11 \times 12}{2} = 66 \text{ kips}$$

Consider the joint at the right support:



$$\sum F_y = 66 - F\sin(9.462°) = 0 \qquad \Rightarrow \qquad F = 401 \text{ kips}$$

(This is the maximum force in the top chord.)

$$K_xL = K_yL = \frac{9}{\cos(9.462°)} = 9.124 \text{ ft.}$$

From the column load tables, for $KL = 9.124$ ft,

Try 2L8 × 6 × $\frac{5}{8}$ SLBB, $\phi_c P_n > 424$ kips (x axis controls), $w = 57.3$ lb/ft

Determine the number of intermediate connectors. To obtain the tabulated strength for the y axis, 3 connectors must be used. Also, from AISC E6.2,

$$\frac{Ka}{r_i} \le \frac{3}{4}\frac{KL}{r}$$

$$a = \text{spacing} = \frac{9.124 \times 12}{n+1}$$

$$r_i = r_z = 1.29 \text{ in.,} \qquad \frac{Ka}{r_i} = \frac{9.124 \times 12}{(n+1)(1.29)}$$

For the member, use the maximum slenderness ratio:

$$\frac{KL}{r} = \frac{K_xL}{r_x} = \frac{9.124 \times 12}{1.77} = 61.86$$

For $\frac{Ka}{r_i} \le \frac{3}{4}\frac{KL}{r}$,

$$\frac{9.124 \times 12}{(n+1)(1.29)} \le \frac{3}{4}(61.86) \quad \Rightarrow \quad n \ge 0.829, \quad \text{1 required for the } x \text{ axis.}$$

Use 2L8 × 6 × $\frac{5}{8}$ SLBB with 3 intermediate connectors

5.2-1

(a) Flange area = $0.5(7.5) = 3.75$ in.^{2} Half web area = $(3/8)(8.5) = 3.188$ in.^{2}

From mid-depth of the cross section,

$$\bar{y} = \frac{3.75(8.5+0.25)+3.188(8.5/2)}{3.75+3.188} = 6.682 \text{ in.}$$

$$Z = \frac{A}{2} \cdot a = (3.75+3.188)(2\times 6.682) = 92.72 \text{ in.}^3$$

$$M_p = F_y Z = 50(92.72) = 4636 \text{ in-kips} = 386 \text{ ft-kips}$$

$\underline{Z = 92.7 \text{ in.}^3,\ M_p = 386 \text{ ft-kips}}$

(b) Moment of inertia:

Component	A	$\bar{I}$	d	$\bar{I}+Ad^2$
top fl	3.75	0.07813	8.750	287.2
web	6.00	154	0.000	154.0
bot fl	3.75	0.07813	8.750	287.2
Sum	13.50			728.4

$$S = \frac{I}{c} = \frac{728.4}{8.5+0.5} = 80.93 \text{ in.}^3$$

$$M_y = F_y S = 50(80.93) = 4047 \text{ in.-kips} = 337 \text{ ft-kips}$$

$\underline{S = 80.9 \text{ in.}^3,\ M_y = 337 \text{ ft-kips}}$

5.2-2

(a) Area above PNA = Area below PNA

$$12(1/2)+(\bar{y}-1/2)(3/8) = (1/2+16-\bar{y})(3/8)+7(1/2) \quad \Rightarrow \quad \bar{y} = 5.167$$

$\underline{\bar{y} = 5.17 \text{ in.}}$

(b) To locate centroids of areas, take moments about plastic neutral axis. For area above PNA,

Component	A	y	Ay
top fl	6.000	4.917	29.50
web	1.750	2.334	4.085
Sum	7.750		33.59

$$\bar{y}_1 = \frac{\sum Ay}{\sum A} = \frac{33.59}{7.750} = 4.334 \text{ in.}$$

For area below PNA,

Component	A	y	Ay
bot. fl	3.500	11.58	40.53
web	4.250	5.667	24.08
Sum	7.750		64.61

$$\bar{y}_2 = \frac{\sum Ay}{\sum A} = \frac{64.61}{7.750} = 8.337 \text{ in.}$$

Plastic moment:

$$M_p = C \times \text{moment arm} = T \times \text{moment arm}$$
$$= AF_y(\bar{y}_1 + \bar{y}_2) = 7.750(50)(4.334 + 8.337)$$
$$= 387.5(12.67) = 4910 \text{ in.-kips} = 409 \text{ ft-kips} \qquad \underline{M_p = 409 \text{ ft-kips}}$$

(c) Because of symmetry, $\bar{x}_t = \bar{x}_c$

Component	A	y	Ay
top fl	3.000	3.000	9.000
web	3.000	0.09380	0.2814
bot. fl	1.750	1.750	3.0625
Sum	7.750		12.34

$$\bar{x}_t = \bar{x}_c = \frac{\sum Ax}{\sum A} = \frac{12.34}{7.750} = 1.592$$

$$Z_y = \frac{A}{2}(\bar{x}_t + \bar{x}_c) = 7.750(1.592 + 1.592) = 24.68 \text{ in.}^3 \qquad \underline{Z_y = 24.7 \text{ in.}^3}$$

5.2-3

For the centroid of half of a W18 × 50, use the centroid of a WT9 × 25:

$$a = d - 2(2.12) = 18.00 - 2(2.12) = 13.76 \text{ in.}$$

$$Z_x = \frac{A}{2}a = \left(\frac{14.7}{2}\right)(13.76) = 101.1 \text{ in.}^3 \qquad \underline{Z_x = 101 \text{ in.}^3}$$

5.2-4

For the centroid of half of an S10 × 35, use the centroid of a ST5 × 17.5:

$$a = d - 2(1.56) = 10.0 - 2(1.56) = 6.880 \text{ in.}$$

$$Z_x = \frac{A}{2}a = \left(\frac{10.3}{2}\right)(6.880) = 35.43 \text{ in.}^3 \qquad \underline{Z_x = 35.4 \text{ in.}^3}$$

5.4-1

(a) The flange is noncompact for flexure when

$$0.38\sqrt{\frac{E}{F_y}} < \frac{b_f}{2t_f} \le 1.0\sqrt{\frac{E}{F_y}}$$

$$0.38\sqrt{\frac{29,000}{60}} < \frac{b_f}{2t_f} \le 1.0\sqrt{\frac{29,000}{60}} \quad \Rightarrow \quad 8.35 < \frac{b_f}{2t_f} \le 22.0$$

The following shapes meet these criteria and are noncompact for $F_y = 60$ ksi:

W30 × 90, W24 × 104, W21 × 48, W14 × 109, W14 × 99, W14 × 90, W14 × 30, W12 × 72, W12 × 65, W12 × 53, W12 × 26, W12 × 14, W10 × 49, W10 × 33, W10 × 12, W8 × 31, W8 × 10, W6 × 15, W6 × 9, W6 × 8.5, M12 × 10, M4 × 6, M4 × 3.45,M4 × 3.2, and M3 × 2.9

The web is noncompact for flexure when

$$0.376\sqrt{\frac{E}{F_y}} < \frac{h}{t_w} \le 5.70\sqrt{\frac{E}{F_y}}$$

$$0.376\sqrt{\frac{29,000}{60}} < \frac{h}{t_w} \le 5.70\sqrt{\frac{29,000}{60}} \quad \Rightarrow \quad 82.7 < \frac{h}{t_w} \le 125$$

No shapes are noncompact because of the web.

(b) The flange is slender for flexure when $\frac{b_f}{2t_f} > 1.0\sqrt{\frac{E}{F_y}} = 22.0$

and the web is slender when $\frac{h}{t_w} > 5.70\sqrt{\frac{E}{F_y}} = 125$

No W, M, or S shapes in Part 1 of the *Manual* are slender.

5.4-2

(a) The flange is noncompact for flexure when

$$0.38\sqrt{\frac{E}{F_y}} < \frac{b_f}{2t_f} \le 1.0\sqrt{\frac{E}{F_y}}$$

$$0.38\sqrt{\frac{29{,}000}{65}} < \frac{b_f}{2t_f} \le 1.0\sqrt{\frac{29{,}000}{65}} \quad \Rightarrow \quad 8.03 < \frac{b_f}{2t_f} \le 21.1$$

The following shapes meet these criteria and are noncompact for $F_y = 65$ ksi:

W30 × 90, W24 × 104, W21 × 48, W18 × 76, W16 × 36, W14 × 109, W14 × 99, W14 × 90, W14 × 30, W12 × 79, W12 × 72, W12 × 65, W12 × 53, W12 × 26, W12 × 14, W10 × 54, W10 × 49, W10 × 33, W10 × 12, W8 × 35, W8 × 31, W18 × 24, W8 × 10, W6 × 20, W6 × 15, W6 × 9, W6 × 8.5, M12.5 × 12.4, M12.5 × 11.6, M12 × 10, M4 × 6, M4 × 3.45,M4 × 3.2, and M3 × 2.9

The web is noncompact for flexure when

$$3.76\sqrt{\frac{E}{F_y}} < \frac{h}{t_w} \le 5.70\sqrt{\frac{E}{F_y}}$$

$$3.76\sqrt{\frac{29{,}000}{65}} < \frac{h}{t_w} \le 5.70\sqrt{\frac{29{,}000}{65}} \quad \Rightarrow \quad 79.4 < \frac{h}{t_w} \le 120$$

No shapes are noncompact because of the web.

(b) The flange is slender for flexure when $\frac{b_f}{2t_f} > 1.0\sqrt{\frac{E}{F_y}} = 21.1$

and the web is slender when $\frac{h}{t_w} > 5.70\sqrt{\frac{E}{F_y}} = 120$

No W, M, or S shapes in Part 1 of the *Manual* are slender.

5.4-3

Flange: maximum $\frac{b_f}{2t_f} = 11.0$ for an M4 × 6

$$\text{Let } 11.9 = 1.0\sqrt{\frac{E}{F_y}} = 1.0\sqrt{\frac{29{,}000}{F_y}} \quad \Rightarrow \quad F_y = 205 \text{ ksi}$$

Web: maximum $\frac{h}{t_w} = 74.8$ for both an M12.5 × 12.4 and an M12.5 × 11.6

$$\text{Let } 74.8 = 5.7\sqrt{\frac{E}{F_y}} = 5.7\sqrt{\frac{29{,}000}{F_y}} \quad \Rightarrow \quad F_y = 168 \text{ ksi (controls)}$$

$F_y = 168$ ksi

M12.5 × 12.4 and M12.5 × 11.6

Conclusion: No W, M, or S shape in the *Manual* is slender for any of the steels listed in Table 2-3 of the *Manual*

5.5-1

Check for compactness. From Part 1 of the Manual,

$$\frac{b_f}{2t_f} = 5.86$$

$$0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29,000}{50}} = 9.15 > 5.86 \qquad \therefore \text{ the flange is compact.}$$

(This shape can also be identified as compact because there is no footnote in the dimensions and properties tables to indicate otherwise.)

$\frac{h}{t_w} < 3.76\sqrt{\frac{E}{F_y}}$ (for all shapes in the Manual for $F_y \leq 65$ ksi), so the web is compact.

$\therefore$ a W16 × 31 is compact for $F_y = 50$ ksi.

Because the beam is compact and laterally supported, the nominal flexural strength is

$$M_n = M_p = F_y Z_x = 50(97.6) = 4880 \text{ in.-kips} = 406.7 \text{ ft kips}$$

(a) Let $M_u = \phi_b M_n$:

$$\frac{1.2(0.077)(30)^2}{8} + \frac{1.6P(30)}{4} = 0.90(406.7), \text{ Solution is: } \{P = 29.64\}$$

$\underline{P = 29.6 \text{ kips}}$

(b) Let $M_a = \frac{M_n}{\Omega_b}$:

$$\frac{0.077(30)^2}{8} + \frac{P(30)}{4} = \frac{406.7}{1.67}, \text{ Solution is: } \{P = 31.32\}$$

$\underline{P = 31.3 \text{ kips}}$

5.5-2

Check for compactness. From Part 1 of the Manual,

$$\frac{b_f}{2t_f} = 7.11$$

$$0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29,000}{50}} = 9.15 > 7.11 \qquad \therefore \text{ the flange is compact.}$$

(This shape can also be identified as compact because there is no footnote in the dimensions and properties tables to indicate otherwise.)

$\frac{h}{t_w} < 3.76\sqrt{\frac{E}{F_y}}$ (for all shapes in the Manual for $F_y \leq 65$ ksi), so the web is compact.

$\therefore$ a W40 × 149 is compact for $F_y = 50$ ksi.

Because the beam is compact and laterally supported, the nominal flexural strength is

$M_n = M_p = F_y Z_x = 50(598) = 2.99 \times 10^4$ in.-kips = 2492 ft kips

(a) Let $M_u = \phi_b M_n$:

$$\frac{w_u(30)^2}{8} = 0.90(2492), \text{ Solution is: } \{w_u = 19.94\}$$

$$1.2w_D + 1.6(2w_D) = 19.94, \text{ Solution is: } \{w_D = 4.532\}$$

$$w_D + w_L = 4.532 + 2(4.532) = 13.60 \text{ kips/ft}$$

If the beam weight is deducted, the maximum service load is

$13.60 - 0.149 = 13.4$ kips/ft $\quad$ $\underline{w = 13.4 \text{ kips/ft in addition to the beam weight}}$

(b) Let $M_a = \dfrac{M_n}{\Omega_b}$:

$$\frac{w_a(30)^2}{8} = \frac{2492}{1.67}, \text{ Solution is: } \{w_a = 13.26\}$$

If the beam weight is deducted, the maximum service load is

$13.26 - 0.149 = 13.1$ kips/ft $\quad$ $\underline{w = 13.1 \text{ kips/ft in addition to the beam weight}}$

5.5-3

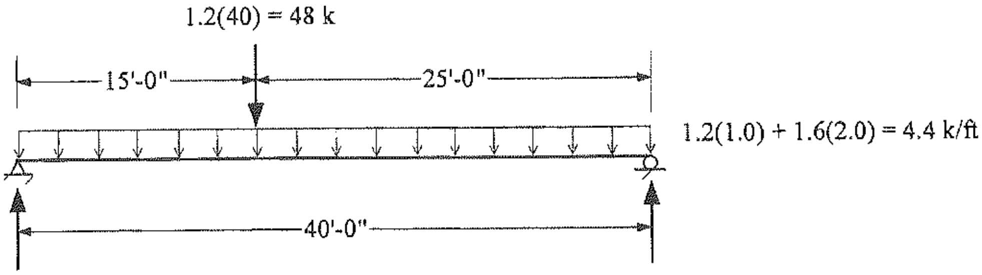
Verify that this shape is compact. For the flange,

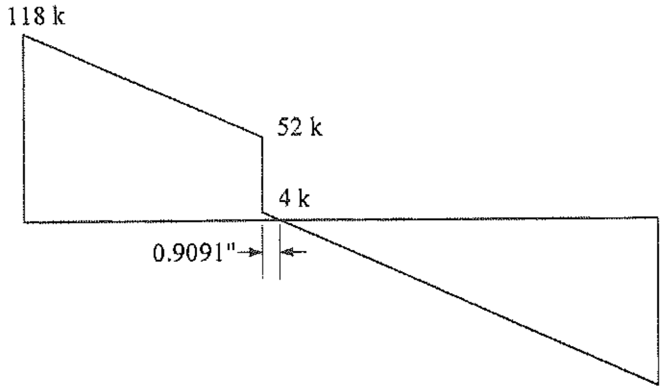
$$\lambda_p = 0.38\sqrt{\frac{29,000}{50}} = 9.15, \qquad \lambda = \frac{b_f}{2t_f} = 6.89 < \lambda_p \ \therefore \text{ flange is compact}$$

For the web, $\lambda_p = 3.76\sqrt{\dfrac{29,000}{50}} = 90.6$, $\quad \lambda = \dfrac{h}{t_w} = 49.6 < \lambda_p \ \therefore$ web is compact, and the shape is compact.

$$M_n = F_y Z_x = \frac{50(346)}{12} = 1442 \text{ ft-kips}$$

(a) Load combination 2 controls:





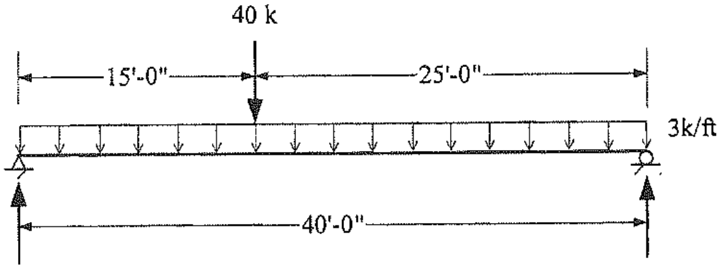
Design strength = $\phi_b M_n = 0.90(1442) = 1298$ ft-kips

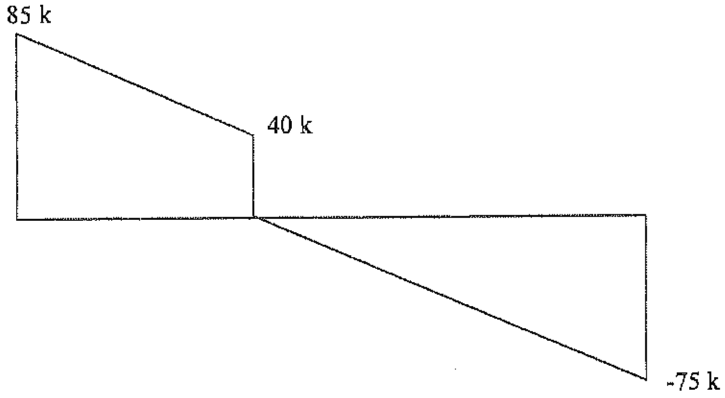
$M_u = 118(15.91) - 4.4(15.91)^2/2 - 48(0.9091)$

$= 1277$ ft-kips < 1298 ft-kips (OK)

A W30 × 108 is adequate.

(b) ASD load combination 2 controls:





Allowable strength $= \dfrac{M_n}{\Omega_b} = \dfrac{1442}{1.67} = 864$ ft-kips

$M_a = 85(15) - 3(15)^2/2 = 938$ ft-kips $>$ 864 ft-kips (N.G.)

A W30 × 108 is not adequate.

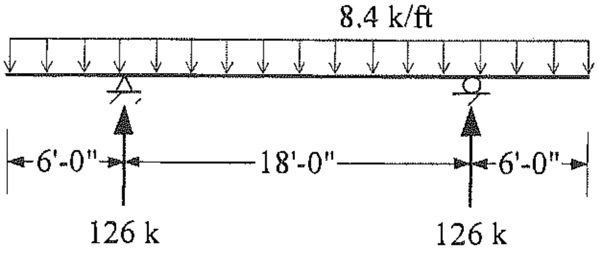
5.5-4

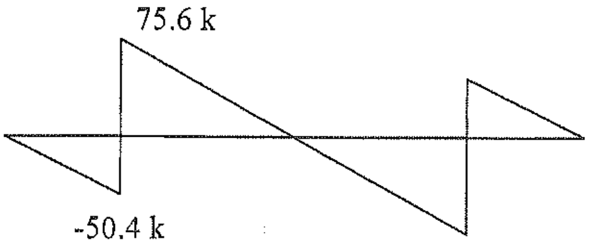
Verify that this shape is compact. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29,000}{50}} = 9.15, \qquad \lambda = \frac{b_f}{2t_f} = 6.31 < \lambda_p \quad \therefore \text{ flange is compact}$$

$\dfrac{h}{t_w} < 3.76\sqrt{\dfrac{E}{F_y}}$ (for all shapes in the Manual for $F_y \leq 65$ ksi), so the web is compact, and the shape is compact.

(a) $w_u = 1.2D + 1.6L = 1.2(3) + 1.6(3) = 8.4$ kips/ft





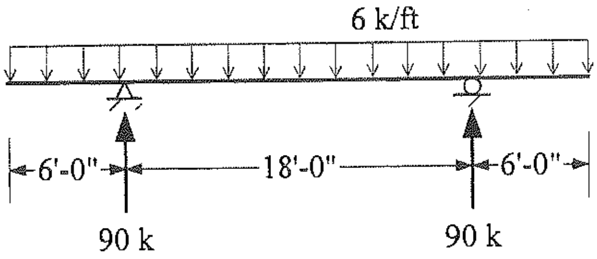
Maximum negative $M_u = -8.4(6)^2/2 = -151$ ft-kips

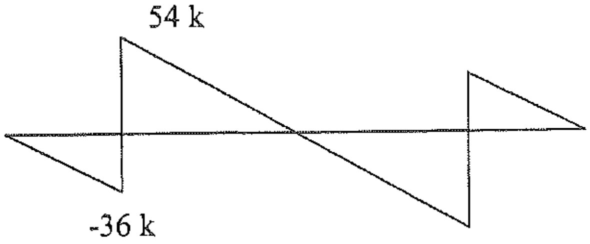
Maximum positive $M_u = -8.4(15)^2/2 + 126(9) = 189$ ft-kips

Compute the design strength: $\phi_b M_n = \phi_b F_y Z_x = 0.9(50)(51.2)/12 = 192$ ft-kips

Since $M_u = 189$ ft-kips $< \phi_b M_n = 192$ ft-kips, a W12 × 35 is adequate.

(b) $w_a = D + L = 6$ kips/ft





Maximum negative $M_a = -6(6)^2/2 = -108$ ft-kips

Maximum positive $M_a = -6(15)^2/2 + 90(9) = 135$ ft-kips

Compute the allowable strength: $\dfrac{M_n}{\Omega_b} = \dfrac{F_y Z_x}{\Omega_b} = \dfrac{50(51.2)}{1.67(12)} = 128$ ft-kips

Since $M_a = 135$ ft-kips $> \dfrac{M_n}{\Omega_b} = 128$ ft-kips, a W12 × 35 is not adequate.

5.5-5

Verify that this shape is compact. For the flange,

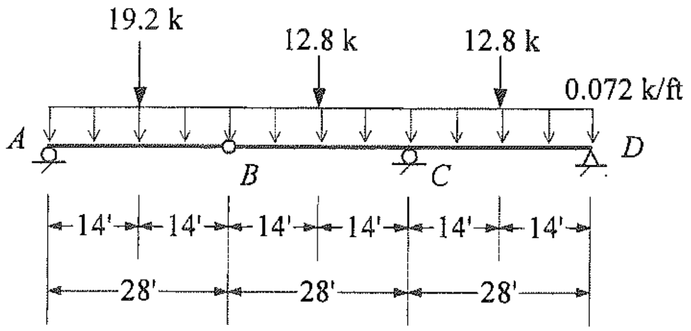
$$\lambda_p = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15, \qquad \lambda = \frac{b_f}{2t_f} = 5.44 < \lambda_p \quad \therefore \text{ flange is compact}$$

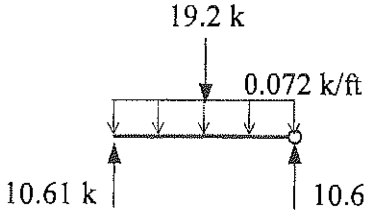
$\dfrac{h}{t_w} < 3.76\sqrt{\dfrac{E}{F_y}}$ (for all shapes in the Manual for $F_y \le 65$ ksi), so the web is compact, and the shape is compact.

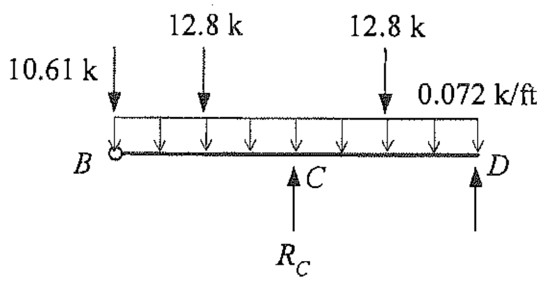
$$M_n = F_y Z_x = \frac{50(123)}{12} = 512.5 \text{ ft-kips}$$

(a) Factored loads, including beam weight:

$1.2(0.060) = 0.072$ kips/ft, $\quad 1.6(12) = 19.2$ kips, $\quad 1.6(8) = 12.8$ kips





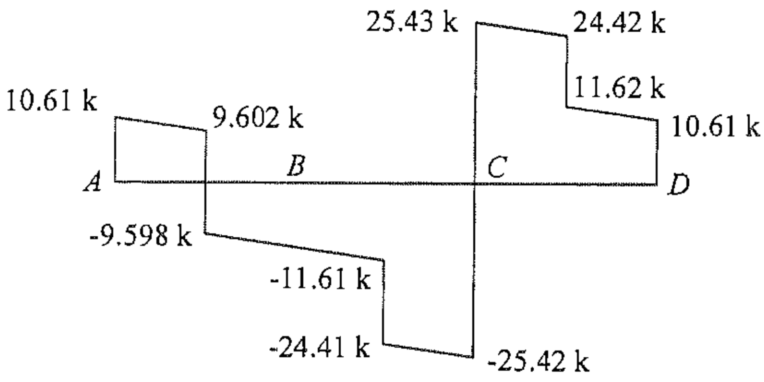


Reaction at C :

$$\sum M_D = -10.61(56) - 12.8(42) - 12.8(14) + R_C(28) - 0.072(56)^2/2 = 0$$

$$R_C = 50.85 \text{ kips}$$

Shear diagram:



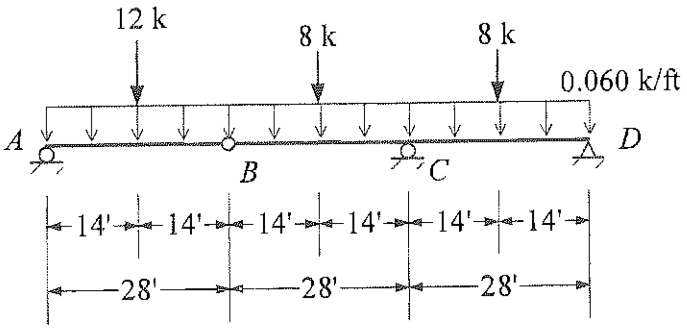
Maximum moment occurs at C. Using the free-body diagram of BCD,

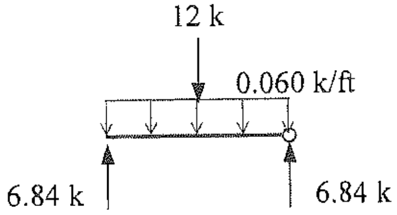
$$M_C = -10.61(28) - 12.8(14) - 0.072(28)^2/2 = -505 \text{ ft-kips}$$

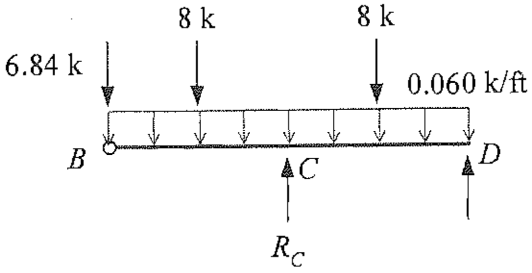
Design strength = $\phi_b M_n = 0.90(512.5) = 461$ ft-kips

$M_u = 505 \text{ ft-kips} > \phi_b M_n = 461 \text{ ft-kips}$ (N.G.) A W18 × 60 is not adequate.

(b) Service loads:





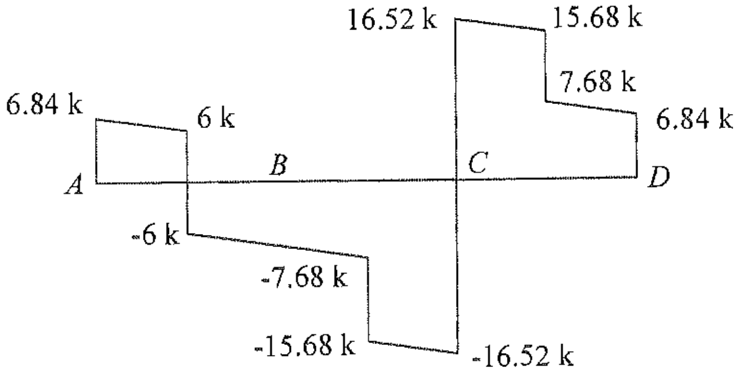


Reaction at C :

$$\sum M_D = -6.84(56) - 8(42) - 8(14) + R_C(28) - 0.060(56)^2/2 = 0$$

$$R_C = 33.04 \text{ kips}$$

Shear diagram:



Maximum moment occurs at C. Refer to the free-body diagram of BCD.

$$M_a = M_C = -6.84(28) - 8(14) - 0.060(28)^2/2 = -327 \text{ ft-kips}$$

$$\text{Allowable strength} = \frac{M_n}{\Omega_b} = \frac{512.5}{1.67} = 307 \text{ ft-kips}$$

$M_a = 327$ ft-kips > 307 ft-kips (N.G.) A W18 × 60 is not adequate.

5.5-6

(a) For a W12 × 30, $A = 8.79 \text{ in.}^2$, $d = 12.3$ in., $t_f = 0.440$ in., $S_x = 38.6 \text{ in.}^3$, $I_y = 20.3 \text{ in.}^4$, $r_y = 1.52$ in., $J = 0.457 \text{ in.}^4$, $C_W = 720 \text{ in.}^6$

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76(1.52)\sqrt{\frac{29{,}000}{50}} = 64.43 \text{ in.} = 5.37 \text{ ft.}$$

The following terms will be needed in the computation of L_r :

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} = \frac{\sqrt{20.3(720)}}{38.6} = 3.132 \text{ in.}^2$$

$$r_{ts} = \sqrt{3.132} = 1.770 \text{ in.}$$

(r_{ts} can also be found in the dimensions and properties tables. For a W12 × 30, it is given as 1.77 in.)

$$h_o = d - t_f = 12.3 - 0.440 = 11.86 \text{ in.}$$

(h_o can also be found in the dimensions and properties tables. For a W12 × 30, it is given as 11.9 in.)

For a doubly-symmetric I-shape, $c = 1.0$. From AISC Equation F2-6,

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o}} \sqrt{1 + \sqrt{1 + 6.76\left(\frac{0.7F_y S_x h_o}{EJc}\right)^2}}$$

$$= 1.95(1.769)\frac{29{,}000}{0.7(50)} \sqrt{\frac{0.457(1.0)}{38.6(11.86)}} \sqrt{1 + \sqrt{1 + 6.76\left(\frac{0.7(50)(38.6)(11.86}{29{,}000(0.457)(1.0)}\right)^2}}$$

$$= 187.2 \text{ in.} = 15.6 \text{ ft}$$

$L_p = 5.37$ ft, $L_r = 15.6$ ft

(b) Verify that this shape is compact. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15, \qquad \lambda = \frac{b_f}{2t_f} = 7.41 < \lambda_p \quad \therefore \text{ flange is compact}$$

For the web, $\lambda_p = 3.76\sqrt{\frac{29{,}000}{50}} = 90.6$, $\lambda = \frac{h}{t_w} = 41.8 < \lambda_p$ ∴ web is compact, and the shape is compact. (Also, there is no footnote in the dimensions and properties table to indicate otherwise.)

For $L_b = 10$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

$$M_p = F_yZ_x = 50(43.1) = 2155 \text{ in.-kips}$$

$$M_n = 1.0\left[2155 - (2155 - 0.7 \times 50 \times 38.6)\left(\frac{10 - 5.37}{15.6 - 5.37}\right)\right] = 1791 \text{ in.-kips} < M_p$$

$$\phi_bM_n = 0.9(1791) = 1612 \text{ in.-kips} = 134 \text{ ft-kips} \qquad \underline{\phi_bM_n = 134 \text{ ft-kips}}$$

(c) $\dfrac{M_n}{\Omega_b} = \dfrac{1791/12}{1.67} = 89.4$ ft-kips $\qquad \underline{\dfrac{M_n}{\Omega_b} = 89.4 \text{ ft-kips}}$

5.5-7

Verify that this shape is compact. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15, \qquad \lambda = \frac{b_f}{2t_f} = 5.01 < \lambda_p \ \therefore \text{ flange is compact}$$

For the web, $\lambda_p = 3.76\sqrt{\dfrac{29{,}000}{50}} = 90.6$, $\quad \lambda = \dfrac{h}{t_w} = 44.6 < \lambda_p$ $\therefore$ web is compact, and the shape is compact. (Also, there is no footnote in the dimensions and properties table to indicate otherwise.)

For a W18 × 46, $A = 13.5$ in.^{2}, $d = 18.1$ in., $t_f = 0.605$ in., $S_x = 78.8$ in.^{3}, $I_y = 22.5$ in.^{4}, $r_y = 1.29$ in., $J = 1.22$ in.^{4}, $C_w = 1720$ in.^{6}

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}} = 1.76(1.29)\sqrt{\frac{29{,}000}{50}} = 54.68 \text{ in.} = 4.557 \text{ ft.}$$

The following terms will be needed in the computation of L_r :

$$r_{ts}^2 = \frac{\sqrt{I_yC_w}}{S_x} = \frac{\sqrt{22.5(1720)}}{78.8} = 2.496 \text{ in.}^2$$

$$r_{ts} = \sqrt{2.496} = 1.580 \text{ in.}$$

(r_{ts} can also be found in the dimensions and properties tables. For a W18 × 46, it is given as 1.58 in.)

$$h_o = d - t_f = 18.1 - 0.605 = 17.50 \text{ in.}$$

(h_o can also be found in the dimensions and properties tables. For a W18 × 46, it is given as 17.5 in.)

For a doubly-symmetric I-shape, $c = 1.0$. From AISC Equation F2-6,

$$L_r = 1.95r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_xh_o}}\sqrt{1 + \sqrt{1 + 6.76\left(\frac{0.7F_yS_xh_o}{EJc}\right)^2}}$$

$$= 1.95(1.580)\frac{29,000}{0.7(50)}\sqrt{\frac{1.22(1.0)}{78.8(17.50)}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7(50)(78.8)(17.50)}{29,000(1.22)(1.0)}\right)^2}}$$

$$= 164.4 \text{ in.} = 13.70 \text{ ft}$$

For $L_b = 10$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

$$M_p = F_yZ_x = 50(90.7) = 4535 \text{ in.-kips} = 378 \text{ ft-kips}$$

$$M_n = 1.0\left[4535 - (4535 - 0.7 \times 50 \times 78.8)\left(\frac{10 - 4.557}{13.70 - 4.557}\right)\right]$$

$= 3477$ in.-kips $= 290$ ft-kips $< M_p$ $\qquad$ $\underline{M_n = 290 \text{ ft-kips}}$

5.5-8

Verify that this shape is compact. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29,000}{65}} = 8.03, \qquad \lambda = \frac{b_f}{2t_f} = 4.71 < \lambda_p \;\therefore\; \text{flange is compact}$$

For the web, $\lambda_p = 3.76\sqrt{\frac{29,000}{65}} = 79.4$, $\qquad \lambda = \frac{h}{t_w} = 32.4 < \lambda_p$ $\therefore$ web is compact, and the shape is compact.

For a W18 × 71, $A = 20.8$ in.^{2}, $d = 18.5$ in., $t_f = 0.810$ in., $S_x = 127$ in.^{3}, $Z_x = 146$ in.^{3}, $I_y = 60.3$ in.^{4}, $r_y = 1.70$ in., $r_{ts} = 2.05$ in., $h_o = 17.7$ in., $J = 3.49$ in.^{4}, $C_w = 4700$ in.^{6}

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}} = 1.76(1.70)\sqrt{\frac{29,000}{65}} = 63.20 \text{ in.} = 5.267 \text{ ft.}$$

The following terms will be needed in the computation of L_r :

$$r_{ts}^2 = \frac{\sqrt{I_yC_w}}{S_x} = \frac{\sqrt{60.3(4700)}}{127} = 4.192 \text{ in.}^2$$

$$r_{ts} = \sqrt{4.192} = 2.047 \text{ in.}$$

(r_{ts} can also be found in the dimensions and properties tables. For a W18 × 71, it is given as 2.05 in.)

$$h_o = d - t_f = 18.5 - 0.810 = 17.69 \text{ in.}$$

(h_o can also be found in the dimensions and properties tables. For a W18 × 71, it is given as 17.7 in.)

For a doubly-symmetric I-shape, $c = 1.0$. From AISC Equation F2-6,

$$L_r = 1.95r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_xh_o}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7F_yS_xh_o}{EJc}\right)^2}}$$

$$= 1.95(2.047)\frac{29,000}{0.7(65)}\sqrt{\frac{3.49(1.0)}{127(17.69)}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7(65)(127)(17.69}{29,000(3.49)(1.0)}\right)^2}}$$

$$= 195.7 \text{ in.} = 16.31 \text{ ft}$$

For $L_b = 9$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

$$M_p = F_yZ_x = 65(146) = 9490 \text{ in.-kips} = 791 \text{ ft-kips}$$

$$M_n = 1.0\left[9490 - (9490 - 0.7 \times 65 \times 127)\left(\frac{9 - 5.267}{16.31 - 5.267}\right)\right]$$

$$= 8235 \text{ in.-kips} = 686 \text{ ft-kips} < M_p \qquad \underline{M_n = 686 \text{ ft-kips}}$$

5.5-9

(a) $R_A = R_B = \frac{300}{2} = 150$ kips

Use lower-case subscripts to identify moments at the quarter points of the unbraced length:

$$M_a = M_c = 150(5) = 750 \text{ ft-kips}$$

$$M_b = 150(10) = 1500 \text{ ft-kips} = M_{\max}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_a + 4M_b + 3M_c}$$

$$= \frac{12.5(1500)}{2.5(1500) + 3(750) + 4(1500) + 3(750)} = 1.316 \qquad \underline{C_b = 1.32}$$

(b) $R_A = R_B = \frac{300 + 0.182}{2} = 150.1$ ft-kips

$$M_a = M_c = 150.1(5) = 750.5 \text{ ft-kips}$$

$$M_b = 150.1(10) = 1501 \text{ ft-kips} = M_{\max}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_a + 4M_b + 3M_c}$$

$$= \frac{12.5(1501)}{2.5(1501) + 3(750.5) + 4(1501) + 3(750.5)} = 1.316 \qquad \underline{C_b = 1.32}$$

5.5-10

(a) Using service loads,

$$R_A = R_B = \frac{300}{2} = 150 \text{ ft-kips}$$

Use lower-case letters to identify moments:

$M_a = 150(2.5) = 375$ ft-kips

$M_b = 150(5) = 750$ ft-kips

$M_c = 150(7.5) = 1125$ ft-kips

$M_{\max} = 150(10) = 1500$ ft-kips

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_a + 4M_b + 3M_c}$$

$$= \frac{12.5(1500)}{2.5(1500) + 3(375) + 4(750) + 3(1125)} = 1.667 \qquad \underline{C_b = 1.67}$$

(b) Using factored loads,

$P_u = 1.6(300) = 480.0$ kips

$R_A = R_B = \frac{480}{2} = 240$ ft-kips

Use lower-case letters to identify moments:

$M_a = 240(2.5) = 600$ ft-kips

$M_b = 240(5) = 1200$ ft-kips

$M_c = 240(7.5) = 1800$ ft-kips

$M_{\max} = 240(10) = 2400$ ft-kips

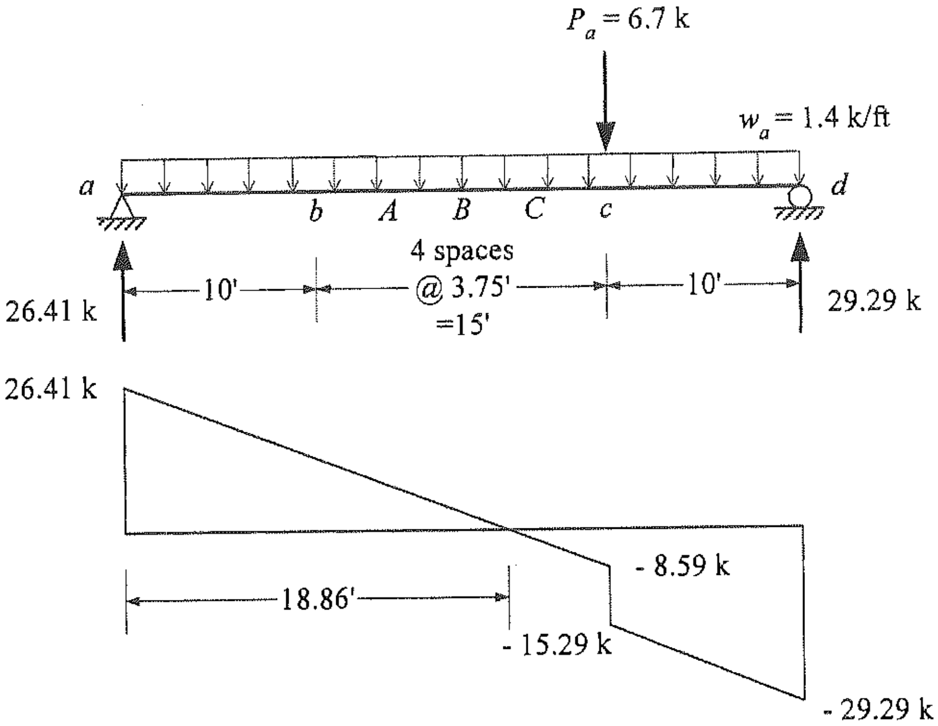
$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_a + 4M_b + 3M_c}$$

$$= \frac{12.5(2400)}{2.5(2400) + 3(600) + 4(1200) + 3(1800)} = 1.667 \qquad \underline{C_b = 1.67}$$

5.5-11

$P_D = 1.7$ kips, $P_L = 5$ kips. $w_D = 0.5$ kips/ft, $w_L = 0.9$ kips/ft

(a) $P_a = 1.7 + 5 = 6.7$ kips. $w_a = 0.5 + 0.9 = 1.4$ kips/ft



$$M_{\max} = 26.41(18.86) - 1.4(18.86)^2/2 = 249.1 \text{ ft-kips}$$

$$M_A = 26.41(13.75) - 1.4(13.75)^2/2 = 230.8 \text{ ft-kips}$$

$$M_B = 26.41(17.5) - 1.4(17.5)^2/2 = 247.8 \text{ ft-kips}$$

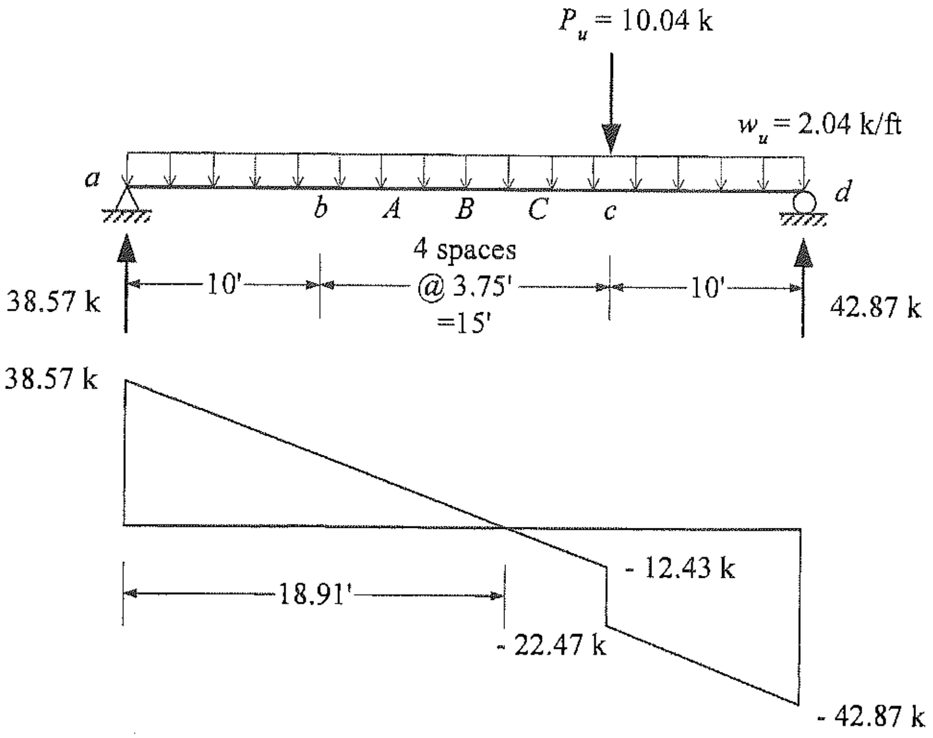
$$M_C = 26.41(21.25) - 1.4(21.25)^2/2 = 245.1 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(249.1)}{2.5(249.1) + 3(230.8) + 4(247.8) + 3(245.1)} = 1.024 \qquad \underline{C_b = 1.02}$$

(b) $P_u = 1.2(1.7) + 1.6(5) = 10.04$ kips

$w_u = 1.2(0.5) + 1.6(0.9) = 2.040$ kips/ft



$$M_{max} = 38.57(18.91) - 2.04(18.91)^2/2 = 364.6 \text{ ft-kips}$$

$$M_A = 38.57(13.75) - 2.04(13.75)^2/2 = 337.5 \text{ ft-kips}$$

$$M_B = 38.57(17.5) - 2.04(17.5)^2/2 = 362.6 \text{ ft-kips}$$

$$M_C = 38.57(21.25) - 2.04(21.25)^2/2 = 359.0 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(364.6)}{2.5(364.6) + 3(337.5) + 4(362.6) + 3(359.0)} = 1.024 \qquad \underline{C_b = 1.02}$$

5.5-12

First, determine the nominal flexural strength. Verify that this shape is compact. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29,000}{50}} = 9.15, \qquad \lambda = \frac{b_f}{2t_f} = 6.04 < \lambda_p \ \therefore \text{ flange is compact}$$

For the web, $\lambda_p = 3.76\sqrt{\frac{29,000}{50}} = 90.6, \qquad \lambda = \frac{h}{t_w} = 43.6 < \lambda_p \ \therefore$ web is compact, and the shape is compact. (Also, there is no footnote in the dimensions and properties table to indicate otherwise.)

For a W21 × 68, $A = 20.0$ in.^{2}, $d = 21.1$ in., $t_f = 0.685$ in., $S_x = 140$ in.^{3}, $Z_x = 160$ in.^{3}, $I_y = 64.7$ in.^{4}, $r_y = 1.80$ in., $r_{ts} = 2.17$ in., $h_o = 20.4$ in., $J = 2.45$ in.^{4}, $C_w = 6760$ in.^{6}

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76(1.80)\sqrt{\frac{29{,}000}{50}} = 76.30 \text{ in.} = 6.358 \text{ ft.}$$

For a doubly-symmetric I-shape, $c = 1.0$. From AISC Equation F2-6,

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_x h_o}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7F_y S_x h_o}{EJc}\right)^2}}$$

$$= 1.95(2.17)\frac{29{,}000}{0.7(50)}\sqrt{\frac{2.45(1.0)}{140(20.4)}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7(50)(140)(20.4}{29{,}000(2.45)(1.0)}\right)^2}}$$

$$= 224.8 \text{ in.} = 18.73 \text{ ft}$$

Lateral support is provided at intervals of $50/10 = 5.0$ ft, (at the fifth points). From Table 3-1 in Part 3 of the *Manual*, $C_b = 1.0$.

For $L_b = 10$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

$$M_p = F_y Z_x = 50(160) = 8000 \text{ in.-kips}$$

$$M_n = 1.0\left[8000 - (8000 - 0.7 \times 50 \times 140)\left(\frac{10 - 6.358}{18.73 - 6.358}\right)\right].$$

$$= 7078 \text{ in.-kips} = 589.8 \text{ ft-kips}$$

(a) Let $M_u = \phi_b M_n$:

$\frac{w_u(50)^2}{8} = 0.90(589.8)$, Solution is: $\{w_u = 1.699\}$

$1.2w_D + 1.6w_L = 1.2(0.068) + 1.6w_L = 1.669$, Solution is: $\{w_L = 0.9921\}$

$\underline{w_L = 0.992 \text{ kips/ft}}$

(b) Let $M_a = \frac{M_n}{\Omega_b}$:

$\frac{w_a(50)^2}{8} = \frac{589.8}{1.67}$, Solution is: $\{w_a = 1.130\}$

If the beam weight is deducted, the maximum service load is

$1.130 - 0.068 = 1.06$ kips/ft $\qquad \underline{w_L = 1.06 \text{ kips/ft}}$

5.5-13

Verify that this shape is compact. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15, \qquad \lambda = \frac{b_f}{2t_f} = 6.57 < \lambda_p \;\therefore \text{ flange is compact}$$

For the web, $\lambda_p = 3.76\sqrt{\frac{29,000}{50}} = 90.6, \qquad \lambda = \frac{h}{t_w} = 39.6 < \lambda_p$ $\therefore$ web is compact, and the shape is compact. (Also, there is no footnote in the dimensions and properties table to indicate otherwise.)

For a W14 × 38, $A = 11.2$ in.^{2}, $d = 14.1$ in., $t_f = 0.515$ in., $S_x = 54.6$ in.^{3}, $I_y = 26.7$ in.^{4}, $r_y = 1.55$ in., $J = 0.798$ in.^{4}, $C_w = 1230$ in.^{6}

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}} = 1.76(1.55)\sqrt{\frac{29,000}{50}} = 65.70 \text{ in.} = 5.475 \text{ ft.}$$

The following terms will be needed in the computation of L_r :

$$r_{ts}^2 = \frac{\sqrt{I_yC_w}}{S_x} = \frac{\sqrt{26.7(1230)}}{54.6} = 3.319 \text{ in.}^2$$

$$r_{ts} = \sqrt{3.319} = 1.822 \text{ in.}$$

(r_{ts} can also be found in the dimensions and properties tables. For a W14 × 38, it is given as 1.82 in.)

$$h_o = d - t_f = 14.1 - 0.515 = 13.59 \text{ in.}$$

(h_o can also be found in the dimensions and properties tables. For a W14 × 38, it is given as 13.6 in.)

For a doubly-symmetric I-shape, $c = 1.0$. From AISC Equation F2-6,

$$L_r = 1.95r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_xh_o}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7F_yS_xh_o}{EJc}\right)^2}}$$

$$= 1.95(1.822)\frac{29,000}{0.7(50)}\sqrt{\frac{0.798(1.0)}{54.6(13.59)}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7(50)(54.6)(13.59}{29,000(0.798)(1.0)}\right)^2}}$$

$$= 195.1 \text{ in.} = 16.26 \text{ ft}$$

. The unbraced length is14 ft. For $L_b = 14$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

From Figure 5.15, $C_b = 1.32$

$$M_p = F_yZ_x = 50(61.5) = 3075 \text{ in.-kips} = 256 \text{ ft-kips}$$

$$M_n = 1.32\left[3075 - (3075 - 0.7 \times 50 \times 54.6)\left(\frac{14 - 5.475}{16.26 - 5.475}\right)\right]$$

$$= 2844 \text{ in.-kips} = 237.0 \text{ ft-kips} < M_p$$

(a) LRFD solution:

$$\phi_bM_n = 0.90(237.0) = 213 \text{ ft-kips}$$

$$M_u = \frac{1}{8}w_uL^2 + \frac{P_uL}{4} = \frac{1}{8}(1.2 \times 0.038)(14)^2 + \frac{1.6(30)(14)}{4}$$

$= 169$ ft-kips < 213 ft-kips (OK) W14 × 38 is adequate.

(b) ASD solution:

$$\frac{M_n}{\Omega_b} = \frac{237.0}{1.67} = 141.9 \text{ ft-kips}$$

$$M_a = \frac{1}{8}w_aL^2 + \frac{P_aL}{4} = \frac{1}{8}(0.038)(14)^2 + \frac{30(14)}{4}$$

$= 106$ ft-kips < 142 ft-kips (OK) W14 × 38 is adequate.

5.5-14.

All channel shapes in the Manual are compact (there are no footnotes to indicate otherwise).

For an MC18 × 58, $A = 17.1$ in.^{2}, $d = 18.0$ in., $t_f = 0.625$ in., $r_{ts} = 1.35$ in., $h_o = 17.4$ in., $S_x = 75.0$ in.^{3}, $Z_x = 95.4$ in.^{3}, $I_y = 17.6$ in.^{4}, $r_y = 1.02$ in., $J = 2.81$ in.^{4}, $C_w = 1070$ in.^{6}

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}} = 1.76(1.02)\sqrt{\frac{29{,}000}{36}} = 50.95 \text{ in.} = 4.246 \text{ ft.}$$

The following terms will be needed in the computation of L_r :

$$r_{ts}^2 = \frac{\sqrt{I_yC_w}}{S_x} = \frac{\sqrt{17.6(1070)}}{75.0} = 1.830 \text{ in.}^2$$

$$r_{ts} = \sqrt{1.830} = 1.353 \text{ in.}$$

(r_{ts} can also be found in the dimensions and properties tables. For an MC18 × 58, it is given as 1.35 in.)

$$h_o = d - t_f = 18.0 - 0.625 = 17.38 \text{ in.}$$

(h_o can also be found in the dimensions and properties tables. For an MC18 × 58, it is given as 17.4 in.)

For channels, from AISC Equation F2-8b,

$$c = \frac{h_o}{2}\sqrt{\frac{I_y}{C_w}} = \frac{17.38}{2}\sqrt{\frac{17.6}{1070}} = 1.115$$

From AISC Equation F2-6,

$$L_r = 1.95r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_xh_o}}\sqrt{1 + \sqrt{1 + 6.76\left(\frac{0.7F_yS_xh_o}{EJc}\right)^2}}$$

$$= 1.95(1.353)\frac{29{,}000}{0.7(36)}\sqrt{\frac{2.81(1.115)}{75.0(17.38)}}\sqrt{1 + \sqrt{1 + 6.76\left(\frac{0.7(36)(75.0)(17.38)}{29{,}000(2.81)(1.115)}\right)^2}}$$

$$= 229.3 \text{ in.} = 19.11 \text{ ft}$$

The unbraced length is14 ft. For $L_b = 14$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p \qquad \text{(inelastic LTB)}$$

From Figure 5.15, $C_b = 1.32$

$$M_p = F_yZ_x = 36(95.4) = 3434 \text{ in.-kips} = 286 \text{ ft-kips}$$

$$M_n = 1.32\left[3434 - (3434 - 0.7 \times 36 \times 75.0)\left(\frac{14 - 4.246}{19.11 - 4.246}\right)\right] = 3195.$$

$$= 3195 \text{ in.-kips} = 266.3 \text{ ft-kips} < M_p$$

(a) LRFD solution:

$$\phi_bM_n = 0.90(266.3) = 240 \text{ ft-kips}$$

$$M_u = \frac{1}{8}w_uL^2 + \frac{P_uL}{4} = \frac{1}{8}(1.2 \times 0.058)(14)^2 + \frac{1.6(30)(14)}{4}$$

$= 170$ ft-kips < 240 ft-kips (OK) An MC18 × 58 is adequate.

(b) ASD solution:

$$\frac{M_n}{\Omega_b} = \frac{266.3}{1.67} = 160 \text{ ft-kips}$$

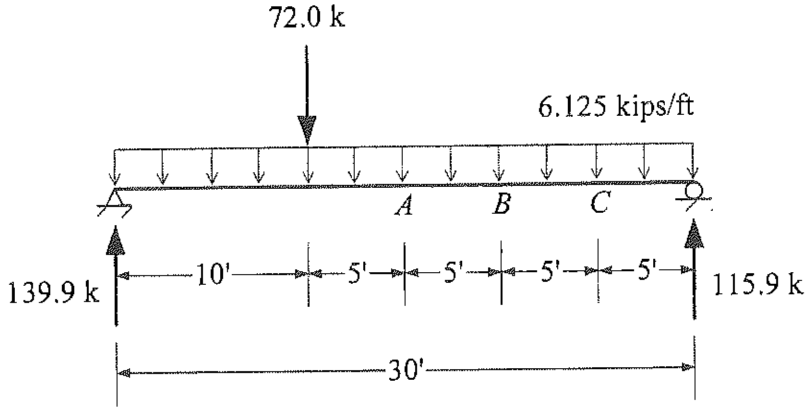
$$M_a = \frac{1}{8}w_aL^2 + \frac{P_aL}{4} = \frac{1}{8}(0.058)(14)^2 + \frac{30(14)}{4}$$

$= 106$ ft-kips < 160 ft-kips (OK) An MC18 × 58 is adequate.

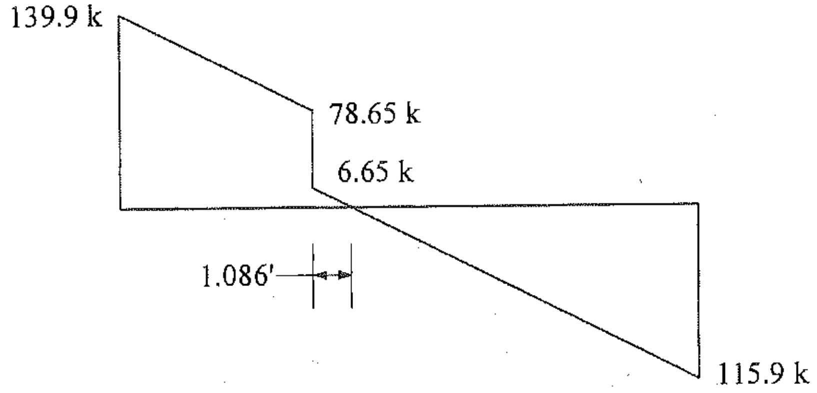
5.5-15

(a) LRFD solution: $P_u = 1.2P_D + 1.6P_L = 1.2(12) + 1.6(36) = 72.0$ kips

$$w_u = 1.2w_D + 1.6w_L = 1.2(1 + 0.104) + 1.6(3) = 6.125 \text{ kips/ft}$$



(Note that points have been relabeled for use in the C_b formula.)



$$M_u = 139.9(11.086) - 6.125(11.086)^2/2 - 72.0(1.086) = 1096 \text{ ft-kips}$$

Verify that this shape is compact. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29,000}{50}} = 9.15, \qquad \lambda = \frac{b_f}{2t_f} = 8.50 < \lambda_p \ \therefore \text{ flange is compact}$$

For the web, $\lambda_p = 3.76\sqrt{\frac{29,000}{50}} = 90.6, \qquad \lambda = \frac{h}{t_w} = 43.1 < \lambda_p \ \therefore$ web is compact, and the shape is compact.

For a W24 × 104, $A = 30.6 \text{ in.}^2$, $S_x = 258 \text{ in.}^3$, $Z_x = 289 \text{ in.}^3$, $I_y = 259 \text{ in.}^4$, $r_y = 2.91$ in., $J = 4.72 \text{ in.}^4$, $C_W = 35,200 \text{ in.}^6$

$$L_p = 1.76 r_y\sqrt{\frac{E}{F_y}} = 1.76(2.91)\sqrt{\frac{29,000}{50}} = 123.3 \text{ in.} = 10.28 \text{ ft.}$$

The following terms will be needed in the computation of L_r :

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} = \frac{\sqrt{259(35200)}}{258} = 11.70 \text{ in.}^2$$

$$r_{ts} = \sqrt{11.70} = 3.421 \text{ in.}$$

(r_{ts} can also be found in the dimensions and properties tables. For a W24 × 104, it is given as 3.42 in.)

$$h_o = d - t_f = 24.1 - 0.750 = 23.35 \text{ in.}$$

(h_o can also be found in the dimensions and properties tables. For a W24 × 104, it is given as 23.3 in.)

For a doubly-symmetric I-shape, $c = 1.0$. From AISC Equation F2-6,

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o}} \sqrt{1 + \sqrt{1 + 6.76\left(\frac{0.7F_y S_x h_o}{EJc}\right)^2}}$$

$$= 1.95(3.421)\frac{29,000}{0.7(50)} \sqrt{\frac{4.72(1.0)}{258(23.35)}} \sqrt{1 + \sqrt{1 + 6.76\left(\frac{0.7(50)(258)(23.35}{29,000(4.72)(1.0)}\right)^2}}$$

$$= 350.4 \text{ in.} = 29.20 \text{ ft}$$

For $L_b = 20$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

Compute C_b : $M_{\max} = M_u = 1096$ ft-kips

$$M_A = 115.9(15) - 6.125(15)^2/2 = 1049 \text{ ft-kips}$$

$$M_B = 115.9(10) - 6.125(10)^2/2 = 852.8 \text{ ft-kips}$$

$$M_C = 115.9(5) - 6.125(5)^2/2 = 502.9 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(1096)}{2.5(1096) + 3(1049) + 4(852.8) + 3(502.9)} = 1.268$$

$$M_p = F_y Z_x = 50(289) = 14,450 \text{ in.-kips}$$

$$M_n = 1.268\left[14,450 - (14,450 - 0.7 \times 50 \times 258)\left(\frac{20 - 10.28}{29.20 - 10.28}\right)\right]$$

$$= 14,790 \text{ in.-kips} > M_p$$

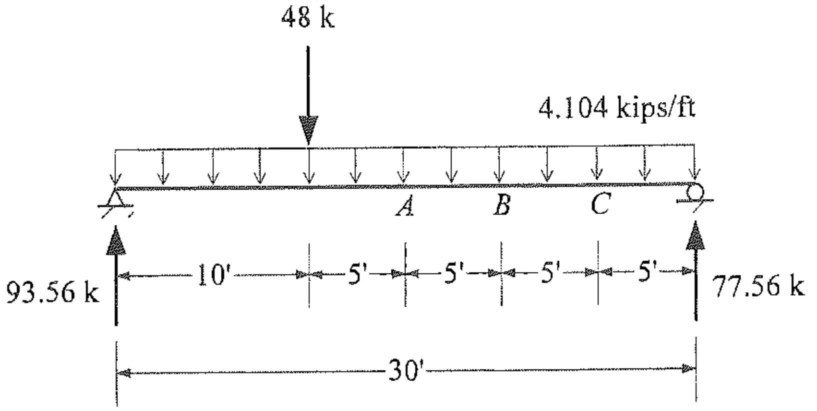
Therefore, use $M_n = M_p = 14,450$ in.-kips and

$$\phi_b M_n = 0.9(14450) = 13,010 \text{ in.-kips} = 1084 \text{ ft-kips}$$

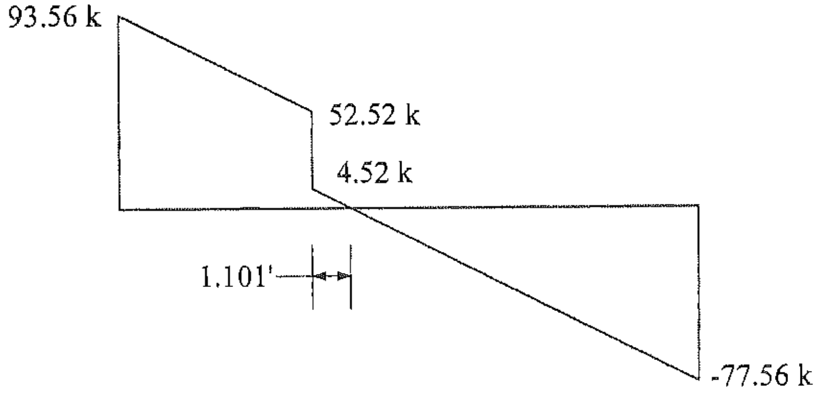
Since $M_u = 1096$ ft-kips $> \phi_b M_n = 1084$ ft-kips, W24 × 104 is not adequate.

(b) ASD solution:

$$P_a = P_D + P_L = 12 + 36 = 48 \text{ kips}, \quad w_a = w_D + w_L = (1 + 0.104) + 3 = 4.104 \text{ kips/ft}$$



(Note that points have been relabeled for use in the C_b formula.)



From the LRFD solution, the shape is compact. Also from the LRFD solution,

$L_p = 10.28$ ft., $L_r = 29.20$ ft

For $L_b = 20$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

Compute C_b : $M_{\max} == 732.9$ ft-kips

$M_A = 77.56(15) - 4.104(15)^2/2 = 701.7$ ft-kips

$M_B = 77.56(10) - 4.104(10)^2/2 = 570.4$ ft-kips

$M_C = 77.56(5) - 4.104(5)^2/2 = 336.5$ ft-kips

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(732.9)}{2.5(732.9) + 3(701.7) + 4(570.4) + 3(336.5)} = 1.267$$

(Note that this is essentially the same as the value found in Part (a) with factored loads.)

$$M_p = F_y Z_x = 50(289) = 14,450 \text{ in.-kips}$$

$$M_n = 1.267\left[14,450 - (14,450 - 0.7 \times 50 \times 258)\left(\frac{20 - 10.28}{29.20 - 10.28}\right)\right]$$

$$= 14,780 \text{ in.-kips} > M_p$$

Therefore, use $M_n = M_p = 14,450$ in.-kips and

$$\frac{M_n}{\Omega_b} = \frac{14,450}{1.67} = 8653 \text{ in.-kips} = 721 \text{ ft-kips}$$

Since $M_a = M_{\max} = 732.9$ ft-kips > 721 ft-kips, W24 × 104 is not adequate.

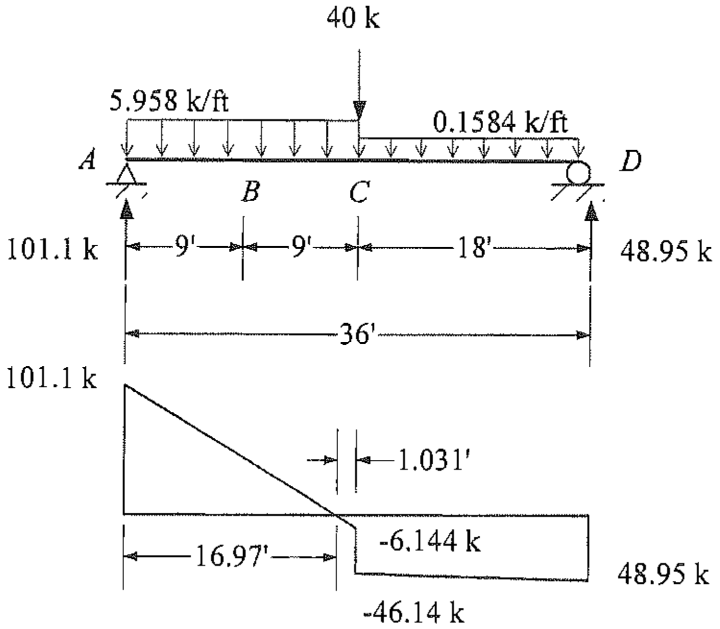
5.5-16

(a) For segment ABC, when beam weight is included, $w_D = 3.5 + 0.132 = 3.632$ kips/ft

$$w_u = 1.2w_D + 1.6w_L = 1.2(3.632) + 1.6(1.0) = 5.958 \text{ kips/ft}$$

For segment CD, $w_u = 1.2(0.132) = 0.1584$ kips/ft

$$P_u = 1.6P_L = 1.6(25) = 40.0 \text{ kips}$$



Maximum moment $= \frac{1}{2}(101.1)(16.97) = 858$ ft-kips

Verify that this shape is compact. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29,000}{50}} = 9.15, \qquad \lambda = \frac{b_f}{2t_f} = 7.15 < \lambda_p \quad \therefore \text{ flange is compact}$$

For the web, $\lambda_p = 3.76\sqrt{\frac{29,000}{50}} = 90.6, \qquad \lambda = \frac{h}{t_w} = 17.7 < \lambda_p$ ∴ web is compact, and the shape is compact.

For a W14 × 132, $A = 38.8 \text{ in.}^2$, $d = 14.7$ in., $t_f = 1.03$ in., $S_x = 209 \text{ in.}^3$, $Z_x = 234 \text{ in.}^3$, $I_y = 548 \text{ in.}^4$, $r_y = 3.76$ in., $r_{ts} = 4.23$ in., $h_o = 13.6$ in., $J = 12.3 \text{ in.}^4$, $C_w = 25{,}500 \text{ in.}^6$

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}} = 1.76(3.76)\sqrt{\frac{29{,}000}{50}} = 159.4 \text{ in.} = 13.28 \text{ ft.}$$

For a doubly-symmetric I-shape, $c = 1.0$. From AISC Equation F2-6,

$$L_r = 1.95r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_xh_o}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7F_yS_xh_o}{EJc}\right)^2}}$$

$$= 1.95(4.23)\frac{29{,}000}{0.7(50)}\sqrt{\frac{12.3(1.0)}{209(13.6)}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7(50)(209)(13.6}{29{,}000(12.3)(1.0)}\right)^2}}$$

$$= 672.2 \text{ in.} = 56.02 \text{ ft}$$

Check unbraced segment BC. For $L_b = 9$ ft, $L_b < L_p$ and

$$M_n = M_p = F_yZ_x = 50(234) = 11{,}700 \text{ in.-kips} = 975 \text{ ft-kips}$$

$$\phi_bM_n = 0.9(11700) = 10{,}530 \text{ in.-kips} = 878 \text{ ft-kips} > 858 \text{ ft-kips} \qquad \text{(OK)}$$

Check unbraced segment CD : For $L_b = 18$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

Compute C_b. Dividing CD into 4 equal segments of length $18/4 = 4.5$ ft and labeling the 3 interior points a, b, and c, we obtain

$$M_a = 48.95(13.5) - 0.1584(13.5)^2/2 = 646.4 \text{ ft-kips}$$

$$M_b = 48.95(9) - 0.1584(9)^2/2 = 434.1 \text{ ft-kips}$$

$$M_c = 48.95(4.5) - 0.1584(4.5)^2/2 = 218.7 \text{ ft-kips}$$

$$M_{\max} = 48.95(18) - 0.1584(18)^2/2 = 855.4 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_a + 4M_b + 3M_c}$$

$$= \frac{12.5(855.4)}{2.5(855.4) + 3(646.4) + 4(434.1) + 3(218.7)} = 1.653$$

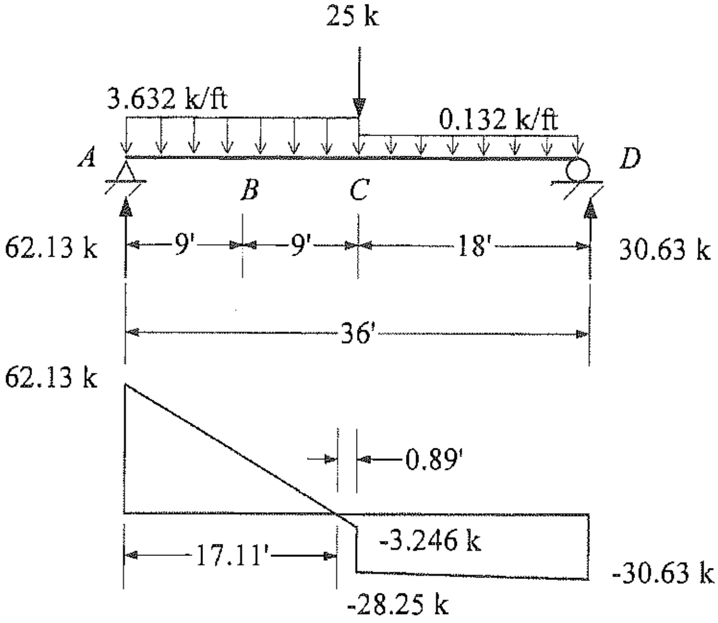
$$M_n = 1.653\left[11700 - (11700 - 0.7 \times 50 \times 209)\left(\frac{18 - 13.28}{56.02 - 13.28}\right)\right]$$

$$= 1.854 \times 10^4 \text{ in.-kips} = 1545 \text{ ft-kips} > M_p$$

$$\therefore\ \phi_bM_n = \phi_bM_p = 878 \text{ ft-kips} > 855 \text{ ft-kips} \qquad \text{(OK)}$$

W14 × 132 is adequate.

(b) For segment ABC, when beam weight is included, $w_D = 3.5 + 0.132 = 3.632$ kips/ft



From Part (a), $L_p = 13.28$ ft, $L_r = 56.02$ ft, $M_p = 11{,}700$ in.-kips = 975.0 ft-kips.

Check unbraced segment BC. For $L_b = 9$ ft, $L_b < L_p$ and

$$M_n = M_p = 975 \text{ ft-kips}$$

$$\frac{M_n}{\Omega_b} = \frac{975}{1.67} = 584 \text{ ft-kips}$$

Maximum moment $= 0 + \frac{1}{2}(17.11)(62.13) = 532$ ft-kips < 584 ft-kips (OK)

Check unbraced segment CD : For $L_b = 18$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

Compute C_b. Dividing CD into 4 equal segments of length 18/4 = 4.5 ft and labeling the 3 interior points a, b, and c, we obtain

$$M_a = 30.63(13.5) - 0.132(13.5)^2/2 = 401.5 \text{ ft-kips}$$

$$M_b = 30.63(9) - 0.132(9)^2/2 = 270.3 \text{ ft-kips}$$

$$M_c = 30.63(4.5) - 0.132(4.5)^2/2 = 136.5 \text{ ft-kips}$$

$$M_{\max} = 30.63(18) - 0.132(18)^2/2 = 530.0 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_a + 4M_b + 3M_c}$$

$$= \frac{12.5(530.0)}{2.5(530.0) + 3(401.5) + 4(270.3) + 3(136.5)} = 1.648$$

$$M_n = 1.648\left[11700 - (11700 - 0.7 \times 50 \times 209)\left(\frac{18 - 13.28}{56.02 - 13.28}\right)\right]$$

$$= 1.848 \times 10^4 \text{ in.-kips} = 1540 \text{ ft-kips} > M_p$$

$$\therefore\ M_n = M_p = 975 \text{ ft-kips}$$

$$\frac{M_n}{\Omega_b} = \frac{975}{1.67} = 584 \text{ ft-kips} > 530 \text{ ft-kips} \qquad \text{(OK)}$$

W14 × 132 is adequate.

5.6-1

Check for compactness. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29{,}000}{50}} = 9.152, \qquad \lambda_r = 1.0\sqrt{\frac{29{,}000}{50}} = 24.08$$

$$\lambda = \frac{b_f}{2t_f} = 9.92, \qquad \lambda_p < \lambda < \lambda_r \ \therefore \text{ flange is noncompact}$$

(In the Dimensions and Properties tables, there is a footnote indicating that the flange of a W12 × 65 is noncompact.) The web is compact for all shapes in Part 1 of the *Manual* for $F_y \leq 65$ ksi.

Compute the strength based on the limit state of flange local buckling.

$$M_p = F_y Z_x = 50(96.8) = 4840 \text{ in.-kips}$$

$$M_n = M_p - (M_p - 0.7F_yS_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

$$= 4840 - (4840 - 0.7 \times 50 \times 87.9)\left(\frac{9.92 - 9.152}{24.08 - 9.152}\right)$$

$$= 4749 \text{ in.-kips} = 395.8 \text{ ft-kips}$$

(a) Available strength = design strength = $\phi_b M_n = 0.90(395.8) = 356.2$ ft-kips

Factored load moment:

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}w_u(50)^2 = 356.2 \qquad \Rightarrow \qquad w_u = 1.140 \text{ kips/ft}$$

$$1.2w_D + 1.6w_L = 1.2w_D + 1.6(3w_D) = 1.140 \qquad \Rightarrow \qquad w_D = 0.1900 \text{ kips/ft}$$

$$w = w_D + w_L = 0.19 + 3(0.19) = 0.760 \text{ kips/ft}$$

$w = 0.760$ kips/ft

(b) Available strength = allowable strength = $\dfrac{M_n}{\Omega_b} = \dfrac{395.8}{1.67} = 237.0$ ft-kips

$$M_a = \frac{1}{8}w_aL^2 = \frac{1}{8}w_a(50)^2 = 237.0 \qquad \Rightarrow \qquad w_a = 0.7584 \text{ kips/ft}$$

$w = 0.758$ kips/ft

5.6-2

Check for compactness. For the flange,

$$\lambda_p = 0.38\sqrt{\frac{29,000}{50}} = 9.152, \qquad \lambda_r = 1.0\sqrt{\frac{29,000}{50}} = 24.08$$

$$\lambda = \frac{b_f}{2t_f} = 9.34, \qquad \lambda_p < \lambda < \lambda_r \;\therefore \text{ flange is noncompact}$$

(In the Dimensions and Properties tables, there is a footnote indicating that the flange of a W14 × 99 is noncompact.) The web is compact for all shapes in Part 1 of the *Manual* for $F_y \le 65$ ksi.

Compute the strength based on the limit state of flange local buckling.

$$M_p = F_y Z_x = 50(173) = 8650 \text{ in.-kips}$$

$$M_n = M_p - (M_p - 0.7F_y S_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

$$= 8650 - (8650 - 0.7 \times 50 \times 157)\left(\frac{9.34 - 9.152}{24.08 - 9.152}\right)$$

$$= 8610 \text{ in.-kips} = 717.5 \text{ ft-kips}$$

Check for lateral-torsional buckling. From the Z_x table, $L_p = 13.5$ ft. Since $L_b = 10$ ft < L_b, lateral-torsional buckling does not have to be investigated.

$M_n = 718$ ft-kips

5.6-3

Verify that the shape is noncompact: For the flange, $\lambda = \frac{b_f}{2t_f} = \frac{18}{2(3/4)} = 12.0$

$$\lambda_p = 0.38\sqrt{\frac{29,000}{50}} = 9.152$$

$$\lambda_r = 0.95\sqrt{\frac{k_c E}{F_L}}, \qquad k_c = \frac{4}{\sqrt{h/t_w}} \qquad (0.35 \le k_c \le 0.763)$$

$$k_c = \frac{4}{\sqrt{52/0.75}} = 0.4804, \qquad F_L = 0.7F_y \text{ (see footnote b in AISC Table B4.1)}$$

$$\lambda_r = 0.95\sqrt{\frac{0.4804(29,000)}{0.7(50)}} = 18.95$$

$$\lambda_p < \lambda < \lambda_r \;\therefore \text{ flange is noncompact}$$

For the web, $\lambda = \frac{h}{t_w} = \frac{52}{3/4} = 69.3 < \lambda_p = 3.76\sqrt{\frac{29,000}{50}} = 90.55$ ∴ web is compact.

Compute properties of the cross section.

Flange area $= 0.75(18) = 13.5$ in.^{2}, Half-web area $= 0.75(52/2) = 19.5$ in.^{2}

Taking moments about the PNA (located at mid-depth of the cross section),

$$\bar{y} = \frac{19.5(13) + 13.5(26.38)}{19.5 + 13.5} = 18.47 \text{ in.}$$

$$Z_x = \frac{A}{2} \cdot a = (19.5 + 13.5)(2 \times 18.47) = 1219 \text{ in.}^3$$

$$M_p = F_y Z_x = 50(1219) = 60,950 \text{ in.-kips}$$

$$I_x = \frac{1}{12}(0.75)(52)^3 + 2\left[\frac{1}{12}(18)(0.75)^3 + 13.5(26.38)^2\right] = 27,580 \text{ in.}^3$$

$$S_x = \frac{I_x}{c} = \frac{27,580}{52/2 + 3/4} = 1031 \text{ in.}^3$$

$$M_n = M_p - (M_p - 0.7F_yS_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

$$= 60,950 - (60,950 - 0.7 \times 50 \times 1031)\left(\frac{12.0 - 9.152}{18.95 - 9.152}\right)$$

$$= 53,720 \text{ in.-kips} = 4480 \text{ ft-kips} \qquad \underline{M_n = 4480 \text{ ft-kips}}$$

5.6-4

Because the beam has continuous lateral support, lateral-torsional buckling does not need to be checked. Verify that the shape is noncompact: For the flange,

$$\lambda = \frac{b_f}{2t_f} = \frac{16}{2(3/4)} = 10.67, \qquad \lambda_p = 0.38\sqrt{\frac{29,000}{50}} = 9.152$$

$$\lambda_r = 0.95\sqrt{\frac{k_cE}{F_L}}, \qquad k_c = \frac{4}{\sqrt{h/t_w}} \qquad (0.35 \le k_c \le 0.763)$$

$$k_c = \frac{4}{\sqrt{40/(1/2)}} = 0.4472, \qquad F_L = 0.7F_y \text{ (see footnote b in AISC Table B4.1)}$$

$$\lambda_r = 0.95\sqrt{\frac{0.4472(29,000)}{0.7(50)}} = 18.29$$

$\lambda_p < \lambda < \lambda_r \;\therefore$ flange is noncompact.

For the web, $\lambda = \frac{h}{t_w} = \frac{40}{1/2} = 80, \qquad \lambda_p = 3.76\sqrt{\frac{29,000}{50}} = 90.55$

Since $\lambda < \lambda_p$, the web is compact.

Compute properties of the cross section.

Flange area = $0.75(16) = 12.0$ in.^{2}, Half-web area = $0.5(0.5 \times 40) = 10.0$ in.^{2}

Taking moments about the PNA (located at mid-depth of the cross section),

$$\bar{y} = \frac{12.0(20 + 3/8) + 10.0(10)}{12.0 + 10.0} = 15.66 \text{ in.}$$

$$Z_x = \frac{A}{2} \cdot a = (12.0 + 10.0)(2 \times 15.66) = 689.0 \text{ in.}^3$$

$$M_p = F_y Z_x = 50(689.0) = 34,450 \text{ in.-kips}$$

$$I_x = \frac{1}{12}(0.5)(40)^3 + 2\left[\frac{1}{12}(16)(0.75)^3 + 12.0(20 + 3/8)^2\right] = 12,630 \text{ in.}^3$$

$$S_x = \frac{I_x}{c} = \frac{12630}{20 + 3/4} = 608.7 \text{ in.}^3$$

$$M_n = M_p - (M_p - 0.7F_y S_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

$$= 34450 - (34450 - 0.7 \times 50 \times 608.7)\left(\frac{10.67 - 9.152}{18.29 - 9.152}\right)$$

$$= 32,270 \text{ in.-kips} = 2690 \text{ ft-kips} \qquad \underline{M_n = 2690 \text{ ft-kips}}$$

5.8-1

$$\frac{h}{t_w} = 71.0, \qquad 2.24\sqrt{\frac{E}{F_y}} = 2.24\sqrt{\frac{29,000}{65}} = 47.31$$

Since $\frac{h}{t_w} > 2.24\sqrt{\frac{E}{F_y}}$, AISC Section G2.1(b) applies.

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{5(29,000)}{65}} = 51.95$$

$$1.37\sqrt{\frac{k_v E}{F_y}} = 1.37\sqrt{\frac{5(29,000)}{65}} = 64.71$$

$$\frac{h}{t_w} > 64.71, \therefore C_v = \frac{1.51Ek_v}{(h/t_w)^2 F_y} = \frac{1.51(29,000)(5)}{(71.0)^2(65)} = 0.6682$$

$$V_n = 0.6F_y A_w C_v = 0.6(65)(9.99 \times 0.130)(0.6682) = 33.84 \text{ kips}$$

$$\underline{V_n = 33.8 \text{ kips}}$$

5.8-2

For an M12 × 11.8 ($t_f = 0.225$ in.) of A242 steel, $F_y = 50$ ksi

$$\frac{h}{t_w} = 62.5, \qquad 2.24\sqrt{\frac{E}{F_y}} = 2.24\sqrt{\frac{29,000}{50}} = 53.95$$

Since $\frac{h}{t_w} > 2.24\sqrt{\frac{E}{F_y}}$, AISC Section G2.1(b) applies.

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{5(29,000)}{50}} = 59.24$$

$$1.37\sqrt{\frac{k_vE}{F_y}} = 1.37\sqrt{\frac{5(29,000)}{50}} = 73.78$$

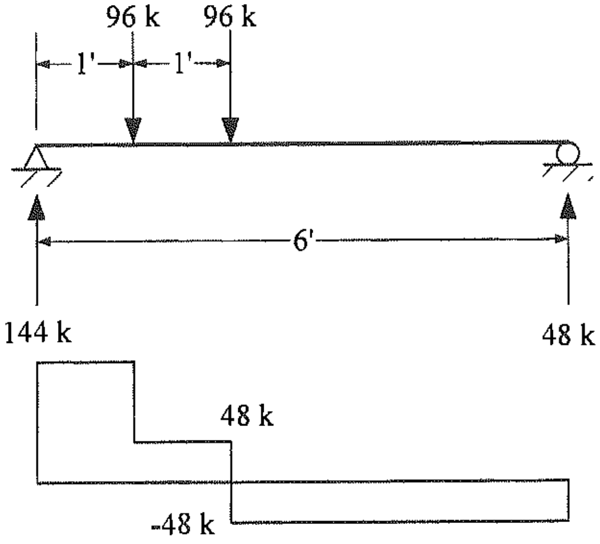
$$59.24 < \frac{h}{t_w} < 73.78, \ \therefore C_v = \frac{1.10\sqrt{\frac{k_vE}{F_y}}}{h/t_w} = \frac{59.24}{62.5} = 0.9478$$

$$V_n = 0.6F_yA_wC_v = 0.6(50)(12.0 \times 0.177)(0.9478) = 60.39 \text{ kips}$$

$\underline{V_n = 60.4 \text{ kips}}$

5.8-3

(a) $P_u = 1.6P_L = 1.6(60) = 96$ kips



$M_u = 144(2) - 96(1) = 192$ ft-kip

From the Z_x table, $\phi_bM_n = \phi_bM_p = 203$ ft-kips > 192 ft-kips (OK)

Check shear:

$$\frac{h}{t_w} = 51.6 < 2.24\sqrt{\frac{E}{F_y}} = 2.24\sqrt{\frac{29,000}{50}} = 53.95$$

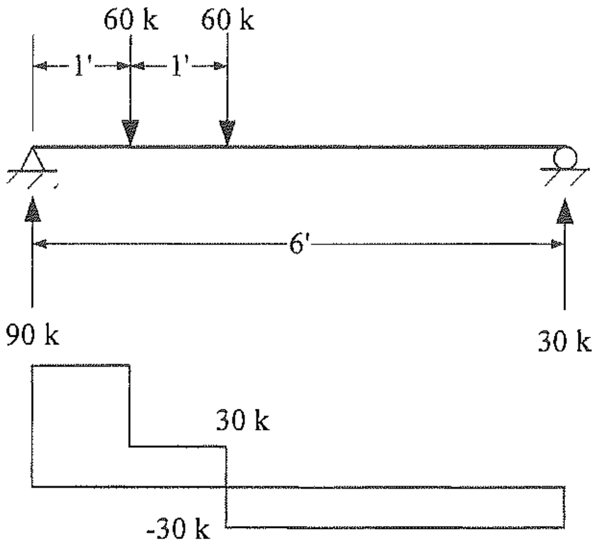
$\therefore V_n = 0.6F_yA_w = 0.6(50)(15.9 \times 0.275) = 131.2$ kips

$\phi_vV_n = 1.0(131.2) = 131$ ksi

$V_u = 144$ kips > 131 kips (N.G.) Beam not adequate

(ϕ_vV_n can also be found in the Z_x table.)

(b) $P_a = 60$ kips



$M_a = 90(2) - 60(1) = 120$ ft-kip

From the Z_x table, $M_n/\Omega_b = M_p/\Omega_b = 135$ ft-kips > 120 ft-kips (OK)

Check shear:

$$\frac{h}{t_w} = 51.6 < 2.24\sqrt{\frac{E}{F_y}} = 2.24\sqrt{\frac{29,000}{50}} = 53.95$$

$$\therefore\ V_n = 0.6F_yA_w = 0.6(50)(15.9 \times 0.275) = 131.2 \text{ kips}$$

$$\frac{V_n}{\Omega_b} = \frac{131.2}{1.50} = 87.5 \text{ ksi}$$

$V_a = 90$ kips > 87.5 kips (N.G.) Beam not adequate

(V_n/Ω_b can also be found in the Z_x table.)

5.8-4

Nominal shear strength: $\frac{h}{t_w} = 41.1 < 2.24\sqrt{\frac{E}{F_y}} = 2.24\sqrt{\frac{29,000}{50}} = 53.95$

$$\therefore\ V_n = 0.6F_yA_w = 0.6(50)(16.1 \times 0.345) = 166.6 \text{ kips}$$

$$\phi_v = 1.00,\ \ \Omega_v = 1.50$$

Nominal flexural strength: The unbraced length is $L_b = 10$ ft. From the Z_x table, a W16 × 45 is compact, $L_p = 5.55$ ft, and $L_r = 16.5$ ft.

Since $L_p < L_b < L_r$, lateral-torsional buckling must be checked.

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

For cantilevers, use $C_b = 1.0$ (AISC F1).

$$M_p = F_yZ_x = 50(82.3) = 4115 \text{ in.-kips}$$

$$M_n = 1.0\left[4115 - (4115 - 0.7 \times 50 \times 72.7)\left(\frac{10 - 5.55}{16.5 - 5.55}\right)\right]$$

$$= 3477 \text{ in.-kips} = 289.8 \text{ ft-kips}$$

(a) $P_u = 1.6P_L = 1.6(85) = 136$ kips, $\quad w_u = 1.2w_D = 1.2(0.100) = 0.12$ kips/ft

$$\phi_vV_n = 1.00(166.6) = 167 \text{ kips}$$

$$V_u = 136 + 0.12(10) = 137 \text{ kips} < 167 \text{ kips} \quad \text{(OK)}$$

$$\phi_bM_n = 0.90(289.8) = 261 \text{ ft-kips}$$

$$M_u = 136(1) + 0.12(10)(10/2) = 142 \text{ ft-kip} < 261 \text{ ft-kips} \quad \text{(OK)}$$

W16 × 45 is adequate.

(b) $\dfrac{V_n}{\Omega_v} = \dfrac{166.6}{1.50} = 111$ kips

$$V_a = 85 + 0.100(10) = 86.0 \text{ kips} < 111 \text{ kips} \quad \text{(OK)}$$

$$\frac{M_n}{\Omega_b} = \frac{289.8}{1.67} = 174 \text{ ft-kips}$$

$$M_a = 85(1) + 0.100(10)(10/2) = 90.0 \text{ ft-kip} < 174 \text{ ft-kips} \quad \text{(OK)}$$

W16 × 45 is adequate.

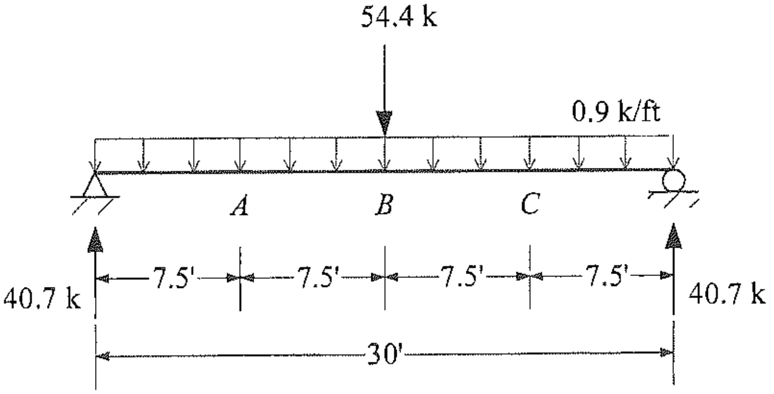
5.10-1

(a) Neglect beam weight and check it later.

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.75) = 0.9 \text{ kips/ft}$$

$$P_u = 1.2P_D + 1.6P_L = 1.6(34) = 54.4 \text{ kips}$$

$$M_u = \frac{1}{8}w_uL^2 + \frac{P_uL}{4} = \frac{1}{8}(0.9)(30)^2 + \frac{54.4(30)}{4} = 509.3 \text{ ft-kips}$$



Compute C_b : $M_{\max} = M_B = 509.3$ ft-kips

$$M_A = M_C = 40.7(7.5) - 0.9(7.5)^2/2 = 279.9 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(509.3)}{2.5(509.3) + 3(279.9) + 4(509.3) + 3(279.9)} = 1.276$$

Enter the beam design charts with $L_b = 30$ ft and $\frac{M_u}{C_b} = \frac{509.3}{1.276} = 399$ ft-kips

Try a W16 × 89

$\phi_b M_n = 410$ ft-kips for $C_b = 1.0$. For $C_b = 1.276$,

$\phi_b M_n = 1.276(410) = 523$ ft-kips > 509.3 ft-kips (OK)

$\phi_b M_p = 656$ ft-kips > 523 ft-kips (OK)

Check beam weight. $w_D = 0.75 + 0.089 = 0.839$ kips/ft

$w_u = 1.2(0.839) = 1.007$ kips/ft

$$M_u = \frac{1}{8}w_u L^2 + \frac{P_u L}{4} = \frac{1}{8}(1.007)(30)^2 + \frac{54.4(30)}{4}$$

$$= 521 \text{ ft-kips} < 523 \text{ ft-kips} \quad \text{(OK)}$$

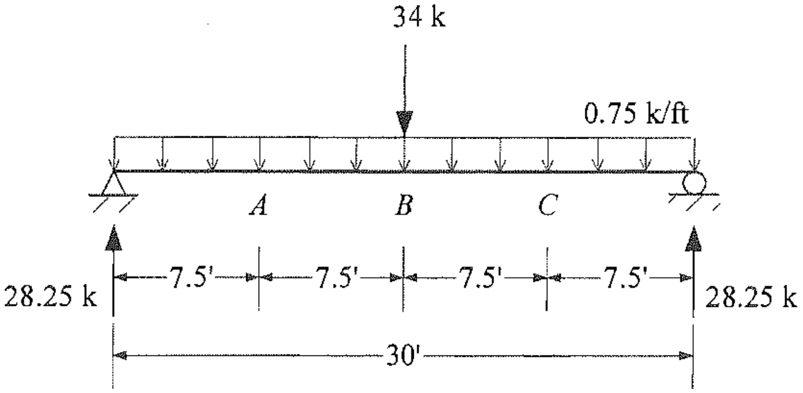
The inclusion of the beam weight changes the total load only slightly, and C_b does not need to be recomputed.

Check shear: From the Z_x table,

$\phi_v V_n = 264$ kips $> V_u \approx 40.7$ kips (OK) Use a W16 × 89

(b) Neglect beam weight and check it later.

$$M_a = \frac{1}{8}w_a L^2 + \frac{P_a L}{4} = \frac{1}{8}(0.75)(30)^2 + \frac{34(30)}{4} = 339.4 \text{ ft-kips}$$



Compute C_b : $M_{\max} = M_B = 339.4$ ft-kips

$$M_A = M_C = 28.25(7.5) - 0.75(7.5)^2/2 = 190.8 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max}+3M_A+4M_B+3M_C}$$

$$= \frac{12.5(339.4)}{2.5(339.4)+3(190.8)+4(339.4)+3(190.8)} = 1.266$$

Enter the beam design charts with $L_b = 30$ ft and $\frac{M_a}{C_b} = \frac{339.4}{1.266} = 268$ ft-kips

Try a W14 × 90

$\frac{M_n}{\Omega_b} = 311$ ft-kips for $C_b = 1.0$. For $C_b = 1.266$,

$\frac{M_n}{\Omega_b} = 1.266(311) = 394$ ft-kips > 339.4 ft-kips (OK)

$\frac{M_p}{\Omega_b} = 382$ ft-kips < 394 ft-kips $\therefore$ use $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 382$ ft-kips

Check beam weight. $w_D = 0.75 + 0.090 = 0.84$ kips/ft

$$M_a = \frac{1}{8}w_aL^2 + \frac{P_aL}{4} = \frac{1}{8}(0.84)(30)^2 + \frac{34(30)}{4} = 350 \text{ ft-kips} < 382 \text{ ft-kips (OK)}$$

The inclusion of the beam weight changes the total load only slightly, and C_b does not need to be recomputed.

Check shear: From the Z_x table,

$\frac{V_n}{\Omega_v} = 123$ kips $> V_a \approx 28.25$ kips (OK)

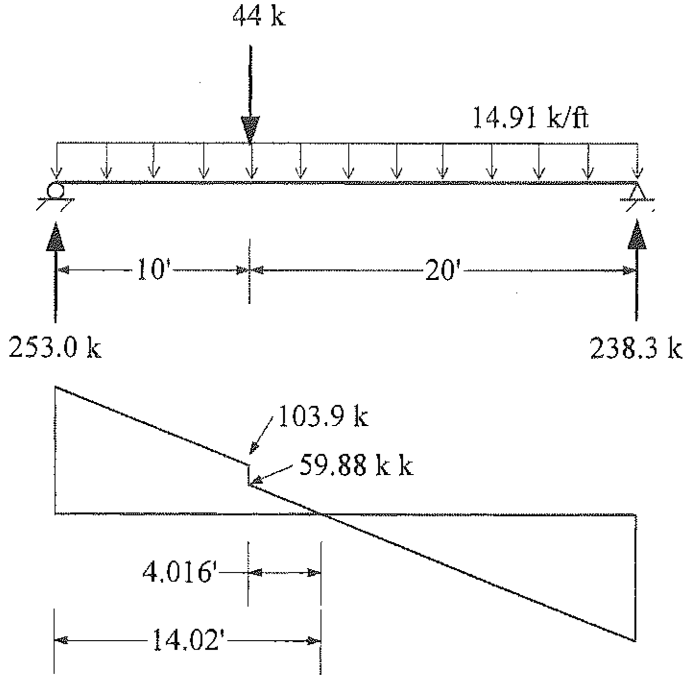
Use a W14 × 90

5.10-2

(a) Assume a beam weight of 200 lb/ft

$$w_u = 1.2w_D + 1.6w_L = 1.2(3.33+0.2) + 1.6(6.67) = 14.91 \text{ kips/ft}$$

$$P_u = 1.2P_D + 1.6P_L = 1.2(10) + 1.6(20) = 44.0 \text{ kips}$$



$M_u = 253.0(14.02) - 14.91(14.02)^2/2 - 44(4.017) = 1905$ ft-kips

(a) Try W36 × 135, $\phi_b M_n = \phi_b M_p = 1910$ ft-kips > 1905 ft-kips (OK)

Beam weight = 135 lb/ft < assumed value of 200 lb/ft (OK)

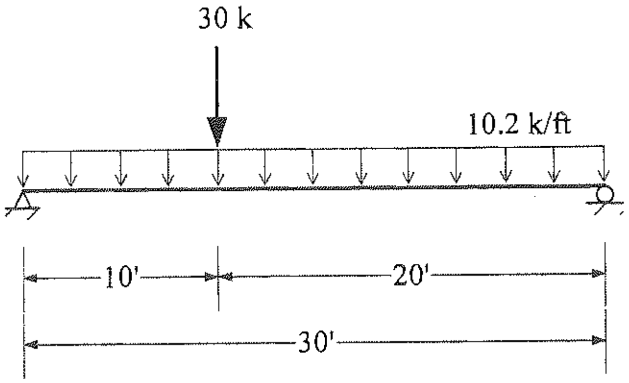
Check shear: From Z_x table,

$\phi_v V_n = 576$ kips > 253 kips (OK) Use a W36 × 135

(b) Assume a beam weight of 200 lb/ft.

$P_a = P_D + P_L = 10 + 20 = 30$ kips

$w_a = w_D + w_L = (3.33 + 0.200) + 6.67 = 10.2$ kips/ft



Take moments about right end to determine left reaction, R_L :

$$\sum M_{right} = R_L(30) - 30(20) - 10.2(30)(30/2) = 0, \qquad R_L = 173.0 \text{ kips}$$

Shear is zero when $173.0 - 30 - 10.2x = 0, \qquad x = 14.02$ ft

Maximum moment is $M_a = 173(14.02) - 30(14.02 - 10) - 10.2(14.02)^2/2 = 1302$ ft-kips

Try W40 × 149, $\dfrac{M_n}{\Omega_b} = \dfrac{M_p}{\Omega_b} = 1490$ ft-kips > 1302 ft-kips (OK)

Beam weight = 149 lb/ft < assumed value of 200 lb/ft (OK)

Check shear: From Z_x table,

$\dfrac{V_n}{\Omega_v} = 432$ kips > 173 kips (OK) Use a W40 × 149

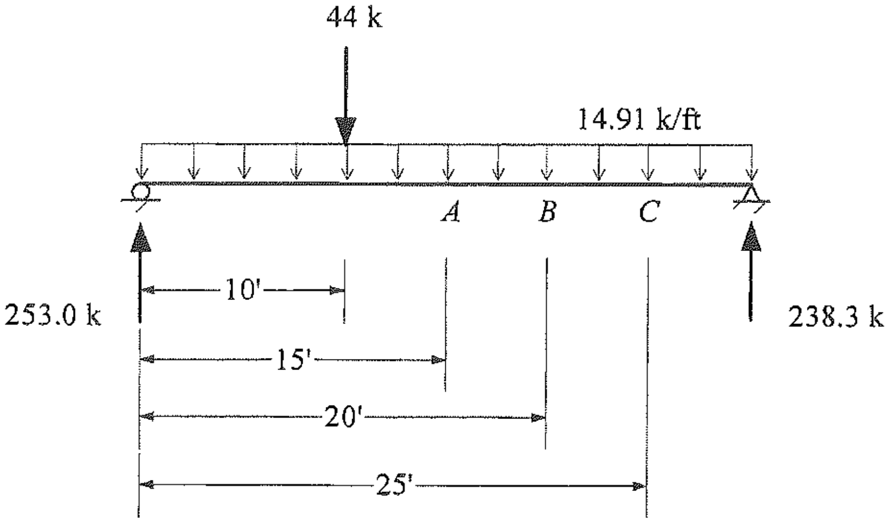
5.10-3

(a) Assume a beam weight of 200 lb/ft

$$w_u = 1.2w_D + 1.6w_L = 1.2(3.33 + 0.2) + 1.6(6.67) = 14.91 \text{ kips/ft}$$

$$P_u = 1.2P_D + 1.6P_L = 1.2(10) + 1.6(20) = 44.0 \text{ kips}$$

The 20-ft unbraced length will control. Compute C_b :



Shear is zero when $253.0 - 44 - 14.91x = 0, \qquad x = 14.02$ ft

$$M_{\max} = 253(14.02) - 44(14.02 - 10) - 14.91(14.02)^2/2 = 1905 \text{ ft-kips}$$

$$M_A = 253(15) - 14.91(15)^2/2 - 44(5) = 1898 \text{ ft-kips}$$

$$M_B = 253(20) - 14.91(20)^2/2 - 44(10) = 1638 \text{ ft-kips}$$

$$M_A = 253(25) - 14.91(25)^2/2 - 44(15) = 1006 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(1905)}{2.5(1905) + 3(1898) + 4(1638) + 3(1006)} = 1.189$$

Enter the beam design charts with $L_b = 20$ ft and $\frac{M_u}{C_b} = \frac{1905}{1.189} = 1602$ ft-kips

Try a W36 × 160

$\phi_b M_n = 1740$ ft-kips for $C_b = 1.0$. For $C_b = 1.189$,

$\phi_b M_n = 1.189(1740) = 2069 \text{ ft-kips} > 1905 \text{ ft-kips}$ (OK)

$\phi_b M_p = 2340 \text{ ft-kips} > 2069 \text{ ft-kips}$ (OK)

Beam weight = 160 lb/ft < assumed value of 200 lb/ft (OK)

Check shear: From Z_x table,

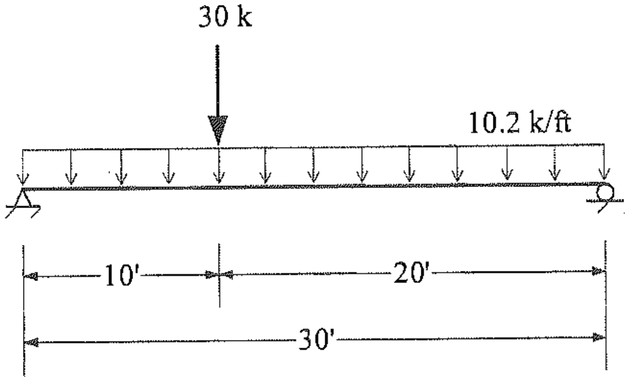
$\phi_v V_n = 702 \text{ kips} > 253 \text{ kips}$ (OK)

Use a W36 × 160

(b) Assume a beam weight of 200 lb/ft.

$$P_a = P_D + P_L = 10 + 20 = 30 \text{ kips}$$

$$w_a = w_D + w_L = (3.33 + 0.200) + 6.67 = 10.2 \text{ kips/ft}$$



Take moments about right end to determine left reaction, R_L :

$$\sum M_{right} = R_L(30) - 30(20) - 10.2(30)(30/2) = 0, \qquad R_L = 173.0 \text{ kips}$$

Shear is zero when $173.0 - 30 - 10.2x = 0, \qquad x = 14.02$ ft

Maximum moment is $M_a = 173(14.02) - 30(14.02 - 10) - 10.2(14.02)^2/2 = 1302$ ft-kips

The 20-ft unbraced length will control. Compute C_b :

Divide the 20-foot unbraced length into four equal segments of length 20/4 = 5 ft. This places $A, B,$ and C at distances of 15, 20, and 25 feet from the left end.

$$M_A = 173(15) - 10.2(15)^2/2 - 30(5) = 1298 \text{ ft-kips}$$

$$M_B = 173(20) - 10.2(20)^2/2 - 30(10) = 1120 \text{ ft-kips}$$

$$M_A = 173(25) - 10.2(25)^2/2 - 30(15) = 687.5 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(1302)}{2.5(1302) + 3(1298) + 4(1120) + 3(687.5)} = 1.189$$

Enter the beam design charts with $L_b = 20$ ft and $\frac{M_a}{C_b} = \frac{1302}{1.189} = 1095$ ft-kips

Try W36 × 160

$$\frac{M_n}{\Omega_b} = 1156 \text{ ft-kips for } C_b = 1.0. \text{ For } C_b = 1.189,$$

$$\frac{M_n}{\Omega_b} = 1.189(1156) = 1374 \text{ ft-kips} > 1302 \text{ ft-kips} \qquad \text{(OK)}$$

$$\frac{M_p}{\Omega_b} = 1560 \text{ ft-kips} > 1374 \text{ ft-kips}$$

Beam weight = 160 lb/ft < assumed value of 200 lb/ft (OK)

Check shear: From the Z_x table,

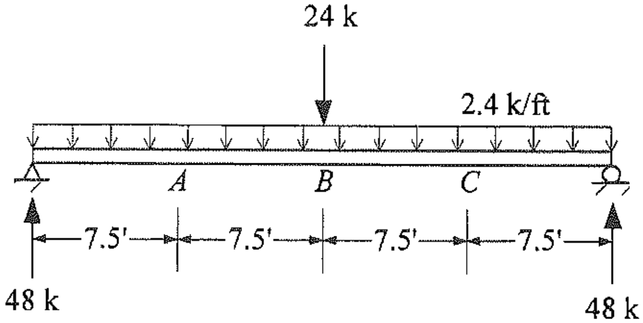
$$\frac{V_n}{\Omega_v} = 468 \text{ kips} > 173 \text{ kips} \quad \text{(OK)}$$

Use a W36 × 160

5.10-4

(a) $w_u = 1.2w_D + 1.6w_L = 1.2(2) = 2.4$ kips/ft (neglect beam wt. and check it later.)

$P_u = 1.2P_D + 1.6P_L = 1.6(15) = 24.0$ kips



For $L_b = 30$ ft, Compute C_b :

$$M_A = M_C = 48(7.5) - 2.4(7.5)^2/2 = 292.5 \text{ ft-kips}$$

$$M_B = M_{\max} = 48(15) - 2.4(15)^2/2 = 450.0 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(450.0)}{2.5(450.0) + 3(292.5) + 4(450.0) + 3(292.5)} = 1.202$$

Enter the beam design charts with $L_b = 30$ ft and $\frac{M_u}{C_b} = \frac{450.0}{1.202} = 374$ ft-kips

Try a W18 × 86

$\phi_b M_n = 407$ ft-kips for $C_b = 1.0$. For $C_b = 1.202$,

$\phi_b M_n = 1.202(407) = 489$ ft-kips > 450 ft-kips (OK)

$\phi_b M_p = 698$ ft-kips > 489 ft-kips (OK)

Check beam weight:

$$M_u = 450.0 + \frac{1}{8}(1.2 \times 0.086)(30)^2 = 462 \text{ ft-kips} < 489 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: $V_u = \frac{24}{2} + \frac{1.2(2 + 0.086)(30)}{2} = 49.5$ kips

From the Z_x table, $\phi_v V_n = 265$ kips > 49.5 kips (OK)

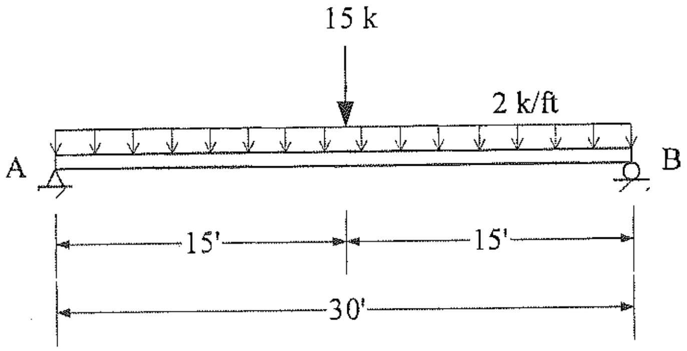
Check deflection: Maximum $\Delta_L = \frac{L}{360} = \frac{30(12)}{360} = 1.0$ in.

$$\Delta_L = \frac{P_L L^3}{48EI} = \frac{15(30 \times 12)^3}{48(29,000)(1070)} = 0.470 \text{ in.} < 1.0 \text{ in.} \quad \text{(OK)}$$

Use a W18 × 86

(b) $w_a = w_D + w_L = 2$ kips/ft (neglect beam wt. and check it later.)

$P_a = P_D + P_L = 15$ kips



Left reaction $= \frac{15 + 2(30)}{2} = 37.5$ kips

For $L_b = 30$ ft, Compute C_b : Divide the 30-foot unbraced length into four equal segments of length 30/4 = 7.5 ft. This places $A, B,$ and C at distances of 7.5, 15, and 22.5 feet from the left end.

$$M_A = M_C = 37.5(7.5) - 2(7.5)^2/2 = 225.0 \text{ ft-kips}$$

$$M_B = M_{\max} = 37.5(15) - 2(15)^2/2 = 337.5 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(337.5)}{2.5(337.5) + 3(225.0) + 4(337.5) + 3(225.0)} = 1.190$$

Enter the beam design charts with $L_b = 30$ ft and $\frac{M_a}{C_b} = \frac{337.5}{1.190} = 284$ ft-kips

Try a W14 × 90

$$\frac{M_n}{\Omega_b} = 311 \text{ ft-kips for } C_b = 1.0. \text{ For } C_b = 1.190,$$

$$\frac{M_n}{\Omega_b} = 1.190(311) = 370 \text{ ft-kips} > 337.5 \text{ ft-kips} \quad \text{(OK)}$$

$$\frac{M_p}{\Omega_b} = 382 \text{ ft-kips} > 370 \text{ ft-kips}$$

(This value of M_p/Ω_b is based on FLB.)

Check beam weight:

$$M_a = 337.5 + \frac{1}{8}(0.090)(30)^2 = 348 \text{ ft-kips} < 370 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the Z_x table,

$$\frac{V_n}{\Omega_v} = 123 \text{ kips} > 37.5 \text{ kips} \qquad \text{(OK)}$$

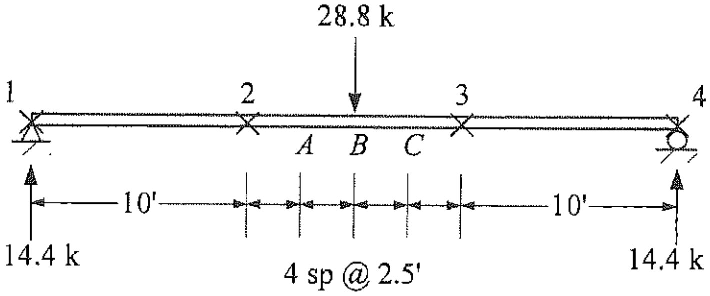
Check deflection: $\quad$ Maximum $\Delta_L = \dfrac{L}{360} = \dfrac{30(12)}{360} = 1.0$ in.

$$\Delta_L = \frac{P_L L^3}{48EI} = \frac{15(30 \times 12)^3}{48(29,000)(999)} = 0.503 \text{ in.} < 1.0 \text{ in.} \qquad \text{(OK)}$$

Use a W14 × 90

5.10-5

(a) $\quad P_u = 1.2P_D + 1.6P_L = 1.6(18) = 28.8$ kips (neglect beam wt. and check it later.)



Compute C_b for center segment:

$$M_A = M_C = 14.4(12.5) = 180.0 \text{ ft-kips}$$

$$M_B = M_{\max} = 14.4(15) = 216.0 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(216)}{2.5(216) + 3(180) + 4(216) + 3(180)} = 1.087$$

Enter the beam design charts with $L_b = 10$ ft and $\dfrac{M_u}{C_b} = \dfrac{216}{1.087} = 199$ ft-kips

Try a W16 × 40

$\phi_b M_n = 229$ ft-kips for $C_b = 1.0$. For $C_b = 1.087$,

$$\phi_b M_n = 1.087(229) = 249 \text{ ft-kips} > 216 \text{ ft-kips} \qquad \text{(OK)}$$

$$\phi_b M_p = 274 \text{ ft-kips} > 249 \text{ ft-kips} \qquad \text{(OK)}$$

Check beam weight:

$$M_u = 216 + \frac{1}{8}(1.2 \times 0.040)(30)^2 = 221 \text{ ft-kips} < 249 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: $V_u = \frac{28.8}{2} + \frac{1.2(0.040)(30)}{2} = 15.1$ kips

From the Z_x table, $\phi_v V_n = 146 \text{ kips} > 15.1 \text{ kips}$ (OK) Use a W16 × 40

(b) $P_a = P_D + P_L = 18$ kips (neglect beam wt. and check it later.)

Left reaction $= \frac{18}{2} = 9$ kips

Segment 2-3 is critical. Divide this 10-foot unbraced length into four equal segments of length 10/4 = 2. 5 ft. This places $A, B,$ and C at distances of 12.5, 15, and 17.5 feet from the left end.

$$M_A = M_C = 9(12.5) = 112.5 \text{ ft-kips}$$

$$M_B = M_{\max} = 9(15) = 135.0 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(135.0)}{2.5(135.0) + 3(112.5) + 4(135.0) + 3(112.5)} = 1.087$$

Enter the beam design charts with $L_b = 10$ ft and $\frac{M_a}{C_b} = \frac{135.0}{1.087} = 124$ ft-kips

Try a W16 × 36

$\frac{M_n}{\Omega_b} = 131$ ft-kips for $C_b = 1.0$. For $C_b = 1.087$,

$$\frac{M_n}{\Omega_b} = 1.087(131) = 142 \text{ ft-kips} > 135 \text{ ft-kips} \qquad \text{(OK)}$$

$$\frac{M_p}{\Omega_b} = 160 \text{ ft-kips} > 142 \text{ ft-kips}$$

Check beam weight:

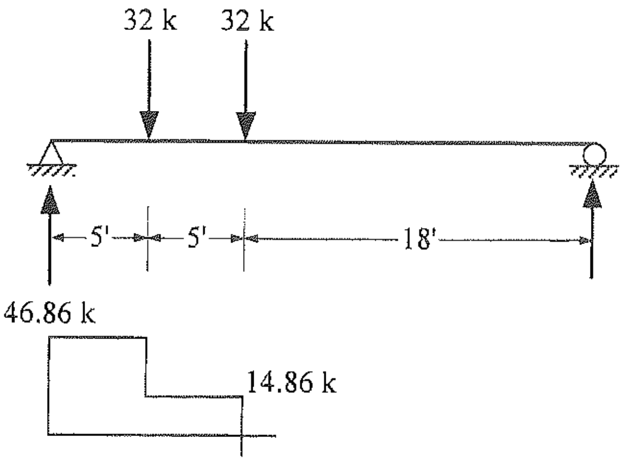
$$M_a = 135.0 + \frac{1}{8}(0.036)(30)^2 = 139 \text{ ft-kips} < 142 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the Z_x table,

$\frac{V_n}{\Omega_v} = 140 \text{ kips} > 9 \text{ kips}$ (OK) Use a W16 × 36

5.10-6

(a) $P_u = 1.2P_D + 1.6P_L = 1.6(20) = 32.0$ kips (neglect beam wt. and check it later.)



$M_u = 46.86(5) + 14.86(5) = 309$ ft-kips

Enter the beam design charts with $L_b = 28$ ft $(C_b = 1.0)$

Try a W14 × 74 $\phi_b M_n = 318$ ft-kips > 309 ft-kips (OK)

Check beam weight. $w_u = 1.2(0.074) = 0.0888$ kips/ft

Conservatively, we can use $M_u = 309 + 0.0888(28)^2/8 = 318$ ft-kips $= \phi_b M_n$ (OK)

Check shear: From the Z_x table, $\phi_v V_n = 191$ kips > 48.86 kips (OK)

Use a W14 × 74

(b) Neglect beam weight and check it later.

$$\text{Left reaction} = 20\left(\frac{18}{28}\right) + 20\left(\frac{23}{28}\right) = 29.29 \text{ kips}$$

The maximum moment occurs at the second load from the left:

$M_a = 29.29(10) - 20(5) = 193$ ft-kips

Enter the beam design charts with $L_b = 28$ ft $(C_b = 1.0)$

Try a W12 × 72

$$\frac{M_n}{\Omega_b} = 205 \text{ ft-kips} > 193 \text{ ft-kips} \quad \text{(OK)}$$

Check beam weight. Conservatively, we can use

$$M_a = 193 + 0.072(28)^2/8 = 200 \text{ ft-kips} < 205 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the Z_x table, $\dfrac{V_n}{\Omega_v} = 105$ kips > 29.29 kips (OK)

Use a W12 × 72

5.10-7

(a) Assume a beam weight of 40 lb/ft: $w_D = 170 + 40 = 210$ lb/ft

Load combination 3 controls: $w_u = 1.2D + 1.6S = 1.2(210) + 1.6(280) = 700.0$ lb/ft

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(0.700)(25)^2 = 54.69 \text{ ft-kips}$$

For lateral support at the ends and at midspan, $C_b = 1.30$ (Fig. 5.15, textbook)

Enter the beam design charts with $L_b = 25/2 = 12.5$ ft and $\dfrac{M_u}{C_b} = \dfrac{54.69}{1.30} = 42.1$ ft-kips:

Try a W8 × 18 :

For $C_b = 1.0$, $\phi_b M_n = 42.5$ ft-kips. For $C_b = 1.30$,

$$\phi_b M_n = 1.30(42.5) = 55.3 \text{ ft-kips} > 54.69 \text{ ft-kips} \quad \text{(OK)}$$

$$\phi_b M_p = 63.8 \text{ ft-kips} > 55.3 \text{ ft-kips}$$

Beam weight = 18 lb/ft < assumed value of 40 lb/ft (OK)

Check shear: From the Z_x table, $\phi_v V_n = 56.2$ kips

$$V_u = \frac{w_uL}{2} \approx \frac{0.700(25)}{2} = 8.75 \text{ kips} < 56.2 \text{ kips} \quad \text{(OK)}$$

Check deflection: $\Delta_{\text{max}} = \dfrac{L}{180} = \dfrac{25(12)}{180} = 1.67$ in.

$$\Delta = \frac{5(w_D + w_S)L^4}{384EI} = \frac{5[(0.170 + 0.018 + 0.280)/12](25 \times 12)^4}{384(29{,}000)(61.9)}$$

$$= 2.29 \text{ in.} > 1.67 \text{ in.} \quad \text{(N.G.)}$$

$$\text{Required } I_x = \frac{5(w_D + w_S)L^4}{384E\Delta_{\text{max}}} = \frac{5[(0.170 + 0.018 + 0.280)/12](25 \times 12)^4}{384(29{,}000)(1.67)}$$

$$= 84.9 \text{ in.}^4$$

Try a W10 × 22:

$$\phi_b M_n = 66 \text{ ft-kips (for } C_b = 1) > 54.69 \text{ ft-kips} \quad \text{(OK)}$$

$$I_x = 118 \text{ in.}^4 > 84.9 \text{ in.}^4 \quad \text{(OK)}$$

$$\phi_v V_n = 73.2 \text{ kips} > 8.75 \text{ kips} \quad \text{(OK)}$$

Use a W10 × 22

(b) Assume a beam weight of 40 lb/ft: $w_D = 170 + 40 = 210$ lb/ft

Load combination 3 controls: $w_a = D + S = 210 + 280 = 490$ lb/ft

$$M_a = \frac{1}{8}w_aL^2 = \frac{1}{8}(0.490)(25)^2 = 38.28 \text{ ft-kips}$$

For lateral support at the ends and at midspan, $C_b = 1.30$ (Fig. 5.15, textbook)

Enter the beam design charts with $L_b = 25/2 = 12.5$ ft and $\dfrac{M_a}{C_b} = \dfrac{38.28}{1.30} = 29.4$ ft-kips:

Try a W10 × 22:

For $C_b = 1.0$, $M_n/\Omega_b = 44$ ft-kips > 38.28 ft-kips. The strength will be higher for $C_b = 1.30$ and need not be investigated.

Beam weight = 22 lb/ft < assumed value of 40 lb/ft (OK)

Check shear: From the Z_x table, $V_n/\Omega_v = 48.8$ kips

$$V_a = \frac{w_aL}{2} \approx \frac{0.490(25)}{2} = 6.13 \text{ kips} < 48.8 \text{ kips} \quad \text{(OK)}$$

Check deflection: $\Delta_{\text{max}} = \dfrac{L}{180} = \dfrac{25(12)}{180} = 1.67$ in.

$$\Delta = \frac{5(w_D + w_S)L^4}{384EI} = \frac{5[(0.170 + 0.018 + 0.280)/12](25 \times 12)^4}{384(29{,}000)(118)}$$

$$= 1.20 \text{ in.} < 1.67 \text{ in.} \quad \text{(OK)}$$

Use a W10 × 22

5.11-1

Dead loads: Slab weight = $\dfrac{5}{12}(150) = 62.5$ psf

$w_D = (62.5 + 10)(6) = 435.0$ lb/ft (neglect beam wt. and check it later.)

Live load: $w_L = (60 + 20)(6) = 480$ lb/ft

(a) $w_u = 1.2w_D + 1.6w_L = 1.2(435) + 1.6(480) = 1290$ lb/ft

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(1.290)(30)^2 = 145 \text{ ft-kips}$$

From the Z_x table,

Try a W16 × 26, $\phi_bM_n = \phi_bM_p = 166$ ft-kips > 145 ft-kips (OK)

Check beam weight:

$$M_u = 145 + \frac{1}{8}(1.2 \times 0.026)(30)^2 = 149 \text{ ft-kips} < 166 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the Z_x table, $\phi_vV_n = 106$ kips

$$V_u \approx \frac{1.290(30)}{2} = 19.4 \text{ kips} < 106 \text{ kips} \quad \text{(OK)}$$

Check deflection: Maximum $\Delta_L = \frac{L}{180} = \frac{30(12)}{180} = 2.0$ in.

$$\Delta_L = \frac{5w_L L^4}{384EI} = \frac{5(0.480/12)(30 \times 12)^4}{384(29,000)(301)} = 1.0 \text{ in.} < 2.0 \text{ in.} \qquad \text{(OK)}$$

Use a W16 × 26

(b) $w_a = w_D + w_L = 435 + 480 = 915$ kips/ft

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(0.915)(30)^2 = 103 \text{ ft-kips}$$

From the Z_x table,

Try a W16 × 26, $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 110$ ft-kips > 103 ft-kips (OK)

Check beam weight:

$$M_a = 103 + \frac{1}{8}(0.026)(30)^2 = 106 \text{ ft-kips} < 110 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the Z_x table, $\frac{V_n}{\Omega_v} = 70.5$ kips

$$V_a \approx \frac{0.915(30)}{2} = 13.7 \text{ kips} < 70.5 \text{ kips} \qquad \text{(OK)}$$

Check deflection: Maximum $\Delta_L = \frac{L}{180} = \frac{30(12)}{180} = 2.0$ in.

$$\Delta_L = \frac{5w_L L^4}{384EI} = \frac{5(0.480/12)(30 \times 12)^4}{384(29,000)(301)} = 1.0 \text{ in.} < 2.0 \text{ in.} \qquad \text{(OK)}$$

Use a W16 × 26

5.11-2

Dead loads: Slab weight $= \frac{4.5}{12}(150) = 56.25$ psf

$w_D = (56.25 + 5)(5.5) = 336.9$ lb/ft (neglect beam wt. and check it later.)

Live load: $w_L = (150 + 20)(5.5) = 935.0$ lb/ft

(a) $w_u = 1.2w_D + 1.6w_L = 1.2(336.9) + 1.6(935.0) = 1900$ lb/ft

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(1.900)(30)^2 = 214 \text{ ft-kips}$$

From the Z_x table,

Try a W18 ×35, $\phi_b M_n = \phi_b M_p = 249$ ft-kips > 214 ft-kips (OK)

Check beam weight:

$$M_u = 214 + \frac{1}{8}(1.2 \times 0.035)(30)^2 = 219 \text{ ft-kips} < 249 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the Z_x table, $\phi_v V_n = 159$ kips

$$V_u \approx \frac{1.900(30)}{2} = 28.5 \text{ kips} < 159 \text{ kips} \qquad \text{(OK)}$$

Check deflection: Maximum $\Delta_L = \frac{L}{360} = \frac{30(12)}{360} = 1.0$ in.

$$\Delta_L = \frac{5w_L L^4}{384EI} = \frac{5(0.935/12)(30 \times 12)^4}{384(29,000)(510)} = 1.15 \text{ in.} > 1.0 \text{ in.} \qquad \text{(N.G.)}$$

$$\text{Required } I = \frac{5w_L L^4}{384E\Delta_{\max}} = \frac{5(0.935/12)(30 \times 12)^4}{384(29,000)(1.0)} = 588 \text{ in.}^4$$

From the I_x table, try a W18 × 40, $I_x = 612 \text{ in.}^4 > 588 \text{ in.}^4$ (OK)

$\phi_b M_n = 294$ ft-kips > 219 ft-kips (weight is OK)

$\phi_v V_n = 169$ kips > 28.5 kips (OK)

Use a W18 × 40

(b) $w_a = w_D + w_L = 336.9 + 935.0 = 1272$ kips/ft

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(1.272)(30)^2 = 143 \text{ ft-kips}$$

From the Z_x table,

Try a W18 × 40, $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 196$ ft-kips > 143 ft-kips (OK)

Check beam weight:

$$M_a = 143 + \frac{1}{8}(0.040)(30)^2 = 148 \text{ ft-kips} < 196 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the Z_x table, $\frac{V_n}{\Omega_v} = 113$ kips

$$V_a \approx \frac{1.272(30)}{2} = 19.1 \text{ kips} < 113 \text{ kips} \qquad \text{(OK)}$$

Check deflection: Maximum $\Delta_L = \frac{L}{360} = \frac{30(12)}{360} = 1.0$ in.

$$\Delta_L = \frac{5w_L L^4}{384EI} = \frac{5(0.935/12)(30 \times 12)^4}{384(29,000)(612)} = 0.960 \text{ in.} > 1.0 \text{ in.} \qquad \text{(OK)}$$

Use a W18 × 40

5.11-3

Dead loads: Slab and deck weight = 43 psf

$w_D = (43 + 5 + 2)(12) = 600.0$ lb/ft (neglect beam wt. and check it later.)

Live load: $w_L = (160 + 20)(12) = 2160$ lb/ft

(a) $w_u = 1.2w_D + 1.6w_L = 1.2(600) + 1.6(2160) = 4176$ lb/ft

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(4.176)(25)^2 = 326 \text{ ft-kips}$$

From the Z_x table,

Try a W21 × 44, $\phi_b M_n = \phi_b M_p = 358$ ft-kips > 326 ft-kips (OK)

Check beam weight:

$$M_u = 326 + \frac{1}{8}(1.2 \times 0.044)(25)^2 = 330 \text{ ft-kips} < 358 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the Z_x table, $\phi_v V_n = 217$ kips

$$V_u \approx \frac{4.176(25)}{2} = 52.2 \text{ kips} < 217 \text{ kips} \quad \text{(OK)}$$

Check deflection: Maximum $\Delta_L = \frac{L}{360} = \frac{25(12)}{360} = 0.833$ in.

$$\Delta_L = \frac{5w_LL^4}{384EI} = \frac{5(2.160/12)(25 \times 12)^4}{384(29{,}000)(843)} = 0.777 \text{ in.} < 0.833 \text{ in.} \quad \text{(OK)}$$

Use a W21 × 44

(b) $w_a = w_D + w_L = 600 + 2160 = 2760$ kips/ft

$$M_a = \frac{1}{8}w_aL^2 = \frac{1}{8}(2.760)(25)^2 = 216 \text{ ft-kips}$$

From the Z_x table,

Try a W21 × 44, $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 238$ ft-kips > 216 ft-kips (OK)

Check beam weight:

$$M_a = 216 + \frac{1}{8}(0.044)(25)^2 = 219 \text{ ft-kips} < 238 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the Z_x table, $\frac{V_n}{\Omega_v} = 145$ kips

$$V_a \approx \frac{2.760(25)}{2} = 34.5 \text{ kips} < 145 \text{ kips} \quad \text{(OK)}$$

Check deflection: Maximum $\Delta_L = \frac{L}{360} = \frac{25(12)}{360} = 0.833$ in.

$$\Delta_L = \frac{5w_LL^4}{384EI} = \frac{5(2.160/12)(25 \times 12)^4}{384(29{,}000)(843)} = 0.777 \text{ in.} < 0.833 \text{ in.} \quad \text{(OK)}$$

Use a W21 × 44

5.11-4

Dead loads: Slab and deck weight = 51 psf

$w_D = (51 + 10)(10) = 610.0$ lb/ft (neglect beam wt. and check it later.)

Live load: $w_L = (80 + 20)(10) = 1000$ lb/ft

(a) $w_u = 1.2w_D + 1.6w_L = 1.2(610) + 1.6(1000) = 2332$ lb/ft

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(2.332)(20)^2 = 117 \text{ ft-kips}$$

From the Z_x table,

Try a W14 × 22, $\phi_bM_n = \phi_bM_p = 125$ ft-kips > 117 ft-kips (OK)

Check beam weight:

$$M_u = 117 + \frac{1}{8}(1.2 \times 0.022)(20)^2 = 118 \text{ ft-kips} < 125 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the Z_x table, $\phi_vV_n = 94.8$ kips

$$V_u \approx \frac{2.332(20)}{2} = 23.3 \text{ kips} < 94.8 \text{ kips} \quad \text{(OK)}$$

Check deflection: Maximum $\Delta_L = \frac{L}{360} = \frac{20(12)}{360} = 0.667$ in.

$$\Delta_L = \frac{5w_LL^4}{384EI} = \frac{5(1.000/12)(20 \times 12)^4}{384(29,000)(199)} = 0.624 \text{ in.} < 0.667 \text{ in.} \quad \text{(OK)}$$

Use a W14 × 22

(b) $w_a = w_D + w_L = 610 + 1000 = 1610$ kips/ft

$$M_a = \frac{1}{8}w_aL^2 = \frac{1}{8}(1.610)(20)^2 = 80.5 \text{ ft-kips}$$

From the Z_x table,

Try a W14 × 22, $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 82.8$ ft-kips > 80.5 ft-kips (OK)

Check beam weight:

$$M_a = 80.5 + \frac{1}{8}(0.022)(20)^2 = 81.6 \text{ ft-kips} < 82.8 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the Z_x table, $\frac{V_n}{\Omega_v} = 63.2$ kips

$$V_a \approx \frac{1.610(20)}{2} = 16.1 \text{ kips} < 63.2 \text{ kips} \quad \text{(OK)}$$

Check deflection: Maximum $\Delta_L = \frac{L}{360} = \frac{20(12)}{360} = 0.667$ in.

$$\Delta_L = \frac{5w_L L^4}{384EI} = \frac{5(1.000/12)(20 \times 12)^4}{384(29{,}000)(199)} = 0.624 \text{ in.} < 0.667 \text{ in.} \qquad \text{(OK)}$$

Use a W14 × 44

5.11-5

Dead loads: Slab weight $= \frac{5}{12}(150) = 62.5$ psf

$w_D = 62.5(5) = 312.5$ lb/ft (neglect beam wt. and check it later.)

Live load: $w_L = (80 + 20)(5) = 500$ lb/ft

(a) $w_u = 1.2w_D + 1.6w_L = 1.2(312.5) + 1.6(500) = 1175$ lb/ft

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(1.175)(30)^2 = 132 \text{ ft-kips}$$

From the Z_x table,

Try a W16 × 26, $\phi_b M_n = \phi_b M_p = 166$ ft-kips > 132 ft-kips (OK)

Check beam weight:

$$M_u = 132 + \frac{1}{8}(1.2 \times 0.026)(30)^2 = 136 \text{ ft-kips} < 166 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear: From the Z_x table, $\phi_v V_n = 106$ kips

$$V_u \approx \frac{1.175(30)}{2} = 17.6 \text{ kips} < 106 \text{ kips} \qquad \text{(OK)}$$

Check deflection: Maximum total $\Delta = \frac{L}{240} = \frac{30(12)}{240} = 1.5$ in.

$$\Delta = \frac{5wL^4}{384EI} = \frac{5[(0.3125 + 0.026 + 0.500)/12](30 \times 12)^4}{384(29{,}000)(301)}$$

$$= 1.75 \text{ in.} > 1.5 \text{ in.} \qquad \text{(N.G.)}$$

$$\text{Required } I = \frac{5w_L L^4}{384E\Delta_{\max}} = \frac{5[(0.3125 + 0.026 + 0.500)/12](30 \times 12)^4}{384(29{,}000)(1.5)} = 351 \text{ in.}^4$$

From the I_x table, try a W16 × 31, $I_x = 375$ in.^{4} > 351 in.^{4} (OK)

$\phi_b M_n = 203$ ft-kips > 136 ft-kips (weight is OK)

$\phi_v V_n = 131$ kips > 17.6 kips (OK)

Use a W16 × 31

(b) $w_a = w_D + w_L = 312.5 + 500 = 812.5$ kips/ft

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(0.8125)(30)^2 = 91.41 \text{ ft-kips}$$

Suspecting that deflection could be a problem, we will compute the required moment of inertia.

$$\text{Maximum total } \Delta = \frac{L}{240} = \frac{30(12)}{240} = 1.5 \text{ in.}$$

$$\text{Required } I = \frac{5wL^4}{384E\Delta_{\max}} = \frac{5(0.8125/12)(30 \times 12)^4}{384(29,000)(1.5)} = 340 \text{ in.}^4$$

Check the Z_x table for $M_n/\Omega_b > 91.4$ ft-kips and $I_x > 340$ in.^{4}

Try a W16 × 31, $\dfrac{M_n}{\Omega_b} = \dfrac{M_p}{\Omega_b} = 135$ ft-kips > 91.4 ft-kips (OK)

$I_x = 375$ in.$^4 > 340$ in.^{4} (OK)

Total deflection, including beam weight is

$$\Delta = \frac{5wL^4}{384EI} = \frac{5[(0.3125 + 0.026 + 0.500)/12](30 \times 12)^4}{384(29,000)(375)}$$

$$= 1.41 \text{ in.} < 1.5 \text{ in.} \quad \text{(OK)}$$

Check moment for beam weight:

$$M_a = 91.4 + \frac{1}{8}(0.031)(30)^2 = 94.9 \text{ ft-kips} < 135 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the Z_x table, $\dfrac{V_n}{\Omega_v} = 87.3$ kips

$$V_a \approx \frac{0.8125(30)}{2} = 12.2 \text{ kips} < 87.3 \text{ kips} \quad \text{(OK)}$$

Use a W16 × 31

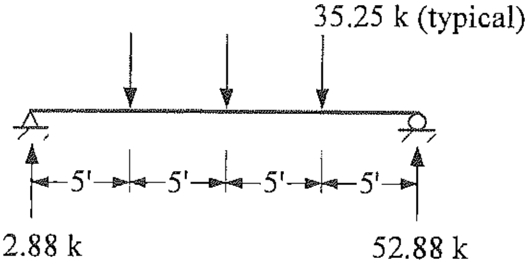
5.11-6

Slab weight $= \dfrac{5}{12}(150) = 62.5$ psf. For beams, use 35 lb/ft

$w_D = 62.5(5) = 312.5$ lb/ft, $w_L = (80 + 20)(5) = 500$ lb/ft

(a) $w_u = 1.2w_D + 1.6w_L = 1.2(312.5) + 1.6(500) = 1175$ lb/ft

Beam reaction $= \dfrac{1.175(30)}{2} \times 2 = 35.25$ kips



Girder reaction $= \dfrac{35.25(3)}{2} = 52.88$ kips

$M_u = 52.88(10) - 35.25(5) = 353$ ft-kips

Try a W21 × 44, $\phi_b M_n = 358$ ft-kips > 353 ft-kips (OK)

Check weight:

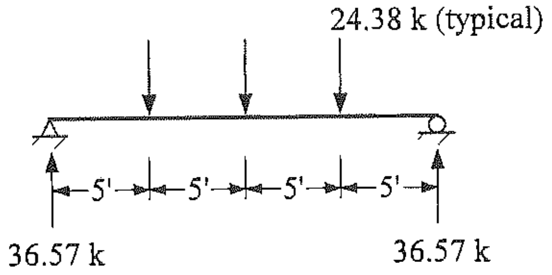
$M_u = 353 + \dfrac{1}{8}(1.2 \times 0.044)(20)^2 = 355$ ft-kips < 358 ft-kips (OK)

Check shear: From the Z_x table, $\phi_v V_n = 217$ kips

$V_u = 52.88 + \dfrac{1.2(0.044)(20)}{2} = 53.4$ kips < 217 kips (OK) Use a W21 × 44

(b) $w_a = w_D + w_L = 312.5 + 500 = 812.5$ lb/ft

Beam reaction $= \dfrac{0.8125(30)}{2} \times 2 = 24.38$ kips



Girder reaction $= \dfrac{24.38(3)}{2} = 36.57$ kips

$M_a = 36.57(10) - 24.38(5) = 244$ ft-kips

Try a W21 × 48, $\dfrac{M_n}{\Omega_b} = \dfrac{M_p}{\Omega_b} = 265$ ft-kips > 244 ft-kips (OK)

Check beam weight:

$M_a = 244 + \dfrac{1}{8}(0.048)(20)^2 = 246$ ft-kips < 265 ft-kips (OK)

Check shear: From the Z_x table, $\dfrac{V_n}{\Omega_v} = 144$ kips

$V_a = 36.57 + \dfrac{0.048(20)}{2} = 37.1$ kips < 144 kips (OK) Use a W21 × 48

5.11-7

Slab weight $= \dfrac{5}{12}(150) = 62.5$ psf. For beams, use 35 lb/ft × 30 ft = 1050 lb/beam

Expressed as a distributed load, the weight of the beams is $\dfrac{1050 \times 3}{20} = 157.5$ lb/ft

$w_D = 62.5(30) + 157.5 = 2033$ lb/ft, $w_L = (80 + 20)(30) = 3000$ lb/ft

(a) $w_u = 1.2w_D + 1.6w_L = 1.2(2.033) + 1.6(3.000) = 7.240 \text{ kips/ft}$

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(7.240)(20)^2 = 362 \text{ ft-kips}$$

Try a W21 × 48, $\phi_b M_n = 398 \text{ ft-kips} > 362 \text{ ft-kips}$ (OK)

Check weight:

$$M_u = 362 + \frac{1}{8}(1.2 \times 0.048)(20)^2 = 365 \text{ ft-kips} < 398 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the Z_x table, $\phi_v V_n = 217$ kips

$$V_u \approx \frac{7.240(20)}{2} = 72.4 \text{ kips} < 217 \text{ kips} \quad \text{(OK)}$$

Use a W21 × 48

(b) $w_a = w_D + w_L = 2033 + 3000 = 5033 \text{ lb/ft}$

$$M_a = \frac{1}{8}w_aL^2 = \frac{1}{8}(5.033)(20)^2 = 252 \text{ ft-kips}$$

Try a W21 × 48, $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 265 \text{ ft-kips} > 252 \text{ ft-kips}$ (OK)

Check beam weight:

$$M_a = 252 + \frac{1}{8}(0.048)(20)^2 = 254 \text{ ft-kips} < 265 \text{ ft-kips} \quad \text{(OK)}$$

Check shear: From the Z_x table, $\frac{V_n}{\Omega_v} = 144$ kips

$$V_a \approx \frac{5.033(20)}{2} = 50.3 \text{ kips} < 144 \text{ kips} \quad \text{(OK)}$$

Use a W21 × 48

5.12-1

(a) For a W16 × 31, $b_f = 5.53$ in., $t_f = 0.440$ in., and $S_x = 47.2 \text{ in.}^3$

Check to see if the flange holes need to be accounted for. The gross area of one flange is

$$A_{fg} = t_f b_f = 0.440(5.53) = 2.433 \text{ in.}^2$$

The effective hole diameter is

$$d_h = \frac{7}{8} + \frac{1}{8} = 1 \text{ in.}$$

and the net flange area is

$$A_{fn} = A_{fg} - t_f \sum d_h = 2.433 - 0.440(2 \times 1) = 1.553 \text{ in.}^2$$

$$F_u A_{fn} = 65(1.553) = 100.9 \text{ kips}$$

Determine Y_t. For A992 steel, the maximum F_y/F_u ratio is 0.85. Since this is greater than 0.8, use $Y_t = 1.1$.

$$Y_t F_y A_{fg} = 1.1(50)(2.433) = 133.8 \text{ kips}$$

Since $F_uA_{fn} < Y_tF_yA_{fg}$, the holes must be accounted for. From AISC Equation F13-1,

$$M_n = \frac{F_uA_{fn}}{A_{fg}}S_x = \frac{100.9}{2.433}(47.2) = 1957 \text{ in.-kips} = 163.1 \text{ ft-kips.} \qquad \underline{M_n = 163 \text{ ft-kips}}$$

(b) Unreduced strength $= M_p = F_yZ_x = 50(54.0) = 2700$ in.-kips (compact shape)

$$\text{Change} = \frac{1957 - 2700}{2700} \times 100 = -27.5\% \qquad \underline{\text{Reduction} = 27.5\%}$$

5.12-2

(a) For a W21 × 48, $b_f = 8.14$ in., $t_f = 0.430$ in., and $S_x = 93.0$ in.^{3}

Check to see if the flange holes need to be accounted for. The gross area of one flange is

$$A_{fg} = t_fb_f = 0.430(8.14) = 3.500 \text{ in.}^2$$

The effective hole diameter is

$$d_h = \frac{3}{4} + \frac{1}{8} = \frac{7}{8} \text{ in.}$$

and the net flange area is

$$A_{fn} = A_{fg} - t_f\sum d_h = 3.500 - 0.430\left(2 \times \frac{7}{8}\right) = 2.748 \text{ in.}^2$$

$$F_uA_{fn} = 65(2.748) = 178.6 \text{ kips}$$

Determine Y_t. For A992 steel, the maximum F_y/F_u ratio is 0.85. Since this is greater than 0.8, use $Y_t = 1.1$.

$$Y_tF_yA_{fg} = 1.1(50)(3.500) = 192.5 \text{ kips}$$

Since $F_uA_{fn} < Y_tF_yA_{fg}$, the holes must be accounted for. From AISC Equation F13-1,

$$M_n = \frac{F_uA_{fn}}{A_{fg}}S_x = \frac{178.6}{3.500}(93.0) = 4746 \text{ in.-kips} = 395.5 \text{ ft-kips.} \qquad \underline{M_n = 396 \text{ ft-kips}}$$

(b) This is a noncompact shape (see footnote in Dimensions and Properties tables). Since there is continuous lateral support, the unreduced strength will be based on flange local buckling.

$$M_n = M_p - (M_p - 0.7F_yS_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

For the flange, $\lambda_p = 0.38\sqrt{\frac{29,000}{50}} = 9.152, \qquad \lambda_r = 1.0\sqrt{\frac{29,000}{50}} = 24.08$

$$\lambda = \frac{b_f}{2t_f} = 9.47 \qquad (\lambda_p < \lambda < \lambda_r \ \therefore \text{ flange is noncompact})$$

$M_p = F_y Z_x = 50(107) = 5350$ in.-kips

$$M_n = 5350 - (5350 - 0.7 \times 50 \times 93.0)\left(\frac{9.47 - 9.152}{24.08 - 9.152}\right)$$

$= 5305$ in.-kips $= 442.1$ ft-kips

Change $= \dfrac{395.5 - 442.1}{442.1} \times 100 = -10.54\%$ Reduction = 10.5%

5.12-3

(a) For a W18 × 35, $b_f = 6.00$ in., $t_f = 0.425$ in., and $S_x = 57.6$ in.^{3}

Check to see if the flange holes need to be accounted for. The gross area of one flange is

$$A_{fg} = t_f b_f = 0.425(6.00) = 2.55 \text{ in.}^2$$

The effective hole diameter is

$$d_h = \frac{3}{4} + \frac{1}{8} = \frac{7}{8} \text{ in.}$$

and the net flange area is

$$A_{fn} = A_{fg} - t_f \sum d_h = 2.55 - 0.425\left(2 \times \frac{7}{8}\right) = 1.806 \text{ in.}^2$$

$$F_u A_{fn} = 65(1.806) = 117.4 \text{ kips}$$

Determine Y_t. For A992 steel, the maximum F_y/F_u ratio is 0.85. Since this is greater than 0.8, use $Y_t = 1.1$.

$$Y_t F_y A_{fg} = 1.1(50)(2.55) = 140.3 \text{ kips}$$

Since $F_u A_{fn} < Y_t F_y A_{fg}$, the holes must be accounted for. From AISC Equation F13-1,

$M_n = \dfrac{F_u A_{fn}}{A_{fg}} S_x = \dfrac{117.4}{2.55}(57.6) = 2652$ in.-kips = 221 ft-kips. $M_n = 221$ ft-kips

(b) Unreduced strength = $M_p = F_y Z_x = 50(66.5) = 3325$ in.-kips (compact shape)

Change $= \dfrac{2652 - 3325}{3325} \times 100 = -20.2\%$ Reduction = 20.2%

5.13-1

Dead loads: Slab weight $= \dfrac{4}{12}(150) = 50.0$ psf

Assuming a joist weight of 3 psf, $w_D = (50 + 3 + 5)(3) = 174$ lb/ft

$$w_L = 80(3) = 240 \text{ lb/ft}$$

$$w_u = 1.2 w_D + 1.6 w_L = 1.2(174) + 1.6(240) = 593 \text{ lb/ft}$$

From Figure 5.35, for a span length of 20 ft, a 16K3 has a capacity of 615 lb/ft >593 lb/ft, and the approximate weight is $6.3/3 = 2.1$ psf < 3 psf estimated.

Use a 14K4

5.13-2

Assuming a joist weight = 4 psf, $w_D = (4+32+5)(4) = 164$ lb/ft

$$w_L = (40+20)(4) = 240 \text{ lb/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(164) + 1.6(240) = 581 \text{ lb/ft}$$

From Figure 5.35, for a span length of 22 ft, a 16K4 has a capacity of 609 lb/ft > 581 lb/ft, and the approximate weight is $7.0/4 = 1.75$ psf < 4 psf estimated.

Use a 16K4

Deflection is not likely to be a problem, because it would take a service live load of 289 kips/ft to cause a live load deflection of $L/360$, and the live load in this problem is only 240 lb/ft (including the partition load).

5.14-1

(a) Factored load = $R_u = 1.6(85) = 136.0$ kips

Determine the length of bearing N required to prevent web yielding. From AISC Equation J10-2, the design strength for this limit state is

$$\phi R_n = \phi(5k+N)F_y t_w$$

For $\phi R_n \geq R_u$,

$$1.0[5(1.24)+N](50)(0.375) \geq 136 \qquad \Rightarrow \qquad N \geq 1.05 \text{ in.}$$

Use AISC Equation J10-4 to determine the value of N required to prevent web crippling. For $\phi R_n \geq R_u$,

$$\phi(0.80)t_w^2\left[1+3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}} \geq R_u$$

$$0.75(0.80)(0.375)^2\left[1+3\left(\frac{N}{13.9}\right)\left(\frac{0.375}{0.645}\right)^{1.5}\right]\sqrt{\frac{29{,}000(50)(0.645)}{0.375}} \geq 136$$

$$N \geq 0.2158 \text{ in.}$$

Since the required N is so small, a minimal plate is needed. Use a plate whose dimensions conform to the details of the applied load.

(b) Service load = $R_a = 85$ kips

Determine the length of bearing N required to prevent web yielding. From AISC Equation J10-2, the allowable strength for this limit state is

$$\frac{R_n}{\Omega} = \frac{(5k+N)F_y t_w}{\Omega}$$

For $\frac{R_n}{\Omega} \geq R_a$

$$\frac{[5(1.24)+N](50)(0.375)}{1.50} \geq 85 \qquad \Rightarrow \qquad N \geq 0.6 \text{ in.}$$

Use AISC Equation J10-4 to determine the value of N required to prevent web crippling. For

$$\frac{R_n}{\Omega} \geq R_a$$

$$\frac{0.80 t_w^2\left[1+3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}}}{\Omega} \geq R_a$$

$$\frac{0.80(0.375)^2\left[1+3\left(\frac{N}{13.9}\right)\left(\frac{0.375}{0.645}\right)^{1.5}\right]\sqrt{\frac{29{,}000(50)(0.645)}{0.375}}}{2.00} \geq 85$$

$N \geq -0.451$ in. (A bearing plate is not needed for this limit state.)

Since the required N is so small, a minimal plate is needed. Use a plate whose dimensions conform to the details of the applied load.

5.14-2

(a) The factored load reaction is

$$R_u = 1.2D + 1.6L = 1.2(28) + 1.6(56) = 123.2 \text{ kips}$$

Determine the length of bearing N required to prevent web yielding. From AISC Equation J10-3, the nominal strength for this limit state is $R_n = (2.5k+N)F_y t_w$

For $\phi R_n \geq R_u$,

$$1.0[2.5(1.32)+N](50)(0.520) \geq 123.2 \qquad \Rightarrow \qquad N \geq 1.438 \text{ in.}$$

Use AISC Equation J10-5 to determine the value of N required to prevent web crippling. Assume that $N/d > 0.2$ and try the second form of the equation, J10-5b. For $\phi R_n \geq R_u$,

$$\phi(0.40)t_w^2\left[1+\left(\frac{4N}{d}-0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}} \geq R_u$$

$$0.75(0.40)(0.520)^2\left[1+\left(\frac{4N}{29.7}-0.2\right)\left(\frac{0.520}{0.670}\right)^{1.5}\right]\sqrt{\frac{29{,}000(50)(0.670)}{0.520}} \geq 123.2$$

$N \geq 2.692$ in.

Check the assumption:

$$\frac{N}{d} = \frac{2.692}{29.7} = 9.064 \times 10^{-2} < 0.2 \qquad \text{(N.G.)}$$

For $N/d < 0.2$, AISC Equation J10-5a applies:

$$\phi(0.40)t_w^2\left[1+3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_yt_f}{t_w}} \geq R_u$$

$$0.75(0.40)(0.520)^2\left[1+3\left(\frac{N}{29.7}\right)\left(\frac{0.520}{0.670}\right)^{1.5}\right]\sqrt{\frac{29,000(50)(0.670)}{0.520}} \geq 123.2$$

$N \geq 1.609$ in., and $\frac{N}{d} = \frac{1.609}{29.7} = 5.418 \times 10^{-2} < 0.2$ (OK)

Try $N = 6$ in. Determine B.

Since the support area will be larger than the bearing plate,

$$P_p = 0.85f_c'A_1\sqrt{\frac{A_2}{A_1}} \leq 1.7f_c'A_1 \qquad \text{(AISC Eq. J8-2)}$$

As an initial trial, assume conservatively that the full area of the support is in contact, and

$$P_p = 0.85f_c'A_1 \qquad \text{(AISC Eq. J8-1)}$$

Let $\phi_cP_p > R_u$: $0.60(0.85)(3)A_1 \geq 123.2 \quad \Rightarrow \quad A_1 \geq 80.52 \text{ in.}^2$

Then $B \geq 80.52/6 = 13.42$ in.

Try a 6 ×14 plate. Support width is then 8 ×16.

$$\sqrt{A_2/A_1} = \sqrt{\frac{8(16)}{6(14)}} = 1.234$$

Let $\phi_cP_p > R_u$: $\phi_c\left(0.85f_c'A_1\sqrt{A_2/A_1}\right) > R_u$

$$0.60(0.85)(3)A_1(1.234) \geq 123.2 \quad \Rightarrow \quad A_1 \geq 65.25 \text{ in.}^2$$

Try $B = 65.25/6 = 10.88$ in. Try a 6 ×11 plate. Support width is then 8 ×13.

$$\sqrt{A_2/A_1} = \sqrt{\frac{8(13)}{6(11)}} = 1.255$$

$0.60(0.85)(3)A_1(1.255) \geq 123.2$, Solution is: $\{64.16 \leq A_1\}$

For a 6 ×11 plate, $A_1 = 66 \text{ in.}^2 > 64.16 \text{ in.}^2$ (OK)

Check upper limit of AISC Equation J8-2:

$$1.7f_c'A_1 = 1.7(3)(66) = 337 \text{ kips}$$

$$0.85f_c'A_1\sqrt{\frac{A_2}{A_1}} = 0.85(3)(66)(1.255) = 211 \text{ kips} < 337 \text{ kips} \qquad \text{(OK)}$$

Plate thickness: $n = \frac{B-2k}{2} = \frac{11-2(1.32)}{2} = 4.18$ in.

$$t \geq \sqrt{\frac{2.22R_un^2}{BNF_y}} = \sqrt{\frac{2.22(123.2)(4.18)^2}{6(11)(36)}} = 1.418 \text{ in., use } 1\tfrac{1}{2} \text{ in.}$$

Use a PL 1½ × 6 × 0′ – 11″

(b) The service load reaction is

$$R_a = D + L = 28 + 56 = 84 \text{ kips}$$

Determine the length of bearing N required to prevent web yielding. From AISC Equation J10-3, the nominal strength for this limit state is $R_n = (2.5k + N)F_y t_w$

For $\frac{R_n}{\Omega} \geq R_a$

$$\frac{1}{1.50}[2.5(1.32) + N](50)(0.520) \geq 84 \qquad \Rightarrow \qquad N \geq 1.546 \text{ in.}$$

Use AISC Equation J10-5 to determine the value of N required to prevent web crippling. Assume that $N/d < 0.2$ and try the first form of the equation, J10-5a. For $\frac{R_n}{\Omega} \geq R_a$,

$$\frac{1}{\Omega}(0.40t_w^2)\left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}} \geq R_a$$

$$\frac{1}{2.00}[0.40(0.520)^2]\left[1 + 3\left(\frac{N}{29.7}\right)\left(\frac{0.520}{0.670}\right)^{1.5}\right]\sqrt{\frac{29{,}000(50)(0.670)}{0.520}} \geq 84$$

$$N \geq 1.975 \text{ in., and } \quad \frac{N}{d} = \frac{1.975}{29.7} = 6.650 \times 10^{-2} < 0.2 \qquad \text{(OK)}$$

Try $N = 6$ in. Determine B.

Since the support area will be larger than the bearing plate,

$$P_p = 0.85 f_c' A_1 \sqrt{\frac{A_2}{A_1}} \leq 1.7 f_c' A_1 \qquad \text{(AISC Eq. J8-2)}$$

As an initial trial, assume conservatively that the full area of the support is in contact, and

$$P_p = 0.85 f_c' A_1 \qquad \text{(AISC Eq. J8-1)}$$

Let $\frac{P_p}{\Omega} > R_a$: $\qquad \frac{0.85(3)A_1}{2.50} \geq 84 \qquad \Rightarrow \qquad A_1 \geq 82.35 \text{ in.}^2$

Then $B \geq 82.35/6 = 13.73$ in.

Try a 6 ×14 plate. Support width is then 8 ×16.

$$\sqrt{A_2/A_1} = \sqrt{\frac{8(16)}{6(14)}} = 1.234$$

Let $\frac{P_p}{\Omega} > R_a$: $\qquad \frac{1}{\Omega}\left(0.85 f_c' A_1 \sqrt{A_2/A_1}\right) > R_a$

$$\frac{0.85(3)A_1(1.234)}{2.50} \geq 84 \qquad \Rightarrow \qquad A_1 \geq 66.74 \text{ in.}^2$$

Try $B = 66.74/6 = 11.12$ in. Try a 6 ×12 plate. Support width is then 8 ×14

$$\sqrt{A_2/A_1} = \sqrt{\frac{8(14)}{6(12)}} = 1.247$$

$$\frac{0.85(3)A_1(1.247)}{2.50} \geq 84 \qquad \Rightarrow \qquad A_1 \geq 66.04 \text{ in.}^2$$

For a 6 ×12 plate, $A_1 = 72 \text{ in.}^2 > 66.04 \text{ in.}^2$ (OK)

Check upper limit of AISC Equation J8-2:

$$1.7f_c'A_1 = 1.7(3)(72) = 367 \text{ kips}$$

$$0.85f_c'A_1\sqrt{\frac{A_2}{A_1}} = 0.85(3)(72)(1.247) = 229 \text{ kips} < 367 \text{ kips} \qquad \text{(OK)}$$

Plate thickness: $n = \dfrac{B-2k}{2} = \dfrac{12-2(1.32)}{2} = 4.68$ in.

$$t \geq \sqrt{\frac{3.34R_a n^2}{BNF_y}} = \sqrt{\frac{3.34(84)(4.68)^2}{6(12)(36)}} = 1.54 \text{ in., use} 1\tfrac{5}{8} \text{ in.}$$

Use a PL $1\frac{5}{8} \times 6 \times 1' - 0"$

5.14-3

(a) The factored load is $P_u = 1.2D + 1.6L = 1.2(65) + 1.6(195) = 390.0$ kips

Compute the required bearing area. From AISC Equation J8-2,

$$P_p = 0.85f_c'A_1\sqrt{\frac{A_2}{A_1}} \leq 1.7f_c'A_1$$

For $\phi_c P_p \geq P_u$,

$$0.60\left[0.85(3.5)A_1\sqrt{\frac{16(16)}{A_1}}\right] \geq 390 \qquad \Rightarrow \qquad A_1 \geq 187 \text{ in.}^2$$

Check upper limit:

$$\phi_c 1.7f_c'A_1 = 0.60(1.7)(3.5)(187) = 668 \text{ kips} > 390 \text{ kips} \qquad \text{(OK)}$$

The plate must be as large as the column: $b_f d = 12.1(12.5) = 151 \text{ in.}^2 < 187 \text{ in.}^2$

Try $B = N = 14$ in., with A_1 provided $= 14(14) = 196 \text{ in.}^2 > 187 \text{ in.}^2$

$$m = \frac{N-0.95d}{2} = \frac{14-0.95(12.5)}{2} = 1.063 \text{ in.}$$

$$n = \frac{B-0.8b_f}{2} = \frac{14-0.8(12.1)}{2} = 2.16 \text{ in.}$$

$$n' = \frac{1}{4}\sqrt{db_f} = \frac{1}{4}\sqrt{12.5(12.1)} = 3.075 \text{ in.}$$

Conservatively, let $\lambda = 1.0$, resulting in

$$\ell = \max(m, n, \lambda n') = \max(1.063, 2.16, 3.075) = 3.075 \text{ in.}$$

$$\text{Required } t = \ell\sqrt{\frac{2P_u}{0.90BNF_y}} = 3.075\sqrt{\frac{2(390)}{0.90(14)(14)(36)}} = 1.08 \text{ in.; use } 1\tfrac{1}{8} \text{ in.}$$

Use a PL $1\frac{1}{8} \times 14 \times 14$

(b) The applied load is $P_a = D + L = 65 + 195 = 260$ kips.

Compute the required bearing area. Using AISC Equation J8-2,

$$\frac{P_p}{\Omega_c} \geq P_a$$

$$\frac{1}{2.50}\left[0.85(3.5)A_1\sqrt{\frac{16(16)}{A_1}}\right] \geq 260 \qquad \Rightarrow \qquad A_1 \geq 187 \text{ in.}^2$$

The upper limit is

$$\frac{1}{\Omega_c}(1.7f_c{}'A_1) = \frac{1}{2.50}[(1.7)(3.5)(187)] = 445 \text{ kips} > 260 \text{ kips} \qquad \text{(OK)}$$

The plate must be as large as the column:

$$b_f d = 12.1(12.5) = 151 \text{ in.}^2 < 187 \text{ in.}^2 \qquad \text{(OK)}$$

Try $B = N = 14$ in., with A_1 provided = 14(14) = 196 in.^{2} > 187 in.^{2}

$$m = \frac{N - 0.95d}{2} = \frac{14 - 0.95(12.5)}{2} = 1.063 \text{ in.}$$

$$n = \frac{B - 0.8b_f}{2} = \frac{14 - 0.8(12.1)}{2} = 2.16 \text{ in.}$$

$$n' = \frac{1}{4}\sqrt{db_f} = \frac{1}{4}\sqrt{12.5(12.1)} = 3.075 \text{ in.}$$

Conservatively, let $\lambda = 1.0$, resulting in

$$\ell = \max(m, n, \lambda n') = \max(1.063, 2.16, 3.075) = 3.075 \text{ in.}$$

From Equation 5.20,

$$t \geq \ell\sqrt{\frac{2P_a}{BNF_y/1.67}} = 3.075\sqrt{\frac{2(260)}{14(14)(36)/1.67}} = 1.08 \text{ in.}$$

Use a PL $1\frac{1}{8} \times 14 \times 14$

5.14-4

(a) The factored load is $P_u = 1.2D + 1.6L = 1.2(20) + 1.6(50) = 104.0$ kips

Compute the required bearing area. From AISC Equation J8-2,

$$P_p = 0.85f_c{}'A_1\sqrt{\frac{A_2}{A_1}} \leq 1.7f_c{}'A_1$$

For $\phi_c P_p \geq P_u$,

$$0.60\left[0.85(3)A_1\sqrt{\frac{12(12)}{A_1}}\right] \geq 104 \qquad \Rightarrow \qquad A_1 \geq 32.1 \text{ in.}^2$$

Check upper limit:

$$\phi_c 1.7f_c{}'A_1 = 0.60(1.7)(3)(32.1) = 98.2 \text{ kips} < 104 \text{ kips} \qquad \text{(N.G.)}$$

Let $\phi_c 1.7 f_c' A_1 = P_u$, $\quad 0.60(1.7)(3)A_1 = 104 \quad \Rightarrow \quad A_1 = 34.0 \text{ in.}^2$

The plate must be as large as the column:

$$b_f d = 7.96(9.73) = 77.5 \text{ in.}^2 > 34.0 \text{ in.}^2 \qquad \text{(N.G.)}$$

Try $B = N = 12$ in., with A_1 provided $= 12(12) = 144 \text{ in.}^2 > 77.5 \text{ in.}^2$

$$m = \frac{N - 0.95d}{2} = \frac{12 - 0.95(9.73)}{2} = 1.378 \text{ in.}$$

$$n = \frac{B - 0.8b_f}{2} = \frac{12 - 0.8(7.96)}{2} = 2.816 \text{ in.}$$

$$n' = \frac{1}{4}\sqrt{db_f} = \frac{1}{4}\sqrt{9.73(7.96)} = 2.2 \text{ in.}$$

Conservatively, let $\lambda = 1.0$, resulting in

$$\ell = \max(m, n, \lambda n') = \max(1.378, 2.816, 2.2) = 2.816 \text{ in.}$$

$$\text{Required } t = \ell\sqrt{\frac{2P_u}{0.90BNF_y}} = 2.816\sqrt{\frac{2(104)}{0.90(12)(12)(36)}} = 0.595 \text{ in.; use } \tfrac{5}{8} \text{ in.}$$

Use a PL $\frac{5}{8} \times 12 \times 12$

(b) The applied load is $P_a = D + L = 20 + 50 = 70$ kips.

Compute the required bearing area. Using AISC Equation J8-2,

$$\frac{P_p}{\Omega_c} \geq P_a$$

$$\frac{1}{2.50}\left[0.85(3)A_1\sqrt{\frac{12(12)}{A_1}}\right] \geq 70 \qquad \Rightarrow \qquad A_1 \geq 32.7 \text{ in.}^2$$

The upper limit is

$$\frac{1}{\Omega_c}(1.7 f_c' A_1) = \frac{1}{2.50}[(1.7)(3)(32.7)] = 66.7 \text{ kips} < 70 \text{ kips} \qquad \text{(N.G.)}$$

The plate must be as large as the column:

$$b_f d = 7.96(9.73) = 77.5 \text{ in.}^2 > 32.7 \text{ in.}^2 \qquad \text{(N.G.)}$$

Try $B = N = 12$ in., with A_1 provided $= 12(12) = 144 \text{ in.}^2 > 77.5 \text{ in.}^2$

$$m = \frac{N - 0.95d}{2} = \frac{12 - 0.95(9.73)}{2} = 1.378 \text{ in.}$$

$$n = \frac{B - 0.8b_f}{2} = \frac{12 - 0.8(7.96)}{2} = 2.816 \text{ in.}$$

$$n' = \frac{1}{4}\sqrt{db_f} = \frac{1}{4}\sqrt{9.73(7.96)} = 2.2 \text{ in.}$$

Conservatively, let $\lambda = 1.0$, resulting in

$$\ell = \max(m, n, \lambda n') = \max(1.378, 2.816, 2.2) = 2.816 \text{ in.}$$

From Equation 5.20,

$$t \ge \ell\sqrt{\frac{2P_a}{BNF_y/1.67}} = 2.816\sqrt{\frac{2(70)}{12(12)(36)/1.67}} = 0.598 \text{ in.; use } \tfrac{5}{8} \text{ in.}$$

Use a PL $\frac{5}{8} \times 12 \times 12$

5.15-1

Compute the nominal flexural strength for x-axis bending. The following data for a W18 × 55 are obtained from the Z_x table. The shape is compact (no footnote to indicate otherwise) and

$$L_p = 5.90 \text{ ft}, \quad L_r = 17.5 \text{ ft}$$

The unbraced length $L_b = 16$ ft, so $L_p < L_b < L_r$, and the controlling limit state is inelastic lateral-torsional buckling.

$$M_{nx} = C_b\left[M_{px} - (M_{px} - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_{px}$$

$$M_{px} = F_yZ_x = 50(112) = 5600 \text{ in.-kips}$$

If the beam weight is neglected, C_b can be obtained from Figure 5.15 as $C_b = 1.32$

$$M_{nx} = 1.32\left[5600 - (5600 - 0.7 \times 50 \times 98.3)\left(\frac{16 - 5.90}{17.5 - 5.90}\right)\right]$$

$$= 4910 \text{ in.-kips} = 409.2 \text{ ft-kips} \qquad (< M_{px})$$

For the y axis, since the shape is compact, there is no flange local buckling and

$$M_{ny} = M_{py} = F_yZ_y = 50(18.5) = 925.0 \text{ in.-kips} = 77.08 \text{ ft-kips}$$

Check the upper limit:

$$\frac{Z_y}{S_y} = \frac{18.5}{11.9} = 1.56 < 1.6 \qquad \therefore\ M_{ny} = M_{py} = 77.08 \text{ ft-kips}$$

(a) LRFD solution:

$$\phi_bM_{nx} = 0.90(409.2) = 368.3 \text{ ft-kips}, \qquad \phi_bM_{ny} = 0.90(77.08) = 69.37 \text{ ft-kips}$$

$$P_u = 1.2(15) + 1.6(15) = 42.0 \text{ kips}, \ w_u = 1.2(0.055) = 0.066 \text{ kips/ft}$$

$$M_{ux} = \frac{42.0(16)}{4} + \frac{0.066(16)^2}{8} = 170.1 \text{ ft-kips}$$

For y-axis bending,

$$1.2(2) + 1.6(2) = 5.6 \text{ kips}$$

$$M_{uy} = \frac{2(5.6)(16)}{4} = 44.8 \text{ ft-kips}$$

Check interaction equation 5.23:

$$\frac{M_{ux}}{\phi_bM_{nx}} + \frac{M_{uy}}{\phi_bM_{ny}} = \frac{170.1}{368.3} + \frac{44.8}{69.37} = 1.11 > 1.0 \qquad \text{(N.G.)}$$

W18 × 55 is unsatisfactory.

(b) ASD solution:

$$\frac{M_{nx}}{\Omega_b} = \frac{409.2}{1.67} = 245.0 \text{ ft-kips}, \qquad \frac{M_{ny}}{\Omega_b} = \frac{77.08}{1.67} = 46.16 \text{ ft-kips}$$

$$M_{ax} = \frac{30(16)}{4} + \frac{0.055(16)^2}{8} = 121.8 \text{ ft-kips}$$

For y-axis bending,

$$M_{ay} = \frac{2(4)(16)}{4} = 32.0 \text{ ft-kips}$$

Check interaction equation 5.24:

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{121.8}{245.0} + \frac{32}{46.16} = 1.19 > 1.0 \qquad \text{(N.G.)}$$

W18 ×55 is unsatisfactory

5.15-2

(a) LRFD solution

Strong-axis bending:

$$P_u = 1.6\left(\frac{4}{5} \times 24\right) = 30.72 \text{ kips}, \qquad M_{ux} = \frac{30.72(30)}{4} = 230.4 \text{ ft-kips}$$

From the beam design charts, for $L_b = 30$ ft and $C_b = 1.0$, $\phi_b M_{nx} = 593$ ft-kips

For $C_b = 1.32$ (from Figure 5.15),

$$\phi_b M_{nx} = 1.32 \times 593 = 782.8 \text{ ft-kips} > \phi_b M_{px} = 720 \text{ ft-kips}$$

$$\therefore \text{ use } \phi_b M_{nx} = \phi_b M_{px} = 720 \text{ ft-kips}$$

Weak-axis bending:

$$P_u = 1.6\left(\frac{3}{5} \times 24\right) = 23.04 \text{ kips}, \qquad M_{uy} = \frac{23.04(30)}{4} = 172.8 \text{ ft-kips}$$

From the Z_y table, $\phi_b M_{ny} = 348$ ft-kips

Check interaction equation 5.23:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{230.4}{720} + \frac{172.8}{348} = 0.817 < 1.0 \qquad \text{(OK)}$$

Satisfactory. Result of interaction equation = 0.817

(b) ASD solution

Strong-axis bending:

$$P_a = \frac{4}{5} \times 24 = 19.2 \text{ kips}, \qquad M_{ax} = \frac{19.2(30)}{4} = 144.0 \text{ ft-kips}$$

From the beam design charts, for $L_b = 30$ ft and $C_b = 1.0$, $\frac{M_{nx}}{\Omega_b} = 395$ ft-kips

For $C_b = 1.32$ (from Figure 5.15),

$$\frac{M_{nx}}{\Omega_b} = 1.32 \times 395 = 521.4 \text{ ft-kips} > \frac{M_{px}}{\Omega_b} = 479 \text{ ft-kips}$$

$$\therefore \text{ use } \frac{M_{nx}}{\Omega_b} = \frac{M_{px}}{\Omega_b} = 479 \text{ ft-kips}$$

For y-axis bending,

$$P_a = \frac{3}{5} \times 24 = 14.4 \text{ kips}, \qquad M_{ay} = \frac{14.4(30)}{4} = 108.0 \text{ ft-kips}$$

From the Z_y table, $\frac{M_{nx}}{\Omega_b} = 231$ ft-kips

Check interaction equation 5.24:

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{144.0}{479} + \frac{108.0}{231} = 0.768 < 1.0 \qquad \text{(OK)}$$

Satisfactory. Result of interaction equation = 0.768

5.15-3

Compute the nominal flexural strength for x-axis bending. The following data for a W21 × 68 are obtained from the Z_x table. The shape is compact (no footnote to indicate otherwise) and

$$L_p = 6.36 \text{ ft}, \; L_r = 18.7 \text{ ft}$$

The unbraced length $L_b = 12$ ft, so $L_p < L_b < L_r$, and the controlling limit state is inelastic lateral-torsional buckling.

$$M_{nx} = C_b\left[M_{px} - (M_{px} - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_{px}$$

$$M_{px} = F_yZ_x = 50(160) = 8000 \text{ in.-kips}$$

From Figure 5.15, $C_b = 1.14$

$$M_{nx} = 1.14\left[8000 - (8000 - 0.7 \times 50 \times 140)\left(\frac{12 - 6.36}{18.7 - 6.36}\right)\right]$$

$$= 7505 \text{ in.-kips} = 625.4 \text{ ft-kips} \qquad (< M_{px})$$

For the y axis, since the shape is compact, there is no flange local buckling and

$$M_{ny} = M_{py} = F_yZ_y = 50(24.4) = 1220 \text{ in.-kips} = 101.7 \text{ ft-kips}$$

Check the upper limit:

$$\frac{Z_y}{S_y} = \frac{24.4}{15.7} = 1.55 < 1.6 \qquad \therefore M_{ny} = M_{py} = 101.7 \text{ ft-kips}$$

(a) LRFD solution:

$$\phi_b M_{nx} = 0.90(625.4) = 562.9 \text{ ft-kips}, \qquad \phi_b M_{ny} = 0.90(101.7) = 91.53 \text{ ft-kips}$$

$$w_{ux} = 1.2(0.068) + 1.6\left(\frac{2}{\sqrt{5}}\right)(4) = 5.806 \text{ kips/ft}$$

$$M_{ux} = \frac{5.806(12)^2}{8} = 104.5 \text{ ft-kips}$$

$$w_{uy} = 1.6\left[\frac{1}{\sqrt{5}}(4)\right] = 2.862 \text{ kips/ft}$$

$$M_{uy} = \frac{2.862(12)^2}{8} = 51.52 \text{ ft-kips}$$

Check interaction equation 5.23 (use 1/2 of weak-axis bending strength):

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{104.5}{562.9} + \frac{51.52}{91.53/2} = 1.31 > 1.0 \qquad \text{(N.G.)}$$

W21 × 68 is unsatisfactory.

(b) ASD solution:

$$\frac{M_{nx}}{\Omega_b} = \frac{625.4}{1.67} = 374.5 \text{ ft-kips}, \qquad \frac{M_{ny}}{\Omega_b} = \frac{101.7}{1.67} = 60.90 \text{ ft-kips}$$

$$w_{ax} = 4\left(2/\sqrt{5}\right) + 0.068 = 3.646 \text{ kips/ft}, \qquad M_{ax} = \frac{3.646(12)^2}{8} = 65.63 \text{ ft-kips}$$

$$w_{ay} = 4\left(1/\sqrt{5}\right) = 1.789 \text{ kips/ft}, \qquad M_{ay} = \frac{1.789(12)^2}{8} = 32.2 \text{ ft-kips}$$

Check interaction equation 5.24 (use 1/2 of weak-axis bending strength):

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{65.63}{374.5} + \frac{32.2}{60.90/2} = 1.23 > 1.0 \qquad \text{(N.G.)}$$

W21 × 68 is unsatisfactory

5.15-4

(a) Strong-axis bending strength:

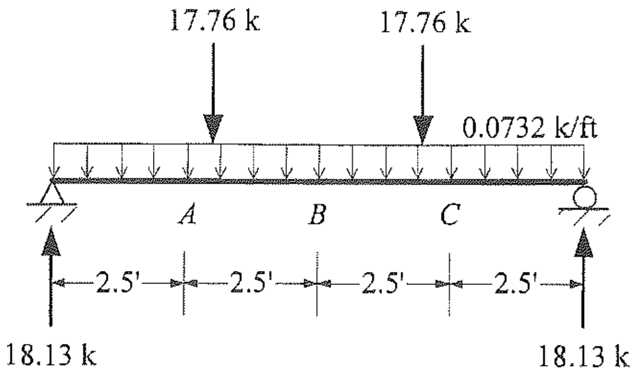
From the beam design charts, for $L_b = 10$ ft and $C_b = 1.0$, $\phi_b M_{nx} = 372$ ft-kips

Compute C_b.

$$w_{ux} = 1.2 w_D = 1.2(0.061) = 0.0732 \text{ kips/ft}$$

$$P_{ux} = 1.2 P_{Dx} + 1.6 P_{Lx} = 1.2(0.3 \times 15)\left(\frac{4}{5}\right) + 1.6(0.7 \times 15)\left(\frac{4}{5}\right) = 17.76 \text{ kips}$$

$$\text{Reaction} = \frac{w_{ux} L}{2} + P_{ux} = \frac{0.0732(10)}{2} + 17.76 = 18.13 \text{ kips}$$



Dividing the unbraced length into 4 equal segments of 10/4 = 2.5 ft each and labeling the 3 interior points A, B, and C, we obtain

$$M_A = M_C = 18.13(2.5) - 0.0732(2.5)^2/2 = 45.10 \text{ ft-kips}$$

$$M_B = M_{\max} = 18.13(5) - 0.0732(5)^2/2 - 17.76(2) = 54.22 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(54.22)}{2.5(54.22) + 3(45.10) + 4(54.22) + 3(45.10)} = 1.088$$

For $C_b = 1.088$,

$$\phi_b M_{nx} = 1.088 \times 372 = 404.7 \text{ ft-kips} > \phi_b M_{px} = 383 \text{ ft-kips}$$

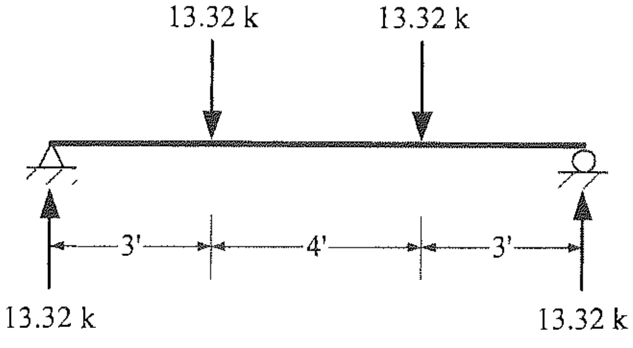
$$\therefore \text{ use } \phi_b M_{nx} = \phi_b M_{px} = 383 \text{ ft-kips}$$

Weak-axis bending strength: From the Z_y table, $\phi_b M_{ny} = 123$ ft-kips

Weak axis bending moment:

$$\text{Reaction} = P_{uy} = 1.2P_{Dy} + 1.6P_{Ly}$$

$$= 1.2(0.3 \times 15)\left(\frac{3}{5}\right) + 1.6(0.7 \times 15)\left(\frac{3}{5}\right) = 13.32 \text{ kips}$$



$$M_{uy} = 13.32(3) = 39.96 \text{ ft-kips}$$

Use 1/2 of weak-axis bending strength in the interaction equation:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{54.22}{383} + \frac{39.96}{123/2} = 0.791 < 1.0 \qquad \text{(OK)}$$

Adequate. Result of interaction equation = 0.791

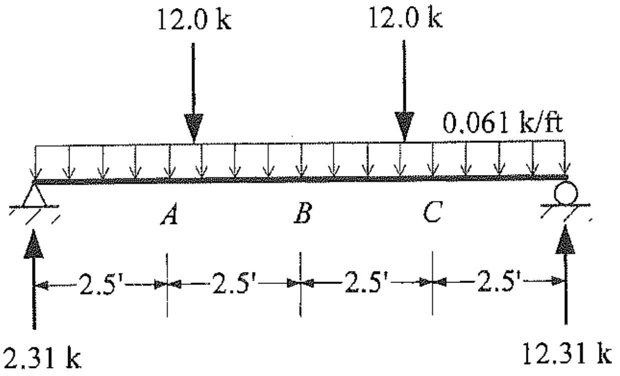
(b) Strong-axis bending strength:

From the beam design charts, for $L_b = 10$ ft and $C_b = 1.0$, $\frac{M_{nx}}{\Omega_b} = 248$ ft-kips

Compute C_b.

$$w_{ax} = w_D = 0.061 \text{ kips/ft}, \qquad P_{ax} = P_{Dx} + P_{Lx} = 15\left(\tfrac{4}{5}\right) = 12.0 \text{ kips}$$

$$\text{Reaction} = \frac{w_{ax}L}{2} + P_{ax} = \frac{0.061(10)}{2} + 12.0 = 12.31 \text{ kips}$$



Dividing the unbraced length into 4 equal segments of 10/4 = 2.5 ft each and labeling the 3 interior points A, B, and C, we obtain

$$M_A = M_C = 12.31(2.5) - 0.061(2.5)^2/2 = 30.58 \text{ ft-kips}$$

$$M_B = M_{\max} = 12.31(5) - 0.061(5)^2/2 - 12.0(2) = 36.79 \text{ ft-kips}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(36.79)}{2.5(36.79) + 3(30.58) + 4(36.79) + 3(30.58)} = 1.088$$

For $C_b = 1.088$,

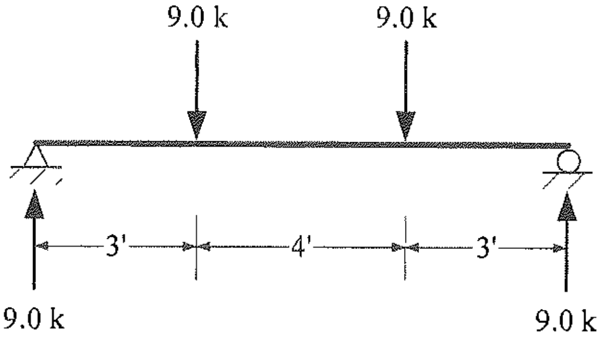
$$\frac{M_{nx}}{\Omega_b} = 1.088 \times 248 = 269.8 \text{ ft-kips} > \frac{M_{px}}{\Omega_b} = 254 \text{ ft-kips}$$

$$\therefore \text{ use } \frac{M_{nx}}{\Omega_b} = \frac{M_{px}}{\Omega_b} = 254 \text{ ft-kips}$$

Weak-axis bending strength: From the Z_y table, $\frac{M_{ny}}{\Omega_b} = 81.8$ ft-kips

Weak axis bending moment:

$$\text{Reaction} = P_{ay} = P_{Dy} + P_{Ly} = 15\left(\tfrac{3}{5}\right) = 9.0 \text{ kips}$$



$$M_{ay} = 9.0(3) = 27.0 \text{ ft-kips}$$

Use 1/2 of weak-axis bending strength in the interaction equation:

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{36.79}{254} + \frac{27.0}{81.8/2} = 0.805 < 1.0 \qquad \text{(OK)}$$

Adequate. Result of interaction equation = 0.805

5.15-5

Compute the nominal flexural strength for x-axis bending. The following data for a W18 × 35 are obtained from the Z_x table. The shape is compact (no footnote to indicate otherwise) and

$$L_p = 4.31 \text{ ft}, \ L_r = 12.4 \text{ ft}$$

The unbraced length $L_b = 10$ ft, so $L_p < L_b < L_r$, and the controlling limit state is inelastic lateral-torsional buckling.

$$M_{nx} = C_b\left[M_{px} - (M_{px} - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_{px}$$

$$M_{px} = F_yZ_x = 50(66.5) = 3325 \text{ in.-kips}$$

From Figure 5.15, $C_b = 1.14$

$$M_{nx} = 1.14\left[3325 - (3325 - 0.7 \times 50 \times 57.6)\left(\frac{10 - 4.31}{12.4 - 4.31}\right)\right]$$

$$= 2741 \text{ in.-kips} = 228.4 \text{ ft-kips} \qquad (< M_{px})$$

For the y axis, since the shape is compact, there is no flange local buckling and

$$M_{ny} = M_{py} = F_yZ_y = 50(8.06) = 403.0 \text{ in.-kips} = 33.58 \text{ ft-kips}$$

Check the upper limit:

$$\frac{Z_y}{S_y} = \frac{8.06}{5.12} = 1.57 < 1.6 \qquad \therefore\ M_{ny} = M_{py} = 33.58 \text{ ft-kips}$$

(a) LRFD solution:

$$\phi_b M_{nx} = 0.90(228.4) = 205.6 \text{ ft-kips}, \qquad \phi_b M_{ny} = 0.90(33.58) = 30.22 \text{ ft-kips}$$

$$w_{ux} = 1.6(1.2)(4/5) = 1.536 \text{ kips/ft}, \qquad M_{ux} = \frac{1.536(10)^2}{8} = 19.2 \text{ ft-kips}$$

$$w_{uy} = 1.6(1.2)(3/5) = 1.152 \text{ kips/ft}, \qquad M_{uy} = \frac{1.152(10)^2}{8} = 14.4 \text{ ft-kips}$$

Check interaction equation 5.23.

For the loading condition of Figure P5.15-5(a), use the full weak-axis bending strength:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{19.2}{205.6} + \frac{14.4}{30.22} = 0.570 < 1.0 \qquad \text{(OK)}$$

Beam is satisfactory for the loading of Fig. P5.15-5(a).

For the loading condition of Figure P5.15-5(b), use 1/2 of the weak-axis bending strength:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{19.2}{205.6} + \frac{14.4}{30.22/2} = 1.05 > 1.0 \qquad \text{(N.G.)}$$

Beam is unsatisfactory for the loading of Fig. P5.15-5(b).

(b) ASD solution:

$$\frac{M_{nx}}{\Omega_b} = \frac{228.4}{1.67} = 136.8 \text{ ft-kips}, \qquad \frac{M_{ny}}{\Omega_b} = \frac{33.58}{1.67} = 20.11 \text{ ft-kips}$$

$$w_{ax} = 1.2(4/5) = 0.96 \text{ kips/ft}, \qquad M_{ax} = \frac{0.96(10)^2}{8} = 12.0 \text{ ft-kips}$$

$$w_{ay} = 1.2(3/5) = 0.72 \text{ kips/ft}, \qquad M_{ay} = \frac{0.72(10)^2}{8} = 9.0 \text{ ft-kips}$$

Check interaction equation 5.24:

For the loading condition of Figure P5.15-5(a), use the full weak-axis bending strength:

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{12.0}{136.8} + \frac{9.0}{20.11} = 0.535 < 1.0 \qquad \text{(OK)}$$

Beam is satisfactory for the loading of Fig. P5.15-5(a)

For the loading condition of Figure P5.15-5(b), use 1/2 of the weak-axis bending strength:

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{12.0}{136.8} + \frac{9.0}{20.11/2} = 0.983 < 1.0 \qquad \text{(OK)}$$

Beam is satisfactory for the loading of Fig. P5.15-5(b)

5.15-6

Before proceeding to the LRFD and ASD solutions, we will compute the nominal flexural strength about the x and y axes. First, determine the strong-axis bending strength. Neither the beam design charts nor the Z tables include shapes smaller than W8, so the flexural strength of the W6 × 12 must be computed. From the dimensions and properties tables, the shape is compact (no footnote). The following properties of a W6 ×12 will be needed:

$A = 3.55\text{ in.}^2$, $S_x = 7.31\text{ in.}^3, Z_x = 8.30\text{ in.}^3\ I_y = 2.99\text{ in.}^4$, $r_y = 0.918\text{ in.}$, $S_y = 1.50\text{ in.}^3$, $r_{ts} = 1.08\text{ in.}$, $h_o = 5.75\text{ in.}$, $J = 0.0903\text{ in.}^4$

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76(0.918)\sqrt{\frac{29{,}000}{50}} = 38.91\text{ in.} = 3.243\text{ ft.}$$

From AISC Equation F2-6,

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_x h_o}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7F_y S_x h_o}{EJc}\right)^2}}$$

$$= 1.95(1.08)\frac{29{,}000}{0.7(50)}\sqrt{\frac{0.0903(1.0)}{7.31(5.75)}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7(50)(7.31)(5.75}{29{,}000(0.0903)(1.0)}\right)^2}}$$

$$= 134.6\text{ in.} = 11.22\text{ ft}$$

For $L_b = 10$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

$C_b = 1.14$ (see Fig. 5.15 in textbook).

$$M_p = F_y Z_x = 50(8.30) = 415.0\text{ in.-kips} = 34.58\text{ ft-kips}$$

$$M_n = 1.14\left[415 - (415 - 0.7 \times 50 \times 7.31)\left(\frac{10 - 3.243}{11.22 - 3.243}\right)\right]$$

$$= 319.4\text{ in.-kips} = 26.62\text{ ft-kips} < M_p$$

For the y axis, since the shape is compact, there is no flange local buckling and

$$M_{ny} = M_{py} = F_y Z_y = 50(2.32) = 116.0\text{ in.-kips} = 9.667\text{ ft-kips}$$

Check the upper limit:

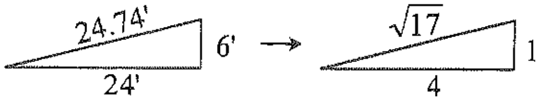
$$\frac{Z_y}{S_y} = \frac{2.32}{1.50} = 1.55 < 1.6 \qquad \therefore\ M_{ny} = M_{py} = 9.667\text{ ft-kips}$$

(a) LRFD solution

$$\phi_b M_{nx} = 0.90(26.62) = 23.96\text{ ft-kips}, \qquad \phi_b M_{ny} = 0.90(9.667) = 8.700\text{ ft-kips}$$

Roof load: Combination 3 controls.

$$w_u = 1.2D + 1.6S = 1.2(40/2) + 1.6(40/2) = 56.0\text{ psf}$$



$$\text{Tributary width} = \frac{\sqrt{17}}{4}(6) = 6.185\text{ ft}$$

Purlin load $= 56.0(6.185) = 346.4$ lb/ft

Component normal to roof $= w_{ux} = \dfrac{4}{\sqrt{17}}(346.4) = 336.1$ lb/ft

Component parallel to roof $= w_{uy} = \dfrac{1}{\sqrt{17}}(346.4) = 84.01$ lb/ft

$$M_{ux} = \frac{1}{8}w_{ux}L^2 = \frac{1}{8}(0.3361)(10)^2 = 4.201 \text{ ft-kips}$$

$$M_{uy} = \frac{1}{8}w_{uy}L^2 = \frac{1}{8}(0.08401)(10)^2 = 1.05 \text{ ft-kips}$$

Use 1/2 of weak-axis bending strength in the interaction equation:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{4.201}{23.96} + \frac{1.05}{8.700/2} = 0.417 < 1.0 \qquad \text{(OK)}$$

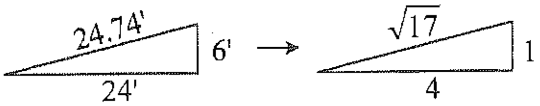
Adequate. Result of interaction equation = 0.415

(b) ASD solution

$$\frac{M_{nx}}{\Omega_b} = \frac{26.62}{1.67} = 15.94 \text{ ft-kips}, \qquad \frac{M_{ny}}{\Omega_b} = \frac{9.667}{1.67} = 5.789 \text{ ft-kips}$$

Roof load: Combination 3 controls.

$$w_a = D + S = 40 \text{ psf}$$



Tributary width $= \dfrac{\sqrt{17}}{4}(6) = 6.185$ ft

Purlin load $= 40(6.185) = 247.4$ lb/ft

Component normal to roof $= w_{ax} = \dfrac{4}{\sqrt{17}}(247.4) = 240.0$ lb/ft

Component parallel to roof $= w_{ay} = \dfrac{1}{\sqrt{17}}(247.4) = 60.0$ lb/ft

$$M_{ax} = \frac{1}{8}w_{ax}L^2 = \frac{1}{8}(0.240)(10)^2 = 3.0 \text{ ft-kips}$$

$$M_{ay} = \frac{1}{8}w_{ay}L^2 = \frac{1}{8}(0.060)(10)^2 = 0.75 \text{ ft-kips}$$

Use 1/2 of weak-axis bending strength in the interaction equation:

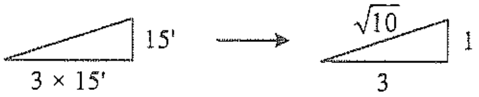
$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{3.0}{15.94} + \frac{0.75}{5.789/2} = 0.447 \qquad \text{(OK)}$$

Adequate. Result of interaction equation = 0.447

5.15-7

Dead load: assume purlin weighs 20 lb/ft.

Other dead load:



$$\text{Tributary width} = \frac{\sqrt{10}}{3}\left(\frac{15}{2}\right) = 7.906 \text{ ft}$$

$$w_D = 7.906(15) + 20 = 138.6 \text{ lb/ft}$$

Snow: $w_S = \left(\frac{15}{2}\right)(20) = 150.0 \text{ lb/ft}$

(a) LRFD solution

Load combination 3 controls.

$$w_u = 1.2w_D + 1.6w_S = 1.2(0.1386) + 1.6(0.1500) = 0.4063 \text{ kips/ft}$$

Component normal to roof $= w_{ux} = \frac{3}{\sqrt{10}}(0.4063) = 0.3855$ kips/ft

Component parallel to roof $= w_{uy} = \frac{1}{\sqrt{10}}(0.4063) = 0.1285$ kips/ft

$$M_{ux} = \frac{1}{8}w_{ux}L^2 = \frac{1}{8}(0.3855)(18)^2 = 15.61 \text{ ft-kips}$$

To compute M_{uy}, use Table 3-22c, "Continuous Beams," in Part 3 of the *Manual*. For a two-span continuous beam, the maximum moment is

$$M_{uy} = 0.125w_{uy}\left(\frac{L}{2}\right)^2 = 0.125(0.1285)\left(\frac{18}{2}\right)^2 = 1.301 \text{ ft-kips}$$

Try a W6 × 12:

Determine the strong-axis bending strength. The beam design charts in the *Manual* do not contain curves for shapes smaller than W8, so the flexural strength of the W6 × 12 must be computed. From the dimensions and properties tables, the shape is compact (no footnote). The following properties of a W6 ×12 will be needed:

$A = 3.55 \text{ in.}^2$, $S_x = 7.31 \text{ in.}^3$, $Z_x = 8.30 \text{ in.}^3$ $I_y = 2.99 \text{ in.}^4$, $r_y = 0.918$ in., $S_y = 1.50 \text{ in.}^3$, $r_{ts} = 1.08$ in., $h_o = 5.75$ in., $J = 0.0903 \text{ in.}^4$

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}} = 1.76(0.918)\sqrt{\frac{29{,}000}{50}} = 38.91 \text{ in.} = 3.243 \text{ ft.}$$

From AISC Equation F2-6,

$$L_r = 1.95r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_xh_o}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7F_yS_xh_o}{EJc}\right)^2}}$$

$$= 1.95(1.08)\frac{29{,}000}{0.7(50)}\sqrt{\frac{0.0903(1.0)}{7.31(5.75)}}\sqrt{1+\sqrt{1+6.76\left(\frac{0.7(50)(7.31)(5.75)}{29{,}000(0.0903)(1.0)}\right)^2}}$$

$$= 134.6 \text{ in.} = 11.22 \text{ ft}$$

For $L_b = 18/2 = 9$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

$C_b = 1.30$ (see Table 3-1 in the *Manual*).

$$M_p = F_yZ_x = 50(8.30) = 415.0 \text{ in.-kips} = 34.58 \text{ ft-kips}$$

$$M_n = 1.30\left[415 - (415 - 0.7 \times 50 \times 7.31)\left(\frac{9 - 3.243}{11.22 - 3.243}\right)\right]$$

$$= 390.2 \text{ in.-kips} = 32.52 \text{ ft-kips} < M_p$$

$$\phi_bM_{nx} = 0.90(32.52) = 29.27 \text{ ft-kips}$$

For the y axis, since the shape is compact, there is no flange local buckling and

$$M_{ny} = M_{py} = F_yZ_y = 50(2.32) = 116.0 \text{ in.-kips} = 9.667 \text{ ft-kips}$$

Check the upper limit:

$$\frac{Z_y}{S_y} = \frac{2.32}{1.50} = 1.55 < 1.6 \qquad \therefore\ M_{ny} = M_{py} = 9.667 \text{ ft-kips}$$

$$\phi_bM_{ny} = 0.90(9.667) = 8.700 \text{ ft-kips}$$

Conservatively, consider all loads, including the purlin weight, to act at the top flange and use 1/2 of weak-axis bending strength in the interaction equation:

$$\frac{M_{ux}}{\phi_bM_{nx}} + \frac{M_{uy}}{\phi_bM_{ny}} = \frac{15.61}{29.27} + \frac{1.301}{8.700/2} = 0.832 < 1.0 \qquad \text{(OK)}$$

Weight assumption and shear strength are OK: Use a W6 × 12

(b) ASD solution

Load combination 3 controls.

$$w_a = w_D + w_S = 0.1386 + 0.1500 = 0.2886 \text{ kips/ft}$$

Component normal to roof $= w_{ax} = \frac{3}{\sqrt{10}}(0.2886) = 0.2738$ kips/ft

Component parallel to roof $= w_{ay} = \frac{1}{\sqrt{10}}(0.2886) = 9.126 \times 10^{-2}$ kips/ft

$$M_{ax} = \frac{1}{8}w_{ax}L^2 = \frac{1}{8}(0.2738)(18)^2 = 11.09 \text{ ft-kips}$$

To compute M_{ay}, use Table 3-22c, "Continuous Beams," in Part 3 of the *Manual*. For a two-span continuous beam, the maximum moment is

$$M_{ay} = 0.125 w_{ay}\left(\frac{L}{2}\right)^2 = 0.125(0.09126)\left(\frac{18}{2}\right)^2 = 0.924 \text{ ft-kips}$$

Try a W6 × 12. From Part (a), $M_{nx} = 30.35$ ft-kips, $M_{ny} = 9.667$ ft-kips

$$\frac{M_{nx}}{\Omega_b} = \frac{32.52}{1.67} = 19.47 \text{ ft-kips}, \qquad \frac{M_{ny}}{\Omega_b} = \frac{9.667}{1.67} = 5.789 \text{ ft-kips}$$

Conservatively, consider all loads, including the purlin weight, to act at the top flange and use 1/2 of weak-axis bending strength in the interaction equation:

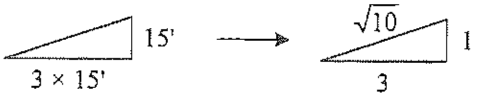
$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{11.09}{19.47} + \frac{0.924}{5.789/2} = 0.889 < 1.0 \qquad \text{(OK)}$$

Weight assumption and shear strength are OK: Use a W6 × 12

5.15-8

Dead load: assume purlin weighs 10 lb/ft.

Other dead load:



$$\text{Tributary width} = \frac{\sqrt{10}}{3}\left(\frac{15}{2}\right) = 7.906 \text{ ft}$$

$$w_D = 7.906(15) + 10 = 128.6 \text{ lb/ft}$$

Snow: $w_S = \left(\frac{15}{2}\right)(20) = 150.0$ lb/ft

(a) LRFD solution

Load combination 3 controls.

$$w_u = 1.2w_D + 1.6w_S = 1.2(0.1286) + 1.6(0.1500) = 0.3943 \text{ kips/ft}$$

$$\text{Component normal to roof} = w_{ux} = \frac{3}{\sqrt{10}}(0.3943) = 0.3741 \text{ kips/ft}$$

$$\text{Component parallel to roof} = w_{uy} = \frac{1}{\sqrt{10}}(0.3943) = 0.1247 \text{ kips/ft}$$

$$M_{ux} = \frac{1}{8}w_{ux}L^2 = \frac{1}{8}(0.3741)(18)^2 = 15.15 \text{ ft-kips}$$

To compute M_{uy}, use Table 3-22c, "Continuous Beams," in Part 3 of the *Manual*. For a three-span continuous beam, the maximum moment is

$$M_{uy} = 0.10 w_{uy}\left(\frac{L}{3}\right)^2 = 0.10(0.1247)\left(\frac{18}{3}\right)^2 = 0.4489 \text{ ft-kips}$$

Try a W6 × 9:

Determine the strong-axis bending strength. The beam design charts in the *Manual* do not contain curves for shapes smaller than W8, so the flexural strength of the W6 × 9 must be computed. From the dimensions and properties tables, the shape is noncompact (see footnote).

Compute the strength based on the limit state of flange local buckling.

$$M_n = M_p - (M_p - 0.7F_yS_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

$$M_p = F_yZ_x = 50(6.23) = 311.5 \text{ in.-kips}$$

$$\lambda = \frac{b_f}{2t_f} = 9.16$$

$$\lambda_p = 0.38\sqrt{\frac{29{,}000}{50}} = 9.152, \qquad \lambda_r = 1.0\sqrt{\frac{29{,}000}{50}} = 24.08$$

$$M_n = 311.5 - (311.5 - 0.7 \times 50 \times 5.56)\left(\frac{9.16 - 9.152}{24.08 - 9.152}\right)$$

$$= 311.4 \text{ in.-kips} = 25.95 \text{ ft-kips}$$

Lateral-torsional buckling:

The following properties of a W6 × 9 will be needed:

$A = 2.68$ in.^{2}, $S_x = 5.56$ in.^{3}, $Z_x = 6.23$ in.^{3} $I_y = 2.20$ in.^{4}, $r_y = 0.905$ in., $S_y = 1.11$ in.^{3}, $Z_y = 1.72$ in.^{3}, $r_{ts} = 1.06$ in., $h_o = 5.69$ in., $J = 0.0405$ in.^{4}

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}} = 1.76(0.905)\sqrt{\frac{29{,}000}{50}} = 38.36 \text{ in.} = 3.197 \text{ ft.}$$

From AISC Equation F2-6,

$$L_r = 1.95r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_xh_o}}\sqrt{1 + \sqrt{1 + 6.76\left(\frac{0.7F_yS_xh_o}{EJc}\right)^2}}$$

$$= 1.95(1.06)\frac{29{,}000}{0.7(50)}\sqrt{\frac{0.0405(1.0)}{5.56(5.69)}}\sqrt{1 + \sqrt{1 + 6.76\left(\frac{0.7(50)(5.56)(5.69}{29{,}000(0.0405)(1.0)}\right)^2}}$$

$$= 117.0 \text{ in.} = 9.75 \text{ ft}$$

For $L_b = 18/3 = 6$ ft, $L_p < L_b < L_r$, so

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(inelastic LTB)}$$

$C_b = 1.01$ (see Table 3-1 in the *Manual*).

$$M_p = F_yZ_x = 50(6.23) = 311.5 \text{ in.-kips} = 25.96 \text{ ft-kips}$$

$$M_n = 1.01\left[311.5 - (311.5 - 0.7 \times 50 \times 5.56)\left(\frac{6 - 3.197}{9.75 - 3.197}\right)\right]$$

$$= 264.1 \text{ in.-kips} = 22.01 \text{ ft-kips} < M_p$$

Lateral-torsional buckling controls.

$$\phi_b M_{nx} = 0.90(22.01) = 19.81 \text{ ft-kips}$$

Weak-axis bending strength: Since the shape is noncompact, use AISC Equation F6-2:

$$M_{ny} = M_{py} - (M_{py} - 0.7F_yS_y)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

$$M_{py} = F_yZ_y = 50(1.72) = 86.0 \text{ in.-kips}$$

$$M_{ny} = 86.0 - (86.0 - 0.7 \times 50 \times 1.11)\left(\frac{9.16 - 9.152}{24.08 - 9.152}\right)$$

$$= 85.97 \text{ in.-kips} = 7.164 \text{ ft-kips}$$

$$\phi_b M_{ny} = 0.90(7.164) = 6.448 \text{ ft-kips}$$

Conservatively, consider all loads, including the purlin weight, to act at the top flange and use 1/2 of weak-axis bending strength in the interaction equation:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{15.15}{19.81} + \frac{0.4489}{6.448/2} = 0.904 < 1.0 \qquad \text{(OK)}$$

Weight and shear strength are OK: Use a W6 × 9

(b) ASD solution

Load combination 3 controls.

$$w_a = w_D + w_S = 0.1286 + 0.1500 = 0.2786 \text{ kips/ft}$$

Component normal to roof $= w_{ax} = \dfrac{3}{\sqrt{10}}(0.2786) = 0.2643$ kips/ft

Component parallel to roof $= w_{ay} = \dfrac{1}{\sqrt{10}}(0.2786) = 0.0881$ kips/ft

$$M_{ax} = \frac{1}{8}w_{ax}L^2 = \frac{1}{8}(0.2643)(18)^2 = 10.7 \text{ ft-kips}$$

To compute M_{ay}, use Table 3-22c, "Continuous Beams," in Part 3 of the *Manual*. For a three-span continuous beam, the maximum moment is

$$M_{ay} = 0.10w_{ay}\left(\frac{L}{3}\right)^2 = 0.10(0.0881)\left(\frac{18}{3}\right)^2 = 0.3172 \text{ ft-kips}$$

Try a W6 × 9. From Part (a), $M_{nx} = 22.01$ ft-kips, $M_{ny} = 7.164$ ft-kips

$$\frac{M_{nx}}{\Omega_b} = \frac{22.01}{1.67} = 13.18 \text{ ft-kips}, \qquad \frac{M_{ny}}{\Omega_b} = \frac{7.164}{1.67} = 4.290 \text{ ft-kips}$$

Conservatively, consider all loads, including the purlin weight, to act at the top flange and use 1/2 of weak-axis bending strength in the interaction equation:

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{10.7}{13.18} + \frac{0.3127}{4.290/2} = 0.958 < 1.0 \qquad \text{(OK)}$$

Weight and shear strength are OK: Use a W6 × 9

6.2-1

(a) LRFD solution:

From the column load tables, the compressive design strength of a W12 × 106 with $F_y = 50$ ksi and $K_yL = 1.0 \times 14 = 14$ feet is

$$\phi_c P_n = 1130 \text{ kips}$$

From the design charts in Part 3 of the *Manual*, for $L_b = 14$ ft and $C_b = 1.0$,

$$\phi_b M_n = 597 \text{ ft-kips} \qquad \text{(Since the bending moment is uniform, } C_b = 1.0.\text{)}$$

The factored axial compressive load is

$$P_u = 1.2P_D + 1.6P_L = 1.2(0.25 \times 250) + 1.6(0.75 \times 250) = 375.0 \text{ kips}$$

The factored bending moment is

$$M_u = 1.2M_D + 1.6M_L = 1.2(0.25 \times 240) + 1.6(0.75 \times 240) = 360.0 \text{ ft-kips}$$

Determine which interaction equation controls:

$$\frac{P_u}{\phi_c P_n} = \frac{375}{1130} = 0.3319 > 0.2 \qquad \therefore \text{ use Equation 6.3 (AISC Equation H1-1a)}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{375}{1130} + \frac{8}{9}\left(\frac{360}{597} + 0\right) = 0.868 < 1.0 \qquad \text{(OK)}$$

This member satisfies the AISC Specification

(b) ASD solution:

From the column load tables, the allowable compressive strength of a W12 × 106 with $F_y = 50$ ksi and $K_yL = 1.0 \times 14 = 14$ feet is

$$\frac{P_n}{\Omega_c} = 754 \text{ kips}$$

From the design charts in Part 3 of the *Manual*, for $L_b = 14$ ft and $C_b = 1.0$,

$$\frac{M_n}{\Omega_b} = 398 \text{ ft-kips} \qquad \text{(Since the bending moment is uniform, } C_b = 1.0.\text{)}$$

The total axial compressive load is $P_a = 250$ kips

Determine which interaction equations controls:

$$\frac{P_a}{P_n/\Omega_c} = \frac{250}{754} = 0.3316 > 0.2 \qquad \therefore \text{ use Equation 6.5 (AISC Equation H1-1a)}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = \frac{250}{754} + \frac{8}{9}\left(\frac{240}{398} + 0\right) = 0.868 < 1.0 \qquad \text{(OK)}$$

This member satisfies the AISC Specification

6.2-2

(a) Compute compressive strength (this shape is not in the column load tables).

For a W18 × 86, $A_g = 25.3 \text{ in.}^2$, $r_y = 2.63$ in., and the shape is not slender (no footnote).

$$\frac{KL}{r_y} = \frac{1.0(20 \times 12)}{2.63} = 91.25 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(91.25)^2} = 34.37 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113.4$$

Since $KL/r = 91.25 < 113.4$, use AISC Eq. E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/34.37)}(50) = 27.20 \text{ ksi}$$

$$P_n = F_{cr}A_g = 27.20(25.3) = 688.2 \text{ kips}$$

$$\phi_c P_n = 0.90(688.2) = 619.4 \text{ kips}$$

From the beam design charts in Part 5 of the *Manual*, for $L_b = 20$ ft, $\phi_b M_n = 552$ ft-kips for $C_b = 1$.

For this case , $C_b = 1.14$ (Figure 5.15, textbook). For $C_b = 1.14$,

$$\phi_b M_n = 1.14 \times 552 = 629.3 \text{ ft-kips} < \phi_b M_p = 698 \text{ ft-kips}$$

Factored axial load $= P_u = 1.2D + 1.6L = 1.2(10) + 1.6(20) = 44.0$ kips

$$\frac{P_u}{\phi_c P_n} = \frac{44.0}{619.4} = 7.104 \times 10^{-2} < 0.2 \therefore \text{ use Eq. 6.4 (AISC Eq. H1-1b):}$$

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{0.07104}{2} + \left(\frac{M_{ux}}{629.3} + 0\right) = 1.0$$

$$M_{ux} = 606.9 \text{ ft-kips}$$

Let $\frac{1}{8}w_u L^2 = M_{ux}$: $\quad \frac{1}{8}w_u(20)^2 = 606.9 \quad \Rightarrow \quad w_u = 12.14$ kips/ft

$$w_u = 1.2w_D + 1.6w_L$$

$12.14 = 1.2(0.086) + 1.6w_L,$ $\qquad$ $\underline{w_L = 7.52 \text{ kips/ft}}$

(b) From part (a), $P_n = 688.2$ kips, $\quad \frac{P_n}{\Omega_c} = \frac{688.2}{1.67} = 412.1$ kips

From the beam design charts in Part 5 of the *Manual*, for $L_b = 20$ ft,

$$\frac{M_n}{\Omega_b} = 368 \text{ ft-kips for } C_b = 1.$$

For this case , $C_b = 1.14$ (Figure 5.15, textbook). For $C_b = 1.14$,

$$\frac{M_n}{\Omega_b} = 1.14 \times 368 = 419.5 \text{ ft-kips} < \frac{M_p}{\Omega_b} = 464 \text{ ft-kips}$$

Axial load $= P_a = D + L = 10 + 20 = 30$ kips

$$\frac{P_a}{P_n/\Omega_c} = \frac{30}{412.1} = 7.280 \times 10^{-2} < 0.2$$

∴ use Equation 6.6 (AISC Equation H1-1b)

$$\frac{P_a}{2P_n/\Omega_c} + \left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = \frac{0.07280}{2} + \left(\frac{M_{ax}}{419.5} + 0\right) = 1.0$$

$$M_{ax} = 404.2 \text{ ft-kips}$$

Let $\frac{1}{8}w_aL^2 = M_{ax}$: $\quad \frac{1}{8}w_a(20)^2 = 404.2 \quad \Rightarrow \quad w_a = 8.084$ kips/ft

$w_D + w_L = 0.086 + w_L = 8.084,$ $\quad \underline{w_L = 8.00 \text{ kips/ft}}$

6.5-1

In the plane of bending,

$$P_{e1} = \frac{\pi^2 EI}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(933)}{(1.0 \times 14 \times 12)^2} = 9462 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{240}{240}\right) = 1.0$$

(a) LRFD solution:

$$P_u = 1.2P_D + 1.6P_L = 1.2(0.25 \times 250) + 1.6(0.75 \times 250) = 375.0 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.00P_u/P_{e1})} = \frac{1.0}{1 - (375/9462)} = 1.041$$

$\underline{B_1 = 1.04}$

(b) ASD solution: $P_a = 250$ kips

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.60P_a/P_{e1})} = \frac{1.0}{1 - (1.60)(375)/9462} = 1.068$$

$\underline{B_1 = 1.07}$

6.5-2

In the plane of bending,

$$P_{e1} = \frac{\pi^2 EI}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(1530)}{(1.0 \times 20 \times 12)^2} = 7603 \text{ kips}$$

$$C_m = 1.0$$

(a) LRFD solution:

$$P_u = 1.2P_D + 1.6P_L = 1.2(10) + 1.6(20) = 44.0 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00P_u/P_{e1})} = \frac{1.0}{1-(44.0/7603)} = 1.006$$

$\underline{B_1 = 1.01}$

(b) ASD solution: $P_a = 10+20 = 30.0$ kips

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{1.0}{1-(1.60)(30.0)/7603} = 1.006$$

$\underline{B_1 = 1.01}$

6.5-3

$K_xL = 0.9(14) = 12.6$ ft, $K_yL = 1.0(14) = 14$ ft.

(a) LRFD solution:

From the column load tables, for $KL = 14$ ft, $\phi_c P_n = 1130$ kips

From the design charts in Part 3 of the *Manual*, for $L_b = 14$ ft and $C_b = 1.0$, $\phi_b M_n = 642$ ft-kips and $\phi_b M_p = 646$ ft-kips. For $C_b = 1.6$,

$\phi_b M_n = 1.6(642) = 1027 \text{ ft-kips} > \phi_b M_p$ $\therefore$ use $\phi_b M_n = \phi_b M_p = 646$ ft-kips

$$P_u = 1.2P_D + 1.6P_L = 1.2(0.33 \times 342) + 1.6(0.67 \times 342) = 502.1 \text{ kips}$$

$$M_{nt} = 1.2M_D + 1.6M_L = 1.2(0.33 \times 246) + 1.6(0.67 \times 246) = 361.1 \text{ ft-kips}$$

For the axis of bending, $C_m = 1.0$ and

$$P_{e1} = \frac{\pi^2 EI}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(1110)}{(12.6 \times 12)^2} = 1.390 \times 10^4 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00P_u/P_{e1})} = \frac{1.0}{1-(502.1/13900)} = 1.037$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.037(361.1) + 0 = 374.5 \text{ ft-kips}$$

Determine which interaction equation controls:

$\frac{P_u}{\phi_c P_n} = \frac{502.1}{1130} = 0.4443 > 0.2$ $\therefore$ use Equation 6.3 (AISC Equation H1-1a)

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.4443 + \frac{8}{9}\left(\frac{374.5}{646} + 0\right) = 0.960 < 1.0 \quad \text{(OK)}$$

This member satisfies the AISC Specification

(b) ASD solution:

From the column load tables, for $KL = 14$ ft, $\frac{P_n}{\Omega_c} = 751$ kips

From the design charts in Part 3 of the *Manual*, for $L_b = 14$ ft and $C_b = 1.0$, $\frac{M_n}{\Omega_b} = 428$

ft-kips and $\frac{M_p}{\Omega_b} = 430$ ft-kips. For $C_b = 1.6$,

$$\frac{M_n}{\Omega_b} = 1.6(428) = 684.8 \text{ ft-kips} > \frac{M_p}{\Omega_b} \quad \therefore \text{ use } \frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 430 \text{ ft-kips}$$

$$P_a = 342 \text{ kips}, \quad M_{nt} = 246 \text{ ft-kips}$$

For the axis of bending, $C_m = 1.0$ and

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29{,}000)(1110)}{(12.6 \times 12)^2} = 1.390 \times 10^4 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r / P_{e1})} = \frac{C_m}{1 - (1.60 P_a / P_{e1})} = \frac{1.0}{1 - 1.60(342/13900)} = 1.041$$

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.041(246) + 0 = 256.1 \text{ ft-kips}$$

Determine which interaction equations controls:

$$\frac{P_a}{P_n/\Omega_c} = \frac{342}{751} = 0.4554 > 0.2 \qquad \therefore \text{ use Equation 6.5 (AISC Equation H1-1a)}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.4554 + \frac{8}{9}\left(\frac{256.1}{430} + 0\right)$$

$$= 0.985 < 1.0 \qquad \text{(OK)}$$

This member satisfies the AISC Specification

6.5-4

(a) LRFD solution:

The factored-load axial force is

$$P_u = 1.2P_D + 1.6P_L = 1.2(0.30 \times 120) + 1.6(0.70 \times 120) = 177.6 \text{ kips}$$

The factored-load end moments are

$$M_{top} = 1.2M_D + 1.6M_L = 1.2(0.30 \times 135) + 1.6(0.70 \times 135) = 199.8 \text{ ft-kips}$$

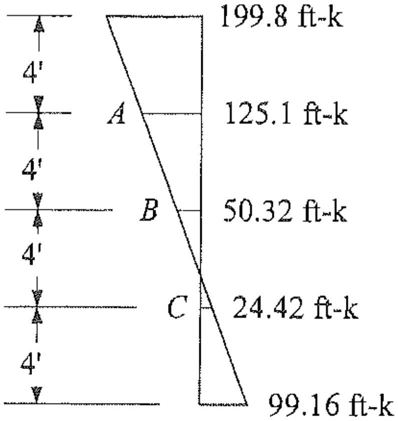
$$M_{bot} = 1.2M_D + 1.6M_L = 1.2(0.30 \times 67) + 1.6(0.70 \times 67) = 99.16 \text{ ft-kips}$$

From the column load tables, for $KL = 16$ ft, $\phi_c P_n = 500$ kips

From the beam design charts in Part 3 of the *Manual*, for $L_b = 16$ ft and $C_b = 1.0$,

$$\phi_b M_n = 283.5 \text{ ft-kips}, \quad \phi_b M_p = 324 \text{ ft-kips}.$$

Compute C_b:



$$C_b = \frac{12.5M_{\text{max}}}{2.5M_{\text{max}} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(199.8)}{2.5(199.8) + 3(125.1) + 4(50.32) + 3(24.42)} = 2.173$$

For $C_b = 2.173, \ \phi_b M_n = 2.173(283.5) = 616$ ft-kips

Since 616 ft-kips > $\phi_b M_p$, use $\phi_b M_n = \phi_b M_p = 324$ ft-kips

For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{99.16}{199.8}\right) = 0.4015$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29,000)(475)}{(16 \times 12)^2} = 3688 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r / P_{e1})} = \frac{C_m}{1 - (1.00 P_u / P_{e1})} = \frac{0.4015}{1 - (177.6/3688)}$$

$$= 0.4218 < 1.0 \therefore \text{ use } B_1 = 1.0$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(199.8) + 0 = 199.8 \text{ ft-kips}$$

Determine which interaction equation controls:

$$\frac{P_u}{\phi_c P_n} = \frac{177.6}{500} = 0.3552 > 0.2 \qquad \therefore \text{ use Equation 6.3 (AISC Equation H1-1a)}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.3552 + \frac{8}{9}\left(\frac{199.8}{324} + 0\right)$$

$$= 0.903 < 1.0 \qquad \text{(OK)}$$

This member satisfies the AISC Specification

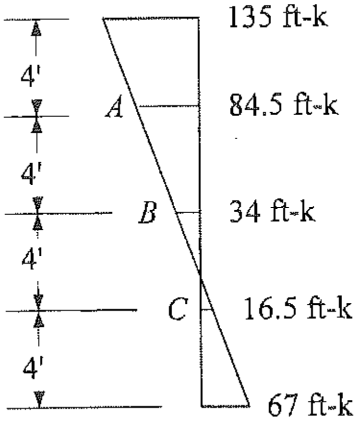
(b) ASD solution:

From the column load tables, for $KL = 16$ ft, $\frac{P_n}{\Omega_c} = 333$ kips

From the design charts in Part 3 of the *Manual*, for $L_b = 16$ ft and $C_b = 1.0$,

$\dfrac{M_n}{\Omega_b} = 189$ ft-kips and $\dfrac{M_p}{\Omega_b} = 216$ ft-kips.

Compute C_b:



$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(135)}{2.5(135) + 3(84.5) + 4(34) + 3(16.5)} = 2.173$$

For $C_b = 2.173$,

$$\frac{M_n}{\Omega_b} = 2.173(189) = 411 \text{ ft-kips} > \frac{M_p}{\Omega_b} \quad \therefore \text{ use } \frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 216 \text{ ft-kips}$$

$$P_a = 120 \text{ kips}, \; M_{nt} = 135 \text{ ft-kips}$$

For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{67}{135}\right) = 0.4015$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(475)}{(16 \times 12)^2} = 3688 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.60P_a/P_{e1})} = \frac{0.4015}{1 - 1.60(120/3688)}$$

$$= 0.4236 < 1.0 \;\therefore\; \text{use } B_1 = 1.0$$

$$M_a = B_1M_{nt} + B_2M_{\ell t} = 1.0(135) + 0 = 135 \text{ ft-kips}$$

Determine which interaction equations controls:

$$\frac{P_a}{P_n/\Omega_c} = \frac{120}{333} = 0.3604 > 0.2 \qquad \therefore \text{ use Equation 6.5 (AISC Equation H1-1a)}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.3604 + \frac{8}{9}\left(\frac{135}{216} + 0\right)$$

$$= 0.916 < 1.0 \qquad \text{(OK)}$$

This member satisfies the AISC Specification

6.5-5

(a) LRFD solution:

$$P_u = 1.2(16) + 1.6(16) = 44.8 \text{ kips}, \qquad M_{nt} = 1.2(30) + 1.6(30) = 84.0 \text{ ft-kips}$$

For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{84}{84}\right) = 1.0$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29,000)(171)}{(10 \times 12)^2} = 3399 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00P_u/P_{e1})} = \frac{1.0}{1-(44.8/3399)} = 1.013$$

$$M_u = B_1 M_{nt} = 1.013(84) = 85.09 \text{ ft-kips}$$

Compute the moment strength. For this loading, $C_b = 1.0$. From the beam design charts with $L_b = 10$ ft and $C_b = 1.0$, $\phi_b M_n = 134$ ft-kips.

From the column load tables with $KL = 10$ ft, $\phi_c P_n = 330$ kips.

$$\frac{P_u}{\phi_c P_n} = \frac{44.8}{330} = 0.1358 < 0.2 \therefore \text{ use Equation 6.4 (AISC Eq. H1-1b):}$$

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{44.8}{2(330)} + \left(\frac{85.09}{134} + 0\right)$$

$$= 0.703 < 1.0 \qquad \text{(OK)}$$

Member is satisfactory.

(b) ASD solution: $P_a = 32$ kips, $M_{nt} = 60$ ft-kips

For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{60}{60}\right) = 1.0$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29,000)(171)}{(10 \times 12)^2} = 3399 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{1.0}{1-1.60(32/3399)} = 1.015$$

$$M_a = B_1 M_{nt} = 1.015(60) = 60.90 \text{ ft-kips}$$

Compute the moment strength. For this loading, $C_b = 1.0$. From the beam design charts with $L_b = 10$ ft and $C_b = 1.0$, $\dfrac{M_n}{\Omega_b} = 89.5$ ft-kips.

From the column load tables with $KL = 10$ ft, $\dfrac{P_n}{\Omega_c} = 220$ kips.

$$\frac{P_a}{P_n/\Omega_c} = \frac{32}{220} = 0.1455 < 0.2 \qquad \therefore \text{ use Equation 6.6 (AISC Equation H1-1b)}$$

$$\frac{P_a}{2P_n/\Omega_c} + \left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = \frac{32}{2(220)} + \left(\frac{60.90}{89.5} + 0\right)$$

$$= 0.753 < 1.0 \qquad \text{(OK)}$$

Member is satisfactory.

6.5-6

(a) LRFD solution:

The factored-load axial force is

$$P_u = 1.2P_D + 1.6P_L = 1.2(0.25 \times 190) + 1.6(0.75 \times 190) = 285.0 \text{ kips}$$

The factored-load end moments are

$$M_{top} = 1.2M_D + 1.6M_L = 1.2(0.25 \times 80) + 1.6(0.75 \times 80) = 120.0 \text{ ft-kips}$$

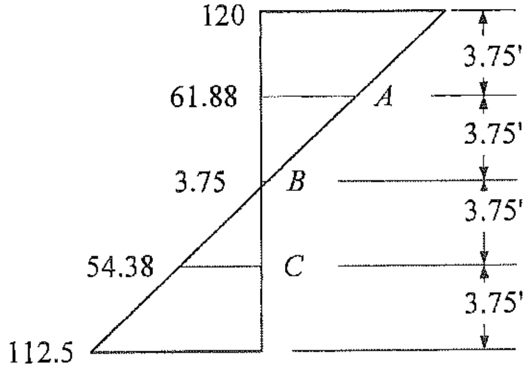
$$M_{bot} = 1.2M_D + 1.6M_L = 1.2(0.25 \times 75) + 1.6(0.75 \times 75) = 112.5 \text{ ft-kips}$$

From the column load tables, for $KL = 15$ ft, $\phi_c P_n = 555$ kips

From the beam design charts in Part 3 of the *Manual*, for $L_b = 15$ ft and $C_b = 1.0$,

$\phi_b M_n = 257$ ft-kips, $\phi_b M_p = 280$ ft-kips.

Compute C_b:



$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(120)}{2.5(120) + 3(61.88) + 4(3.75) + 3(54.38)} = 2.260$$

For $C_b = 2.260$, $\phi_b M_n = 2.260(257) = 581$ ft-kips

Since 581 ft-kips > $\phi_b M_p$, use $\phi_b M_n = \phi_b M_p = 280$ ft-kips

For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{112.5}{120.0}\right) = 0.225$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29{,}000)(341)}{(15 \times 12)^2} = 3012 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r / P_{e1})} = \frac{C_m}{1 - (1.00 P_u / P_{e1})} = \frac{0.225}{1 - (285/3012)}$$

$$= 0.2485 < 1.0 \quad \therefore \text{ use } B_1 = 1.0$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(120.0) + 0 = 120.0 \text{ ft-kips}$$

Determine which interaction equation controls:

$$\frac{P_u}{\phi_c P_n} = \frac{285}{555} = 0.5135 > 0.2 \qquad \therefore \text{ use Equation 6.3 (AISC Equation H1-1a)}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.5135 + \frac{8}{9}\left(\frac{120.0}{280} + 0\right)$$

$$= 0.894 < 1.0 \quad \text{(OK)}$$

This member satisfies the AISC Specification

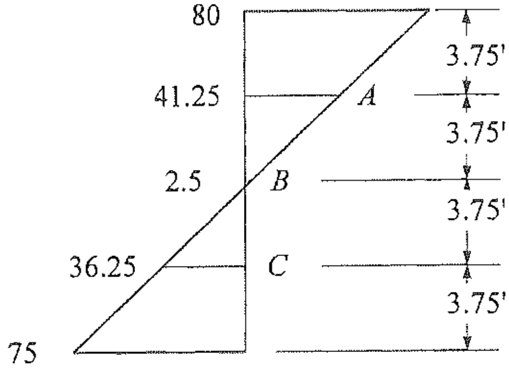
(b) ASD solution:

From the column load tables, for $KL = 15$ ft, $\frac{P_n}{\Omega_c} = 369$ kips

From the design charts in Part 3 of the *Manual*, for $L_b = 15$ ft and $C_b = 1.0$,

$$\frac{M_n}{\Omega_b} = 171 \text{ ft-kips and } \frac{M_p}{\Omega_b} = 186 \text{ ft-kips.}$$

Compute C_b:



$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(80)}{2.5(80) + 3(41.25) + 4(2.5) + 3(36.25)} = 2.260$$

For $C_b = 2.260$,

$$\frac{M_n}{\Omega_b} = 2.260(171) = 386 \text{ ft-kips} > \frac{M_p}{\Omega_b} \quad \therefore \text{ use } \frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 186 \text{ ft-kips}$$

$$P_a = 190 \text{ kips}, \ M_{nt} = 80 \text{ ft-kips}$$

For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{75}{80}\right) = 0.225$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29{,}000)(341)}{(15 \times 12)^2} = 3012 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r / P_{e1})} = \frac{C_m}{1 - (1.60 P_a / P_{e1})} = \frac{0.225}{1 - 1.60(190/3012)}$$

$$= 0.2503 < 1.0 \ \therefore \text{ use } B_1 = 1.0$$

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(80) + 0 = 80.0 \text{ ft-kips}$$

Determine which interaction equations controls:

$$\frac{P_a}{P_n/\Omega_c} = \frac{190}{369} = 0.5149 > 0.2 \qquad \therefore \text{ use Equation 6.5 (AISC Equation H1-1a)}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.5419 + \frac{8}{9}\left(\frac{80.0}{186} + 0\right)$$

$$= 0.924 < 1.0 \qquad \text{(OK)}$$

This member satisfies the AISC Specification

6.5-7

(a) LRFD solution:

$$P_u = 1.2(70) + 1.6(170) = 356.0 \text{ kips}, \qquad w_u = 1.2(1.5) + 1.6(3.5) = 7.4 \text{ kips/ft}$$

$$Q_u = 1.2(7) + 1.6(18) = 37.2 \text{ kips}$$

$$M_{nt} = \frac{1}{8}w_uL^2 + \frac{Q_uL}{4} = \frac{1}{8}(7.4)(16)^2 + \frac{37.2(16)}{4} = 385.6 \text{ ft-kips}$$

Compute the moment strength. From the beam design charts, for $L_b = 8$ ft,

$$\phi_bM_n = \phi_bM_p = 488 \text{ ft-kips.} \ (L_b < L_p)$$

Compute the amplified moment. Use $C_m = 1.0$.

$$P_{e1} = \frac{\pi^2EI}{(K_1L)^2} = \frac{\pi^2EI_x}{(K_xL)^2} = \frac{\pi^2(29,000)(623)}{(16 \times 12)^2} = 4837 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00P_u/P_{e1})} = \frac{1.0}{1-(356/4837)} = 1.079$$

$$M_u = B_1M_{nt} + B_2M_{\ell t} = 1.079(385.6) + 0 = 416.1 \text{ ft-kips}$$

Compressive strength: $K_yL = 8$ ft, $\quad K_xL = 16$ ft, $\quad \dfrac{K_xL}{r_x/r_y} = \dfrac{16}{1.74} = 9.195 \text{ ft} > 8$ ft.

From the column load tables, for $KL = 9.195$ ft, $\phi_cP_n = 1164$ kips

$$\frac{P_u}{\phi_cP_n} = \frac{356}{1164} = 0.3058 > 0.2 \ \therefore \text{ use Eq. 6.3 (AISC Eq. H1-1a)}$$

$$\frac{P_u}{\phi_cP_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_bM_{nx}} + \frac{M_{uy}}{\phi_bM_{ny}}\right) = 0.3058 + \frac{8}{9}\left(\frac{416.1}{488} + 0\right) = 1.06 > 1.0 \quad \text{(N.G.)}$$

Member is unsatisfactory.

(b) ASD solution:

$$P_a = 70 + 170 = 240 \text{ kips}, \qquad w_a = 1.5 + 3.5 = 5.0 \text{ kips/ft}, \qquad Q_a = 7 + 18 = 25 \text{ kips}$$

$$M_{nt} = \frac{1}{8}w_aL^2 + \frac{Q_aL}{4} = \frac{1}{8}(5.0)(16)^2 + \frac{25(16)}{4} = 260.0 \text{ ft-kips}$$

Compute the moment strength. From the beam design charts, for $L_b = 8$ ft,

$$\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 324 \text{ ft-kips.} \ (L_b < L_p)$$

Compute the amplified moment. Use $C_m = 1.0$.

$$P_{e1} = \frac{\pi^2EI}{(K_1L)^2} = \frac{\pi^2EI_x}{(K_xL)^2} = \frac{\pi^2(29,000)(623)}{(16 \times 12)^2} = 4837 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{1.0}{1-1.60(240/4837)} = 1.086$$

$$M_a = B_1M_{nt} + B_2M_{\ell t} = 1.086(260.0) + 0 = 282.4 \text{ ft-kips}$$

Compressive strength: $K_yL = 8$ ft, $\quad K_xL = 16$ ft, $\quad \dfrac{K_xL}{r_x/r_y} = \dfrac{16}{1.74} = 9.195$ ft > 8 ft.

From the column load tables, for $KL = 9.195$ ft, $\dfrac{P_n}{\Omega_c} = 775.7$ kips

$$\frac{P_a}{P_n/\Omega_c} = \frac{240}{775.7} = 0.3094 > 0.2 \qquad \therefore \text{ use Equation 6.5 (AISC Equation H1-1a)}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.3094 + \frac{8}{9}\left(\frac{282.4}{324} + 0\right)$$

$$= 1.08 > 1.0 \qquad \text{(N.G.)}$$

Member is unsatisfactory.

6.5-8

(a) LRFD solution:

The factored-load axial force is

$$P_u = 1.2P_D + 1.6P_L = 1.2(625/2) + 1.6(625/2) = 875.0 \text{ kips}$$

The factored-load end moments are

$$M_{top} = 1.2M_D + 1.6M_L = 1.2(195/2) + 1.6(195/2) = 273.0 \text{ ft-kips}$$

$$M_{bot} = 1.2M_D + 1.6M_L = 1.2(225/2) + 1.6(225/2) = 315.0 \text{ ft-kips}$$

This shape is not in the column load tables, so the axial compressive strength must be computed. Also, the footnote in the dimensions and properties table indicates that a W33 × 118 is a slender element shape. First, compute the flexural buckling strength.

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{1.0(11 \times 12)}{2.32} = 56.90 < 200 \qquad \text{(OK)}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4$$

Since 56.90 < 113.4, use AISC Equation E3-2.

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(56.90)^2} = 88.40 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/88.40)}(50) = 39.46 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 39.46(34.7) = 1369 \text{ kips}$$

Check width-thickness ratios. From the dimensions and properties table in the *Manual*, the width-thickness ratio of the web is

$$\frac{h}{t_w} = 54.5$$

From AISC Table B4.1, case 10 (and Figure 4.9 in this book), the upper limit for nonslender elements is

$$1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29{,}000}{50}} = 35.88$$

Since $\frac{h}{t_w} > 1.49\sqrt{\frac{E}{F_y}}$, the web is slender.

For the flange,

$$\frac{b_f}{2t_f} = 7.76 < 0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000}{50}} = 13.49 \therefore \text{ flange is not slender}$$

Because the web is a stiffened element, $Q_s = 1.0$, and Q_a must be computed from AISC Section E7.2. AISC E7.2(a) applies, provided that

$$\frac{b}{t} \geq 1.49\sqrt{\frac{E}{f}}$$

where b/t is the generic notation for the width-thickness ratio and $f = F_{cr}$ computed with $Q = 1.0$. From the flexural buckling strength computations above, $F_{cr} = 39.46$ ksi.

$$1.49\sqrt{\frac{E}{f}} = 1.49\sqrt{\frac{29{,}000}{39.46}} = 40.39$$

Since $\frac{b}{t} \geq 1.49\sqrt{\frac{E}{f}}$, local buckling must be considered. From AISC Equation E7-17, the reduced width of the web is

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{(b/t)}\sqrt{\frac{E}{f}}\right] \leq b$$

$$= 1.92(0.550)\sqrt{\frac{29{,}000}{39.46}}\left[1 - \frac{0.34}{54.5}\sqrt{\frac{29{,}000}{39.46}}\right] = 23.79 \text{ in.}$$

Unreduced width is

$$b = d - 2k_{des} = 32.9 - 2(1.44) = 30.02 \text{ in.}$$

Reduced area is

$$A_{eff} = A - t_w(b - b_e) = 34.7 - 0.550(30.02 - 23.79) = 31.27 \text{ in.}^2$$

$$Q_a = \frac{A_{eff}}{A} = \frac{31.27}{34.7} = 0.9012$$

$Q = Q_sQ_a = 1.0(0.9012) = 0.9012$

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000}{0.9012(50)}} = 119.5$$

$$\frac{KL}{r} = 56.90 < 119.5 \qquad \therefore \text{ use AISC Equation E7-2.}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y$$

$$= 0.9012\left(0.658^{\frac{0.9012(50)}{88.40}}\right)50 = 36.40 \text{ ksi}$$

The nominal strength is

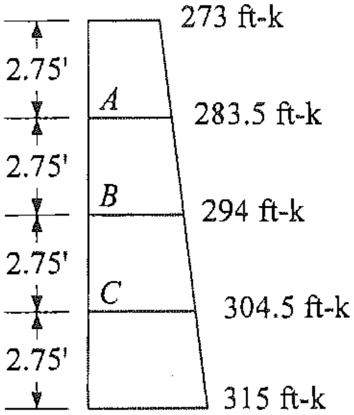
$$P_n = F_{cr}A_g = 36.40(34.7) = 1263 \text{ kips} \;\therefore\; \text{web local buckling controls.}$$

$$\phi_c P_n = 0.90(1263) = 1137 \text{ kips}$$

From the beam design charts in Part 3 of the *Manual*, for $L_b = 11$ ft and $C_b = 1.0$,

$$\phi_b M_n = 1443 \text{ ft-kips}, \;\; \phi_b M_p = 1560 \text{ ft-kips.}$$

Compute C_b:



$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(315)}{2.5(315) + 3(283.5) + 4(294) + 3(304.5)} = 1.056$$

For $C_b = 1.056$, $\phi_b M_n = 1.056(1443) = 1524$ ft-kips $< \phi_b M_p = 1560$ ft-kips.

For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{273}{315}\right) = 0.9467$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29{,}000)(5900)}{(11 \times 12)^2} = 9.692 \times 10^4 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.00P_u/P_{e1})} = \frac{0.9467}{1 - (875/96920)}$$

$$= 0.9553 < 1.0 \;\therefore\; \text{use } B_1 = 1.0$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(315) + 0 = 315 \text{ ft-kips}$$

Determine which interaction equation controls:

$$\frac{P_u}{\phi_c P_n} = \frac{875}{1136} = 0.7702 > 0.2 \qquad \therefore \text{ use Equation 6.3 (AISC Equation H1-1a)}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.7702 + \frac{8}{9}\left(\frac{315}{1524} + 0\right)$$

$$= 0.954 < 1.0 \qquad \text{(OK)}$$

This member satisfies the AISC Specification

(b) ASD solution:

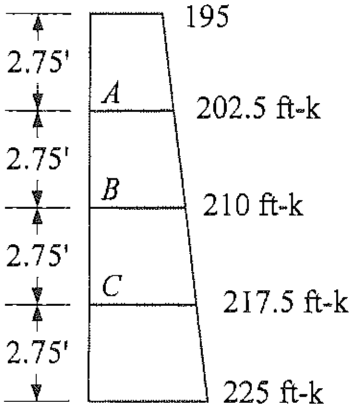
$$P_a = 625 \text{ kips}, \; M_{top} = 195 \text{ ft-kips}, \; M_{bot} = 225 \text{ ft-kips}$$

From part (a), $P_n = 1263$ kips, and $\frac{P_n}{\Omega_c} = \frac{1263}{1.67} = 756.3$ kips

From the design charts in Part 3 of the *Manual*, for $L_b = 11$ ft and $C_b = 1.0$,

$$\frac{M_n}{\Omega_b} = 962 \text{ ft-kips and } \frac{M_p}{\Omega_b} = 1040 \text{ ft-kips.}$$

Compute C_b:



$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(225)}{2.5(225) + 3(202.5) + 4(210) + 3(217.5)} = 1.056$$

For $C_b = 1.056$,

$$\frac{M_n}{\Omega_b} = 1.056(962) = 1016 \text{ ft-kips} < \frac{M_p}{\Omega_b}$$

For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{195}{225}\right) = 0.9467$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29,000)(5900)}{(11 \times 12)^2} = 9.692 \times 10^4 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{0.9467}{1-1.60(625/96920)}$$

$$= 0.9566 < 1.0 \therefore \text{ use } B_1 = 1.0$$

$$M_a = B_1M_{nt} + B_2M_{\ell t} = 1.0(225) + 0 = 225 \text{ ft-kips}$$

Determine which interaction equations controls:

$$\frac{P_a}{P_n/\Omega_c} = \frac{625}{756.3} = 0.8264 > 0.2 \qquad \therefore \text{ use Equation 6.5 (AISC Equation H1-1a)}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.8264 + \frac{8}{9}\left(\frac{225}{1016} + 0\right)$$

$$= 1.02 > 1.0 \qquad \text{(N.G.)}$$

This member does not satisfy the AISC Specification.

6.5-9

(a) LRFD Solution:

The service load moments at each end are

$$M_D = 0.33(150) = 49.50 \text{ ft-kips and } M_L = 0.67(150) = 100.5 \text{ ft-kips}$$

The factored-load moment at each end is

$$1.2M_D + 1.6M_L = 1.2(49.50) + 1.6(100.5) = 220.2 \text{ ft-kips}$$

For the axis of bending,

$$P_{e1} = \frac{\pi^2 EI}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29,000)(833)}{(1.0 \times 15 \times 12)^2} = 7359 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{220.2}{220.2}\right) = 0.2$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.0P_u/P_{e1})} = \frac{0.2}{1-(P_u/7359)}$$

Assume $B_1 = 1.0$ and check it later.

$$\therefore M_{ux} = B_1M_{nt} = 1.0(220.2) = 220.2 \text{ ft-kips}$$

Compute the moment strength. From the beam design charts with $L_b = 15$ ft,

$$\phi_bM_n = 528 \text{ ft-kips for } C_b = 1.0, \text{ and } \phi_bM_p = 551 \text{ ft-kips}$$

From Figure 5.15e in the textbook, $C_b = 2.27$.

For $C_b = 2.27$, $\phi_bM_n = 2.27(528) = 1199$ ft-kips

Since 1199 ft-kips > $\phi_b M_p$, use $\phi_b M_n = \phi_b M_p = 551$ ft-kips

Determine the axial compressive design strength. From the column load tables with $KL = 15$ ft,

$$\phi_c P_n = 990 \text{ kips}$$

Assume that $\dfrac{P_u}{\phi_c P_n} > 0.2$ and use Equation 6.3 (AISC Eq. H1-1a):

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0$$

Let $\dfrac{P_u}{990} + \dfrac{8}{9}\left(\dfrac{220.2}{551} + 0\right) = 1.0$, Solution is: $\{P_u = 638.3\}$

Check assumptions.

$$\frac{P_u}{\phi_c P_n} = \frac{683.3}{990} > 0.2 \qquad \text{(OK)}$$

$$B_1 = \frac{C_m}{1 - (1.0P_u/P_{e1})} = \frac{0.2}{1 - (683.3/7359)} = 0.221 < 1.0$$

$\therefore$ use $B_1 = 1.0$ (as assumed; OK)

Let $1.2D + 1.6L = P_u$

$1.2(0.33P) + 1.6(0.67P) = 638.3$, Solution is: $\{P = 434.8\}$ $\underline{P = 435 \text{ kips}}$

(b) ASD solution:

For the axis of bending,

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29{,}000)(833)}{(1.0 \times 15 \times 12)^2} = 7359 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{150}{150}\right) = 0.2$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.60P_a/P_{e1})} = \frac{0.2}{1 - (1.60P_a/7359)}$$

Assume $B_1 = 1.0$ and check it later.

$\therefore\ M_r = M_{ax} = B_1 M_{nt} = 1.0(150) = 150$ ft-kips

Compute the moment strength. From the beam design charts with $L_b = 15$ ft,

$\dfrac{M_n}{\Omega_b} = 352$ ft-kips for $C_b = 1.0$, and $\dfrac{M_p}{\Omega_b} = 367$ ft-kips

From Figure 5.15e in the textbook, $C_b = 2.27$.

For $C_b = 2.27$, $\dfrac{M_n}{\Omega_b} = 2.27(352) = 799.0$ ft-kips

Since 799.0 ft-kips > $\dfrac{M_p}{\Omega_b}$, use $\dfrac{M_n}{\Omega_b} = \dfrac{M_p}{\Omega_b} = 367$ ft-kips

From the column load tables with $KL = 15$ ft, $P_n/\Omega_c = 659$ kips

Assume that $\frac{P_a}{P_n/\Omega_c} > 0.2$ and use Equation 6.5 (AISC Eq. H1-1a):

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) \leq 1.0$$

Let $\frac{P_a}{659} + \frac{8}{9}\left(\frac{150}{367} + 0\right) = 1.0$, Solution is: $\{P_a = 419.6\}$

Check assumptions.

$$\frac{P_a}{P_n/\Omega_c} = \frac{419.6}{659} > 0.2 \qquad \text{(OK)}$$

$$B_1 = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{0.2}{1-1.60(419.6)/7359} = 0.220 < 1.0$$

$\therefore$ use $B_1 = 1.0$ (as assumed; OK) $\underline{P = 420 \text{ kips}}$

6.5-10

(a) LRFD solution:

$$P_u = 1.2(0.25 \times 53) + 1.6(0.75 \times 53) = 79.5 \text{ kips}$$

$$Q_u = 1.2(0.25 \times 5) + 1.6(0.75 \times 5) = 7.5 \text{ kips}$$

$$M_{nt} = \frac{Q_u L}{4} = \frac{7.5(12)}{4} = 22.5 \text{ ft-kips}$$

Compute the moment strength. From the beam design charts, for $L_b = 12$ ft and $C_b = 1$,

$$\phi_b M_n = 71 \text{ ft-kips}, \ \phi_b M_p = 86.6 \text{ ft-kips}$$

For $C_b = 1.32$ (Fig. 5.15 in the textbook),

$$\phi_b M_n = 1.32(71) = 93.72 \text{ ft-kips} > \phi_b M_p \ \therefore \text{ use } \phi_b M_n = \phi_b M_p = 86.6 \text{ ft-kips}$$

Compute the amplified moment. Use $C_m = 1.0$.

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29{,}000)(82.7)}{(12 \times 12)^2} = 1142 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00P_u/P_{e1})} = \frac{1.0}{1-(79.5/1142)} = 1.075$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.075(22.5) + 0 = 24.19 \text{ ft-kips}$$

Compressive strength: A W8 × 24 is not in the column load tables, so its axial compressive strength must be computed. The shape is not slender for compression (no footnote).

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{1.0(12 \times 12)}{1.61} = 89.44 < 200 \qquad \text{(OK)}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4$$

Since 89.44 < 113.4, use AISC Equation E3-2.

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(89.44)^2} = 35.78 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/35.78)}(50) = 27.86 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 27.86(7.08) = 197.2 \text{ kips}$$

$$\phi_c P_n = 0.90(197.2) = 177.5 \text{ kips}$$

$$\frac{P_u}{\phi_c P_n} = \frac{79.5}{177.5} = 0.4479 > 0.2 \;\therefore\; \text{use Eq. 6.3 (AISC Eq. H1-1a)}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.4479 + \frac{8}{9}\left(\frac{24.19}{86.6} + 0\right) = 0.696 < 1.0 \quad \text{(OK)}$$

Member is satisfactory.

(b) ASD solution:

$$P_a = 53 \text{ kips}, \qquad M_{nt} = \frac{Q_a L}{4} = \frac{5(12)}{4} = 15 \text{ ft-kips}$$

Compute the moment strength. From the beam design charts, for $L_b = 12$ ft,

$$\frac{M_n}{\Omega_b} = 47.5 \text{ ft-kips}, \; \frac{M_p}{\Omega_b} = 57.6 \text{ ft-kips}$$

For $C_b = 1.32$ (Fig. 5.15 in the textbook),

$$\frac{M_n}{\Omega_b} = 1.32(47.5) = 62.7 \text{ ft-kips} > \frac{M_p}{\Omega_b} \;\therefore\; \text{use } \frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 57.6 \text{ ft-kips}$$

Compute the amplified moment. Use $C_m = 1.0$.

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29,000)(82.7)}{(12 \times 12)^2} = 1142 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.60 P_a/P_{e1})} = \frac{1.0}{1 - 1.60(53/1142)} = 1.08$$

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.08(15) + 0 = 16.2 \text{ ft-kips}$$

Compressive strength: A W8 × 24 is not in the column load tables, so its axial compressive strength must be computed. From part (a),

$$P_n = 197.2 \text{ kips} \quad \Rightarrow \quad \frac{P_n}{\Omega_c} = \frac{197.2}{1.67} = 118.1 \text{ kips}$$

$$\frac{P_a}{P_n/\Omega_c} = \frac{53}{118.1} = 0.4488 > 0.2 \;\therefore\; \text{use Equation 6.5 (AISC Equation H1-1a)}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.4488 + \frac{8}{9}\left(\frac{16.2}{57.6} + 0\right) = 0.699 < 1.0 \quad \text{(OK)}$$

Member is satisfactory.

6.5-11

(a) LRFD solution:

$$P_u = 1.2(0.33 \times 410) + 1.6(0.67 \times 410) = 601.9 \text{ kips}$$

$$M_{ntx} = 1.2(0.33 \times 286) + 1.6(0.67 \times 286) = 419.8 \text{ ft-kips}$$

$$M_{nty} = 1.2(0.33 \times 102) + 1.6(0.67 \times 102) = 149.7 \text{ ft-kips}$$

Compute the strong axis bending moments.

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4(0) = 0.6$$

$$P_{e1x} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29,000)(1530)}{(1.0 \times 13 \times 12)^2} = 1.799 \times 10^4 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - (\alpha P_r / P_{e1x})} = \frac{C_{mx}}{1 - (1.00 P_u / P_{e1x})} = \frac{0.6}{1 - (601.9/17990)} = 0.6208$$

$$= 0.621 < 1.0 \ \therefore \text{ use } B_{1x} = 1.0$$

The required moment strength is

$$M_r = M_{ux} = B_{1x} M_{ntx} + B_{2x} M_{\ell tx} = 1.0(419.8) + 0 = 419.8 \text{ ft-kips}$$

From the Beam Design Charts with $C_b = 1.0$ and $L_b = 15$ feet, the moment strength is

$$\phi_b M_{nx} = \phi_b M_{px} = 878 \text{ ft-kips} \ \ (L_b < L_p)$$

Compute the weak axis bending moments.

$$C_{my} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4(0) = 0.6$$

$$P_{e1y} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_y}{(K_y L)^2} = \frac{\pi^2(29,000)(548)}{(1.0 \times 13 \times 12)^2} = 6445 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1 - (\alpha P_r / P_{e1y})} = \frac{C_{my}}{1 - (1.00 P_u / P_{e1y})} = \frac{0.6}{1 - 1.00(601.9/6445)}$$

$$= 0.662 < 1.0 \ \therefore \text{ use } B_{1y} = 1.0$$

The required moment strength is

$$M_r = M_{uy} = B_{1y} M_{nty} + B_{2y} M_{\ell ty} = 1.0(149.7) + 0 = 149.7 \text{ ft-kips}$$

From the Z_y table, $\phi_b M_{ny} = \phi_b M_{py} = 424$ ft-kips.

Determine the compressive strength. For $KL = 1.0(13) = 13$ feet, the axial compressive design strength from the column load tables is $\phi_c P_n = 1540$ kips.

$$\frac{P_u}{\phi_c P_n} = \frac{601.9}{1540} = 0.3908 > 0.2 \qquad \therefore \text{ use Equation 6.3 (AISC Equation H1-1a)}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.3908 + \frac{8}{9}\left(\frac{419.8}{878} + \frac{149.7}{424}\right)$$

$$= 1.13 > 1.0 \qquad \text{(N.G.)}$$

The W12 × 65 is not adequate.

(b) ASD Solution: $P_a = 410$ kips, $M_{ntx} = 286$ ft-kips, $M_{nty} = 102$ ft-kips

Compute the strong axis bending moments:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4(0) = 0.6$$

$$P_{e1x} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29{,}000)(1530)}{(1.0 \times 13 \times 12)^2} = 1.799 \times 10^4 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - (\alpha P_r / P_{e1x})} = \frac{C_{mx}}{1 - (1.60 P_a / P_{e1x})} = \frac{0.6}{1 - 1.60(410)/17990}$$

$$= 0.623 < 1.0 \therefore \text{use } B_{1x} = 1.0$$

$$M_r = M_{ax} = B_{1x} M_{ntx} + B_{2x} M_{\ell tx} = 1.0(286) + 0 = 286 \text{ ft-kips}$$

From the Beam Design Charts with $C_b = 1.0$ and $L_b = 15$ feet, the moment strength is

$$\frac{M_{nx}}{\Omega_b} = \frac{M_{px}}{\Omega_b} = 584 \text{ ft-kips} \qquad (L_b < L_p)$$

$$C_{my} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4(0) = 0.6$$

$$P_{e1y} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_y}{(K_y L)^2} = \frac{\pi^2(29{,}000)(548)}{(1.0 \times 13 \times 12)^2} = 6445 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1 - (\alpha P_r / Py)} = \frac{C_{my}}{1 - (1.60 P_a / P_{e1y})} = \frac{0.6}{1 - 1.60(410)/6445}$$

$$= 0.668 < 1.0 \therefore \text{use } B_{1y} = 1.0$$

$$M_r = M_{ay} = B_{1y} M_{nty} + B_{2y} M_{\ell ty} = 1.0(102) + 0 = 102 \text{ ft-kips}$$

From the Z_y table, $\dfrac{M_{ny}}{\Omega_b} = \dfrac{M_{py}}{\Omega_b} = 282$ ft-kips.

Find the compressive strength. For $KL = 1.0(15) = 13$ feet, the axial compressive strength from the column load tables is $\dfrac{P_n}{\Omega_c} = 1020$ kips.

Check the interaction formula:

$$\frac{P_a}{P_n/\Omega_c} = \frac{410}{1020} = 0.4020 > 0.2 \qquad \therefore \text{ use Equation 6.5 (AISC Equation H1-1a)}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.4020 + \frac{8}{9}\left(\frac{286}{584} + \frac{102}{282}\right)$$

$$= 1.16 > 1.0 \qquad \text{(N.G.)}$$

The W12 × 65 is not adequate.

6.5-12

(a) LRFD solution:

$$P_u = 1.2(20/2) + 1.6(20/2) = 28.0 \text{ kips}$$

$$Q_u = 1.2(40/2) + 1.6(40/2) = 56.0 \text{ kips}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.060) = 0.072 \text{ kips/ft}$$

$$M_{ntx} = \frac{(4/5)Q_uL}{4} + \frac{w_{ux}L^2}{8} = \frac{(4/5)(56.0)(12)}{4} + \frac{0.072(12)^2}{8} = 135.7 \text{ ft-kips}$$

$$M_{nty} = \frac{(3/5)Q_uL}{4} = \frac{(3/5)(56.0)(12)}{4} = 100.8 \text{ ft-kips}$$

Strong-axis bending strength. From the beam design charts, for $L_b = 12$ ft and $C_b = 1$,

$$\phi_b M_{nx} = 268.5 \text{ ft-kips}, \quad \phi_b M_{px} = 280 \text{ ft-kips}$$

For $C_b = 1.32$ (Fig. 5.15 in the textbook),

$$\phi_b M_{nx} = 1.32(268.5) = 354.4 \text{ ft-kips} > \phi_b M_{px} \therefore \text{ use } \phi_b M_{nx} = \phi_b M_{px} = 280 \text{ ft-kips}$$

Weak-axis bending strength: From the Z_y tables, $\phi_b M_{ny} = \phi_b M_{py} = 131$ ft-kips

Compute the amplified moments. Use $C_{mx} = C_{my} = 1.0$.

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(341)}{(12 \times 12)^2} = 4707 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1-(\alpha P_r/P_{e1x})} = \frac{C_{mx}}{1-(1.00P_u/P_{e1x})} = \frac{1.0}{1-(28.0/4707)} = 1.006$$

$$M_{ux} = B_1M_{ntx} + B_2M_{ltx} = 1.006(135.7) + 0 = 136.5 \text{ ft-kips}$$

$$P_{e1y} = \frac{\pi^2 EI_y}{(K_1L)^2} = \frac{\pi^2 EI_y}{(K_yL)^2} = \frac{\pi^2(29{,}000)(116)}{(12 \times 12)^2} = 1601 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1-(\alpha P_r/P_{e1y})} = \frac{C_{my}}{1-(1.00P_u/P_{e1y})} = \frac{1.0}{1-(28.0/1601)} = 1.018$$

$$M_{uy} = B_1M_{nty} + B_2M_{lty} = 1.018(100.8) + 0 = 102.6 \text{ ft-kips}$$

Compressive strength: From the column load tables, for a W10 × 60 with $KL = 12$ ft,

$$\phi_c P_n = 631 \text{ kips}$$

$$\frac{P_u}{\phi_c P_n} = \frac{28.0}{631} = 4.437 \times 10^{-2} > 0.2 \therefore \text{ use Eq. 6.4 (AISC Eq. H1-1b)}$$

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{0.04437}{2} + \left(\frac{136.5}{280} + \frac{102.6}{131}\right)$$

$$= 1.29 > 1.0 \text{ (N.G.)}$$

Member is unsatisfactory.

(b) ASD Solution

$P_a = 20$ kips, $\quad Q_a = 40$ kips

$w_a = w_D = 0.060$ kips/ft

$$M_{ntx} = \frac{(4/5)Q_a L}{4} + \frac{w_{ax}L^2}{8} = \frac{(4/5)(40)(12)}{4} + \frac{0.060(12)^2}{8} = 97.08 \text{ ft-kips}$$

$$M_{nty} = \frac{(3/5)Q_a L}{4} = \frac{(3/5)(40)(12)}{4} = 72.0 \text{ ft-kips}$$

Strong-axis bending strength. From the beam design charts, for $L_b = 12$ ft and $C_b = 1$,

$$\frac{M_{nx}}{\Omega_b} = 179 \text{ ft-kips}, \quad \frac{M_{px}}{\Omega_b} = 186 \text{ ft-kips}$$

For $C_b = 1.32$ (Fig. 5.15 in the textbook),

$$\frac{M_{nx}}{\Omega_b} = 1.32(179) = 236.3 \text{ ft-kips} > \frac{M_{px}}{\Omega_b} \quad \therefore \text{ use } \frac{M_{nx}}{\Omega_b} = \frac{M_{px}}{\Omega_b} = 186 \text{ ft-kips}$$

Weak-axis bending strength: From the Z_y tables, $\dfrac{M_{ny}}{\Omega_b} = \dfrac{M_{py}}{\Omega_b} = 87.3$ ft-kips

Compute the amplified moments. Use $C_{mx} = C_{my} = 1.0$.

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29{,}000)(341)}{(12 \times 12)^2} = 4707 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1-(\alpha P_r/P_{e1x})} = \frac{C_{mx}}{1-(1.60P_a/P_{e1x})} = \frac{1.0}{1-1.60(20)/4707} = 1.007$$

$$M_{ax} = B_1 M_{ntx} + B_2 M_{\ell tx} = 1.007(97.08) + 0 = 97.76 \text{ ft-kips}$$

$$P_{e1y} = \frac{\pi^2 EI_y}{(K_1 L)^2} = \frac{\pi^2 EI_y}{(K_y L)^2} = \frac{\pi^2(29{,}000)(116)}{(12 \times 12)^2} = 1601 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1-(\alpha P_r/P_{e1y})} = \frac{C_{my}}{1-(1.60P_a/P_{e1y})} = \frac{1.0}{1-1.60(20)/1601} = 1.02$$

$$M_{ay} = B_1 M_{nty} + B_2 M_{\ell ty} = 1.02(72.0) + 0 = 73.44 \text{ ft-kips}$$

Find the compressive strength. For $KL = 12$ feet, the axial compressive strength from the column load tables is $\dfrac{P_n}{\Omega_c} = 420$ kips.

Check the interaction formula:

$$\frac{P_a}{P_n/\Omega_c} = \frac{20}{420} = 4.762 \times 10^{-2} < 0.2 \qquad \therefore \text{ use Equation 6.6 (AISC Equation H1-1b)}$$

$$\frac{P_a}{2P_n/\Omega_c} + \left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = \frac{0.04762}{2} + \left(\frac{97.76}{186} + \frac{73.44}{87.3}\right)$$

$$= 1.39 > 1.0 \quad \text{(N.G.)}$$

Member is unsatisfactory.

6.5-13

Since this shape is not in the column load tables, the axial compressive design strength must be computed.

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{0.65(20 \times 12)}{1.80} = 86.67$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(86.67)^2} = 38.1 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4$$

Since $KL/r = 86.67 < 113.4$, use AISC Eq. E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/38.1)}(50) = 28.87 \text{ ksi}$$

$$P_n = F_{cr}A_g = 28.87(20.0) = 577.4 \text{ kips}$$

(a) LRFD solution: $\phi_c P_n = 0.90(577.4) = 519.7$ kips

$$P_u = 1.2D + 1.6L = 1.2(70) + 1.6(200) = 404.0 \text{ kips}$$

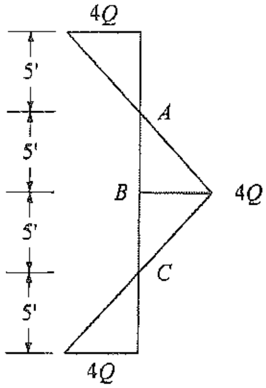
$$\frac{P_u}{\phi_c P_n} = \frac{404}{519.7} = 0.7774 > 0.2 \therefore \text{ use Eq. 6.3 (AISC Eq. H1-1a).}$$

$$\text{Maximum moment} = M_{nt} = \frac{Q_u L}{8} = \frac{1.6Q(20)}{8} = 4Q$$

Determine the moment strength. From the beam design charts, for $L_b = 20$ ft and $C_b = 1$,

$\phi_b M_n = 332$ ft-kips and $\phi_b M_p = 600$ ft-kips.

Compute C_b:



$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(4Q)}{2.5(4Q) + 3(0) + 4(4Q) + 3(0)} = 1.923$$

For $C_b = 1.923,\ \phi_b M_n = 1.923(332) = 638.4$ ft-kips

Since 638.4 ft-kips > $\phi_b M_p$, use $\phi_b M_n = \phi_b M_p = 600$ ft-kips

Compute the amplified moment. Use $C_m = 1.0$

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2 (29{,}000)(1480)}{(0.65 \times 20 \times 12)^2} = 1.741 \times 10^4 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - (\alpha P_r / P_{e1x})} = \frac{C_{mx}}{1 - (1.00 P_u / P_{e1x})} = \frac{1.0}{1 - (404/17410)} = 1.024$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.024(4Q) + 0 = 4.096Q \text{ ft-kips}$$

Eq. 6.3 (AISC Eq. H1-1a):

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.7774 + \frac{8}{9}\left(\frac{4.096Q}{600} + 0\right)$$

Let $0.7774 + \frac{8}{9}\left(\frac{4.096Q}{600} + 0\right) \le 1.0$, Solution is: $\{Q \le 36.68\}$

Maximum $Q = 36.7$ kips

(b) ASD solution: $P_a = 70 + 200 = 270$ kips

$$\frac{P_n}{\Omega_c} = \frac{577.4}{1.67} = 345.7 \text{ kips}$$

$$\frac{P_a}{P_n/\Omega_c} = \frac{270}{345.7} = 0.7810 > 0.2 \therefore \text{ use Eq. 6.5 (AISC Eq. H1-1a).}$$

$$\text{Maximum moment} = M_{nt} = \frac{Q_a L}{8} = \frac{Q(20)}{8} = 2.5Q$$

Determine the moment strength. From the beam design charts, for $L_b = 20$ ft and $C_b = 1$,

$$\frac{M_n}{\Omega_b} = 221 \text{ ft-kips and } \frac{M_p}{\Omega_b} = 400 \text{ ft-kips.}$$

$C_b = 1.923$ (see LRFD solution above)

For $C_b = 1.923$, $\frac{M_n}{\Omega_b} = 1.923(221) = 425.0$ ft-kips

Since 425.0 ft-kips $> \frac{M_p}{\Omega_b}$, use $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 400$ ft-kips

Compute the amplified moment. Use $C_m = 1.0$

$$P_{e1} = \frac{\pi^2 EI}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29,000)(1480)}{(0.65 \times 20 \times 12)^2} = 1.741 \times 10^4 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1-(\alpha P_r/P_{e1x})} = \frac{C_{mx}}{1-(1.60P_a/P_{e1x})} = \frac{1.0}{1-1.60(270)/17410} = 1.025$$

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.025(2.5Q) + 0 = 2.563Q \text{ ft-kips}$$

Equation 6.5 (AISC Equation H1-1a)

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.7810 + \frac{8}{9}\left(\frac{2.563Q}{400} + 0\right)$$

Let $0.7810 + \frac{8}{9}\left(\frac{2.563Q}{400} + 0\right) \leq 1.0$, Solution is: $\{Q \leq 38.45\}$

Maximum $Q = 38.5$ kips

6.6-1

Determine the axial compressive design strength. Use K_x for the unbraced condition.

$$\frac{K_xL}{r_x/r_y} = \frac{1.7(14)}{2.44} = 9.754 \text{ ft} < K_yL = 14 \text{ ft}$$

From the column load tables with $KL = 14$ ft, $\phi_c P_n = 774$ kips

$$\frac{P_u}{\phi_c P_n} = \frac{400}{774} = 0.5168 > 0.2 \therefore \text{ use Eq. 6.3 (AISC Eq. H1-1a).}$$

Check the braced condition first. For the axis of bending,

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{24}{45}\right) = 0.3867$$

$$P_{e1} = \frac{\pi^2 EI}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29,000)(881)}{(1.0 \times 14 \times 12)^2} = 8934 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1-(\alpha P_r/P_{e1x})} = \frac{C_{mx}}{1-(1.00P_u/P_{e1x})} = \frac{0.3867}{1-(400/8934)}$$

$$= 0.405 < 1.0 \qquad \therefore \text{ use } B_1 = 1.0$$

Sway condition: use

$$B_2 = \frac{1}{1 - \frac{\alpha \Sigma P_{nt}}{\Sigma P_{e2}}} = \frac{1}{1 - \frac{1.00 \Sigma P_u}{\Sigma P_{e2}}} = \frac{1}{1 - \frac{1.00(6000)}{40000}} = 1.176$$

The total amplified moment at the top is

$$M_{u\,top} = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(45) + 1.176(40) = 92.04 \text{ ft-kips}$$

The total amplified moment at the bottom is

$$M_{u\,bot} = 1.0(24) + 1.176(95) = 135.7 \text{ ft-kips}$$

Use $M_u = 135.7$ ft-kips. Compute the moment strength. From the beam design charts with $L_b = 14$ ft,

$$\phi_b M_n = 479 \text{ ft-kips for } C_b = 1.0 \text{ and } \phi_b M_p = 521 \text{ ft-kips}$$

Using the total amplified moment, compute C_b:

$$C_b = \frac{12.5 M_{\max}}{2.5 M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(135.7)}{2.5(135.7) + 3(35.11) + 4(21.83) + 3(78.77)} = 2.208$$

For $C_b = 2.208$, $\phi_b M_n = 2.208(479) = 1058$ ft-kips

Since 1058 ft-kips > $\phi_b M_p$, use $\phi_b M_n = \phi_b M_p = 521$ ft-kips

Eq. 6.3 (AISC Eq. H1-1a):

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.5168 + \frac{8}{9}\left(\frac{135.7}{521} + 0\right) = 0.748 < 1.0 \quad \text{(OK)}$$

Member is satisfactory.

6.6-2

(a) LRFD solution

Determine the axial compressive design strength. Use K_x for the unbraced condition.

$$\frac{K_x L}{r_x / r_y} = \frac{1.2(16)}{2.44} = 7.869 \text{ ft} < K_y L = 16 \text{ ft}$$

From the column load tables with $KL = 16$ ft, $\phi_c P_n = 698$ kips

Check load combination 2. For the braced condition,

$$P_u = 1.2 P_D + 1.6 P_L = 1.2(120) + 1.6(240) = 528.0 \text{ kips}$$

$$\frac{P_u}{\phi_c P_n} = \frac{528}{698} = 0.7564 > 0.2 \therefore \text{ use Eq. 6.3 (AISC Eq. H1-1a).}$$

$$M_{nt}(\text{top}) = 1.2(15) + 1.6(40) = 82.0 \text{ ft-kips}$$

$$M_{nt}(\text{bot}) = 1.2(18) + 1.6(48) = 98.4 \text{ ft-kips}$$

For the axis of bending, $C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{82.0}{98.4}\right) = 0.2667$

$$P_{e1} = \frac{\pi^2 EI_x}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(881)}{(0.85 \times 16 \times 12)^2} = 9467 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00P_u/P_{e1})} = \frac{0.2667}{1-(528/9467)}$$

$$= 0.2825 < 1.0 \quad \therefore \text{ use } B_1 = 1.0$$

$$M_u = B_1M_{nt} + B_2M_{\ell t} = 1.0(98.4) + 0 = 98.4 \text{ ft-kips}$$

Compute the moment strength. From the beam design charts with $L_b = 16$ ft,

$$\phi_bM_n = 462 \text{ ft-kips for } C_b = 1.0 \text{ and } \phi_bM_p = 521 \text{ ft-kips}$$

Compute C_b:

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(98.4)}{2.5(98.4) + 3(36.9) + 4(8.20) + 3(53.3)} = 2.239$$

For $C_b = 2.239, \phi_bM_n = 2.239(462) = 1034$ ft-kips

Since 1034 ft-kips > ϕ_bM_p, use $\phi_bM_n = \phi_bM_p = 521$ ft-kips

Equation 6.3 (AISC Eq. H1-1a):

$$\frac{P_u}{\phi_cP_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_bM_{nx}} + \frac{M_{uy}}{\phi_bM_{ny}}\right) = 0.7564 + \frac{8}{9}\left(\frac{98.4}{521} + 0\right) = 0.924 < 1.0 \quad \text{(OK)}$$

Check load combination 4. For the braced condition,

$$P_u = 1.2P_D + 0.5P_L = 1.2(120) + 0.5(240) = 264.0 \text{ kips}$$

$$\frac{P_u}{\phi_cP_n} = \frac{264}{698} = 0.3782 > 0.2 \therefore \text{ use AISC Eq. H1-1a.}$$

$$M_{nt}(\text{top}) = 1.2(15) + 0.5(40) = 38.0 \text{ ft-kips}$$

$$M_{nt}(\text{bot}) = 1.2(18) + 0.5(48) = 45.6 \text{ ft-kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{38.0}{45.6}\right) = 0.2667$$

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00P_u/P_{e1})} = \frac{0.2667}{1-(264/9467)}$$

$$= 0.274 < 1.0 \quad \therefore \text{ use } B_1 = 1.0$$

For the sway condition,

$$M_{\ell t}(\text{top}) = M_{\ell t}(\text{bot}) = 1.6W = 1.6(130) = 208.0 \text{ ft-kips}$$

For the axis of bending,

$$P_{e2} = \frac{\pi^2 EI}{(K_2L)^2} = \frac{\pi^2(29{,}000)(881)}{(1.2 \times 16 \times 12)^2} = 4750 \text{ kips}$$

$$B_2 = \frac{1}{1 - \dfrac{\alpha \Sigma P_{nt}}{\Sigma P_{e2}}} \approx \frac{1}{1 - \dfrac{\alpha P_{nt}}{P_{e2}}}$$

$$= \frac{1}{1 - \dfrac{1.00P_u}{P_{e2}}} = \frac{1}{1 - \dfrac{1.00(264)}{4750}} = 1.059$$

$$M_u = 1.0(45.6) + 1.059(208.0) = 265.9 \text{ ft-kips}$$

AISC Equation H1-1a:

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.3782 + \frac{8}{9}\left(\frac{265.9}{521} + 0\right) = 0.832 < 1.0 \quad \text{(OK)}$$

(Load combination 2 controls.) Member is adequate.

(b) ASD solution

Determine the axial compressive design strength. Use K_x for the unbraced condition.

$$\frac{K_xL}{r_x/r_y} = \frac{1.2(16)}{2.44} = 7.869 \text{ ft} < K_yL = 16 \text{ ft}$$

From the column load tables with $KL = 16$ ft, $P_n/\Omega_c = 465$ kips

Check load combination 2. For the braced condition,

$$P_a = P_D + P_L = 120 + 240 = 360 \text{ kips}$$

$$\frac{P_a}{P_n/\Omega_c} = \frac{360}{465} = 0.7742 > 0.2 \ \therefore \text{ use Eq. 6.5 (AISC Eq. H1-1a).}$$

$$M_{nt}(\text{top}) = 15 + 40 = 55 \text{ ft-kips}$$

$$M_{nt}(\text{bot}) = 18 + 48 = 66 \text{ ft-kips}$$

For the axis of bending, $C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{55}{66}\right) = 0.2667$

$$P_{e1} = \frac{\pi^2 EI_x}{(K_1L)^2} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(881)}{(0.85 \times 16 \times 12)^2} = 9467 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.60P_a/P_{e1})} = \frac{0.2667}{1 - 1.60(360)/9467}$$

$$= 0.2840 < 1.0 \ \therefore \text{ use } B_1 = 1.0$$

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(66) + 0 = 66 \text{ ft-kips}$$

Compute the moment strength. From the beam design charts with $L_b = 16$ ft,

$$\frac{M_n}{\Omega_b} = 308 \text{ ft-kips for } C_b = 1.0 \text{ and } \frac{M_p}{\Omega_b} = 347 \text{ ft-kips}$$

For $C_b = 2.239$ (see part a), $\frac{M_n}{\Omega_b} = 2.239(308) = 689.6$ ft-kips

Since 689.6 ft-kips > $\frac{M_p}{\Omega_b}$, use $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 347$ ft-kips

Equation 6.5 (AISC Eq. H1-1a):

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.7742 + \frac{8}{9}\left(\frac{66}{347} + 0\right) = 0.943 < 1.0 \text{ (OK)}$$

Check load combination 6. For the braced condition,

$$P_a = P_D + 0.75P_L = 120 + 0.75(240) = 300.0 \text{ kips}$$

$$\frac{P_a}{P_n/\Omega_c} = \frac{300}{465} = 0.6452 > 0.2 \therefore \text{ use Eq. 6.5 (AISC Eq. H1-1a).}$$

$$M_{nt}(\text{top}) = 15 + 0.75(40) = 45.0 \text{ ft-kips}$$

$$M_{nt}(\text{bot}) = 18 + 0.75(48) = 54.0 \text{ ft-kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{45}{54}\right) = 0.2667$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.60P_a/P_{e1})} = \frac{0.2667}{1 - 1.60(300)/9467}$$

$$= 0.281 < 1.0 \therefore \text{ use } B_1 = 1.0$$

For the sway condition,

$$M_{\ell t}(\text{top}) = M_{\ell t}(\text{bot}) = 0.75W = 0.75(130) = 97.5 \text{ ft-kips}$$

For the axis of bending,

$$P_{e2} = \frac{\pi^2 EI}{(K_2L)^2} = \frac{\pi^2(29{,}000)(881)}{(1.2 \times 16 \times 12)^2} = 4750 \text{ kips}$$

$$B_2 = \frac{1}{1 - \frac{\alpha \Sigma P_{nt}}{\Sigma P_{e2}}} \approx \frac{1}{1 - \frac{\alpha P_{nt}}{P_{e2}}}$$

$$= \frac{1}{1 - \frac{1.60P_a}{P_{e2}}} = \frac{1}{1 - \frac{1.60(300)}{4750}} = 1.112$$

$$M_a = 1.0(66) + 1.112(97.5) = 174.4 \text{ ft-kips}$$

Equation 6.5 (AISC Eq. H1-1a):

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.6452 + \frac{8}{9}\left(\frac{174.4}{347} + 0\right)$$

$$= 1.09 > 1.0 \text{ (N.G.)}$$

Member is inadequate.

6.7-1

(a) LRFD solution

The factored axial load is

$$P_u = 1.2(0.30 \times 236) + 1.6(0.70 \times 236) = 349.3 \text{ kips}$$

The factored moment is

$$M_{ntx} = 1.2(0.30 \times 168) + 1.6(0.70 \times 168) = 248.6 \text{ ft-kips}$$

The amplification factor B_1 can be estimated as 1.0 for purposes of making a trial selection.

$$M_{ux} = B_{1x}M_{ntx} = 1.0(248.6) = 248.6 \text{ ft-kips}$$

The effective length for compression and the unbraced length for bending are the same:

$$KL = L_b = 15 \text{ ft.}$$

From Table 6-1, Try a W12 × 79, with $p = 1.24 \times 10^{-3}$ and $b_x = 2.10 \times 10^{-3}$.

Determine which interaction equation to use:

$$pP_r = pP_u = (1.24 \times 10^{-3})(349.3) = 0.4331 > 0.2 \quad \therefore \text{ Equation 6.8 controls.}$$

As a preliminary check (remember that B_1 has not yet been computed and C_b has not been accounted for)

$$\begin{aligned} pP_r + b_xM_{rx} + b_yM_{ry} &= pP_u + b_xM_{ux} + b_yM_{uy} \\ &= (1.24 \times 10^{-3})(349.3) + (2.10 \times 10^{-3})(248.6) + 0 \\ &= 0.955 < 1.0 \qquad \text{(OK)} \end{aligned}$$

Calculate B_1:

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{0}{M_2}\right) = 0.6$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29,000)(662)}{(15 \times 12)^2} = 5848 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{P_u}{P_{e1x}}} = \frac{0.6}{1 - \dfrac{349.3}{5848}} = 0.638 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

(Since this shape is adequate for $C_b = 1.0$, the steps shown below, computation of the actual value of C_b and an adjustment of b_x, are not necessary.)

From Figure 5.15g, $C_b = 1.67$. Modify b_x to account for C_b.

$$C_b \times \phi_b M_{nx} = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 1.67 \times \frac{8}{9} \times \frac{1}{2.10 \times 10^{-3}} = 706.9 \text{ ft-kips}$$

From the Z_x table, $\phi_b M_{px} = 446$ ft-kips < 706.9 ft-kips $\therefore\ \phi_b M_{nx} = 446$ ft-kips

$$b_x = \frac{8}{9(\phi_b M_{nx})} = \frac{8}{9(446)} = 1.99 \times 10^{-3}$$

Check Equation 6.8: $p = 1.24 \times 10^{-3},\ b_x = 1.99 \times 10^{-3}$

$$pP_u + b_xM_{ux} + b_yM_{uy} = (1.24 \times 10^{-3})(349.3) + (1.99 \times 10^{-3})(248.6)$$

$$= 0.928 < 1.0 \qquad \text{(OK)}$$

Use a W12 × 79

(b) ASD Solution

The amplification factor B_1 can be estimated as 1.0 for purposes of making a trial selection.

$$M_{ax} = B_{1x}M_{ntx} = 1.0(168) = 168 \text{ ft-kips}$$

The effective length for compression and the unbraced length for bending are the same:

$$KL = L_b = 15 \text{ ft.}$$

From Table 6-1, Try a W12 × 79, with $p = 1.86 \times 10^{-3}$ and $b_x = 3.16 \times 10^{-3}$.

Determine which interaction equation to use:

$$pP_r = pP_a = (1.86 \times 10^{-3})(236) = 0.4390 > 0.2 \ \therefore \text{ Equation 6.8 controls.}$$

As a preliminary check (remember that B_1 has not yet been computed and C_b has not been accounted for),

$$pP_r + b_xM_{rx} + b_yM_{ry} = pP_a + b_xM_{ax} + b_yM_{ay}$$

$$= (1.86 \times 10^{-3})(236) + (3.16 \times 10^{-3})(168) + 0$$

$$= 0.970 < 1.0 \qquad \text{(OK)}$$

Calculate B_1:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{0}{M_2}\right) = 0.6$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(662)}{(15 \times 12)^2} = 5848 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{1.60P_a}{P_{e1x}}} = \frac{0.6}{1 - \dfrac{1.60(236)}{5848}} = 0.6414 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

(Since this shape is adequate for $C_b = 1.0$, the steps shown below, computation of the actual value of C_b and an adjustment of b_x, are not necessary.)

From Figure 5.15g, $C_b = 1.67$. Modify b_x to account for C_b.

$$C_b \times \frac{M_{nx}}{\Omega_b} = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 1.67 \times \frac{8}{9} \times \frac{1}{1.86 \times 10^{-3}} = 798.1 \text{ ft-kips}$$

$$\frac{M_{px}}{\Omega_b} = 297 \text{ ft-kips} < 798.1.0 \text{ ft-kips} \ \therefore\text{use } \frac{M_{nx}}{\Omega_b} = 297 \text{ ft-kips}$$

$$b_x = \frac{8}{9(M_{nx}/\Omega_b)} = \frac{8}{9(297)} = 2.99 \times 10^{-3}$$

Check Equation 6.8: $p = 1.86 \times 10^{-3}$, $b_x = 2.99 \times 10^{-3}$

$$pP_a + b_xM_{ax} + b_yM_{ay} = (1.86 \times 10^{-3})(236) + (2.99 \times 10^{-3})(168)$$

$$= 0.941 < 1.0 \quad \text{(OK)}$$

Use a W12 × 79

6.7-2

(a) LRFD solution

The factored axial load is

$$P_u = 1.2(285/2) + 1.6(285/2) = 399.0 \text{ kips}$$

The factored moments are

$$M_{ntx}\text{ (top)} = 1.2(130/2) + 1.6(130/2) = 182.0 \text{ ft-kips}$$

$$M_{ntx}\text{ (bot)} = 1.2(100/2) + 1.6(100/2) = 140.0 \text{ ft-kips}$$

The amplification factor B_1 can be estimated as 1.0 for purposes of making a trial selection.

$$M_{ux} = B_{1x}M_{ntx} = 1.0(182) = 182 \text{ ft-kips}$$

The effective length for compression and the unbraced length for bending are the same:

$$KL = L_b = 12 \text{ ft.}$$

From Table 6-1, Try a W10 × 77, with $p = 1.23 \times 10^{-3}$ and $b_x = 2.50 \times 10^{-3}$.

Determine which interaction equation to use:

$$pP_r = pP_u = (1.23 \times 10^{-3})(399) = 0.4908 > 0.2 \;\therefore \text{ Equation 6.8 controls.}$$

As a preliminary check (remember that B_1 has not yet been computed and C_b has not been accounted for)

$$pP_r + b_xM_{rx} + b_yM_{ry} = pP_u + b_xM_{ux} + b_yM_{uy}$$

$$= (1.23 \times 10^{-3})(399) + (2.50 \times 10^{-3})(182) + 0$$

$$= 0.946 < 1.0 \quad \text{(OK)}$$

Calculate B_1:

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{140}{182}\right) = 0.9077$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(455)}{(12 \times 12)^2} = 6280 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{P_u}{P_{e1x}}} = \frac{0.9077}{1 - \dfrac{399}{6280}} = 0.9693 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

(Since this shape is adequate for $C_b = 1.0$, the steps shown below, computation of the actual value of C_b and an adjustment of b_x, are not necessary.)

Compute C_b and modify b_x to account for C_b.

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(182)}{2.5(182) + 3(150.5) + 4(161) + 3(171.5)} = 1.102$$

$$C_b \times \phi_b M_{nx} = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 1.102 \times \frac{8}{9} \times \frac{1}{2.50 \times 10^{-3}} = 391.8 \text{ ft-kips}$$

From the Z_x table, $\phi_b M_{px} = 366$ ft-kips < 391.8 ft-kips $\therefore\ \phi_b M_{nx} = 366$ ft-kips

$$b_x = \frac{8}{9(\phi_b M_{nx})} = \frac{8}{9(366)} = 2.43 \times 10^{-3}$$

Check Equation 6.8: $p = 1.23 \times 10^{-3},\ b_x = 2.43 \times 10^{-3}$

$$pP_u + b_x M_{ux} + b_y M_{uy} = (1.23 \times 10^{-3})(399) + (2.43 \times 10^{-3})(182)$$

$$= 0.933 < 1.0 \quad \text{(OK)}$$

Use a W10 × 77

(b) ASD Solution

The amplification factor B_1 can be estimated as 1.0 for purposes of making a trial selection.

$$M_{ax} = B_{1x} M_{ntx} = 1.0(130) = 130 \text{ ft-kips}$$

The effective length for compression and the unbraced length for bending are the same:

$$KL = L_b = 12 \text{ ft.}$$

From Table 6-1, Try a W10 × 77, with $p = 1.85 \times 10^{-3}$ and $b_x = 3.76 \times 10^{-3}$.

Determine which interaction equation to use:

$$pP_r = pP_a = (1.85 \times 10^{-3})(285) = 0.5273 > 0.2 \ \therefore \text{ Equation 6.8 controls.}$$

As a preliminary check (remember that B_1 has not yet been computed and C_b has not been accounted for),

$$pP_r + b_x M_{rx} + b_y M_{ry} = pP_a + b_x M_{ax} + b_y M_{ay}$$

$$= (1.85 \times 10^{-3})(285) + (3.76 \times 10^{-3})(130) + 0$$

$$= 1.02 > 1.0 \quad \text{(but close. Continue with this shape.)}$$

Calculate B_1:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{100}{130}\right) = 0.9077$$

$$P_{e1x} = \frac{\pi^2 E I_x}{(K_x L)^2} = \frac{\pi^2(29{,}000)(455)}{(12 \times 12)^2} = 6280 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \frac{1.60P_a}{P_{e1x}}} = \frac{0.9077}{1 - \frac{1.60(285)}{6280}} = 0.9788 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

Compute C_b and modify b_x to account for C_b.

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max}+3M_A+4M_B+3M_C}$$

$$= \frac{12.5(130)}{2.5(130)+3(107.5)+4(115)+3(122.5)} = 1.102$$

$$C_b \times \frac{M_{nx}}{\Omega_b} = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 1.102 \times \frac{8}{9} \times \frac{1}{3.76 \times 10^{-3}} = 260.5 \text{ ft-kips}$$

$$\frac{M_{px}}{\Omega_b} = 244 \text{ ft-kips} < 260.5 \text{ ft-kips} \quad \therefore \text{use } \frac{M_{nx}}{\Omega_b} = 244 \text{ ft-kips}$$

$$b_x = \frac{8}{9(M_{nx}/\Omega_b)} = \frac{8}{9(244)} = 3.64 \times 10^{-3}$$

Check Equation 6.8: $p = 1.85 \times 10^{-3}$, $b_x = 3.64 \times 10^{-3}$

$$pP_a + b_xM_{ax} + b_yM_{ay} = (1.85 \times 10^{-3})(285) + (3.64 \times 10^{-3})(130)$$

$$= 1.0 \quad \text{(OK)}$$

Use a W10 × 77

6.7-3

(a) LRFD Solution

$$P_u = 1.2(0.25 \times 45) + 1.6(0.75 \times 45) = 67.5 \text{ kips}$$

$$w_u = 1.2(0.25 \times 5) + 1.6(0.75 \times 5) = 7.5 \text{ kips/ft}$$

$$M_{ntx} = \frac{1}{8}(7.5)(12)^2 = 135.0 \text{ ft-kips}$$

The amplification factor B_1 can be estimated as 1.0 for purposes of making a trial selection.

$$M_{ux} = B_{1x}M_{ntx} = 1.0(135) = 135 \text{ ft-kips}$$

The effective length for compression and the unbraced length for bending are the same:

$$KL = L_b = 12 \text{ ft.}$$

From Table 6-1, Try a W10 × 39, with $p = 2.85 \times 10^{-3}$ and $b_x = 5.68 \times 10^{-3}$.

Determine which interaction equation to use:

$$pP_r = pP_u = (2.85 \times 10^{-3})(67.5) = 0.1924 < 0.2 \quad \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check (remember that B_1 has not yet been computed and C_b has not been accounted for)

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry}) = 0.5pP_u + \frac{9}{8}(b_xM_{ux} + b_yM_{uy})$$

$$= 0.5(2.85 \times 10^{-3})(67.5) + \frac{9}{8}((5.68 \times 10^{-3})(135) + 0)$$

$$= 0.959 < 1.0 \quad \text{(OK)}$$

Calculate B_1: Use $C_{mx} = 1.0$ (transversely-loaded member)

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(209)}{(12 \times 12)^2} = 2885 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \frac{P_u}{P_{e1x}}} = \frac{1.0}{1 - \frac{67.5}{2885}} = 1.024$$

$$M_{ux} = B_1 M_{ntx} + B_2 M_{ltx} = 1.024(135) + 0 = 138.2 \text{ ft-kips}$$

From Figure 5.15 in the textbook, $C_b = 1.14$. Modify b_x to account for C_b.

$$C_b \times \phi_b M_{nx} = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 1.14 \times \frac{8}{9} \times \frac{1}{5.68 \times 10^{-3}} = 178.4 \text{ ft-kips}$$

From the Z_x table, $\phi_b M_{px} = 176$ ft-kips < 178.4 ft-kips $\therefore$ $\phi_b M_{nx} = 176$ ft-kips

$$b_x = \frac{8}{9(\phi_b M_{nx})} = \frac{8}{9(176)} = 5.05 \times 10^{-3}$$

Check Equation 6.9: $p = 2.85 \times 10^{-3}$, $b_x = 5.05 \times 10^{-3}$

$$0.5pP_u + \frac{9}{8}(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(2.85 \times 10^{-3})(67.5) + \frac{9}{8}((5.05 \times 10^{-3})(138.2) + 0) = 0.881 < 1.0 \quad \text{(OK)}$$

Try the next lighter shape, a W10 × 33 with $p = 3.42 \times 10^{-3}$ and $b_x = 7.00 \times 10^{-3}$.

$pP_r = pP_u = (3.42 \times 10^{-3})(67.5) = 0.2309 > 0.2$ $\therefore$ Equation 6.8 controls.

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(171)}{(12 \times 12)^2} = 2360 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \frac{P_u}{P_{e1x}}} = \frac{1.0}{1 - \frac{67.5}{2360}} = 1.029$$

$$M_{ux} = B_1 M_{ntx} + B_2 M_{ltx} = 1.029(135) + 0 = 138.9 \text{ ft-kips}$$

Modify b_x to account for C_b.

$$C_b \times \phi_b M_{nx} = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 1.14 \times \frac{8}{9} \times \frac{1}{7.00 \times 10^{-3}} = 144.8 \text{ ft-kips}$$

From the Z_x table, $\phi_b M_{px} = 146$ ft-kips >144.8 ft-kips

$$b_x = \frac{8}{9(\phi_b M_{nx})} = \frac{8}{9(144.8)} = 6.14 \times 10^{-3}$$

Check Equation 6.8: $p = 3.42 \times 10^{-3}$, $b_x = 6.14 \times 10^{-3}$

$$pP_u + b_x M_{ux} + b_y M_{uy} = (3.42 \times 10^{-3})(67.5) + (6.14 \times 10^{-3})(138.9)$$

$$= 1.08 > 1.0 \quad \text{(N.G.)}$$

Use a W10 × 39

(b) ASD Solution

$$P_a = 45 \text{ kips}, \qquad M_{ntx} = \frac{1}{8}(5)(12)^2 = 90.0 \text{ ft-kips}$$

The amplification factor B_1 can be estimated as 1.0 for purposes of making a trial selection.

$$M_{ax} = B_{1x}M_{ntx} = 1.0(90) = 90 \text{ ft-kips}$$

$$KL = L_b = 12 \text{ ft.}$$

From Table 6-1, Try a W10 × 39, with $p = 4.29 \times 10^{-3}$ and $b_x = 8.53 \times 10^{-3}$.

$$pP_r = pP_a = (4.29 \times 10^{-3})(45) = 0.1931 < 0.2 \therefore \text{ Equation 6.9 controls.}$$

preliminary check:

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry}) = 0.5pP_a + \frac{9}{8}(b_xM_{ax} + b_yM_{ay})$$

$$= 0.5(4.29 \times 10^{-3})(45) + \frac{9}{8}((8.53 \times 10^{-3})(90) + 0)$$

$$= 0.960 < 1.0 \quad \text{(OK)}$$

Calculate B_1: Use $C_{mx} = 1.0$ (transversely-loaded member)

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(209)}{(12 \times 12)^2} = 2885 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{1.60P_a}{P_{e1x}}} = \frac{1.0}{1 - \dfrac{1.60(45)}{2885}} = 1.026$$

$$M_{ax} = B_{1x}M_{ntx} = 1.026(90) = 92.34 \text{ ft-kips}$$

From Figure 5.15 in the textbook, $C_b = 1.14$. Modify b_x to account for C_b.

$$C_b \times \frac{M_{nx}}{\Omega_b} = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 1.14 \times \frac{8}{9} \times \frac{1}{8.53 \times 10^{-3}} = 118.8 \text{ ft-kips}$$

$$\frac{M_{px}}{\Omega_b} = 117 \text{ ft-kips} < 118.8 \text{ ft-kips} \quad \therefore \text{use } \frac{M_{nx}}{\Omega_b} = 117 \text{ ft-kips}$$

$$b_x = \frac{8}{9(M_{nx}/\Omega_b)} = \frac{8}{9(117)} = 7.60 \times 10^{-3}$$

Check Equation 6.9: $p = 4.29 \times 10^{-3}$, $b_x = 7.60 \times 10^{-3}$

$$0.5pP_a + \frac{9}{8}(b_xM_{ax} + b_yM_{ay}) = 0.5(4.29 \times 10^{-3})(45) + \frac{9}{8}((7.60 \times 10^{-3})(92.34) + 0)$$

$$= 0.886 < 1.0 \quad \text{(OK)}$$

Use a W10 × 39

6.7-4

(a) LRFD Solution

$$P_u = 1.2(0.33 \times 95) + 1.6(0.67 \times 95) = 139.5 \text{ kips}$$

The factored moments at the top are

$$M_{ntx} = M_{nty} = 1.2(0.33 \times 102) + 1.6(0.67 \times 102) = 149.7 \text{ ft-kips}$$

The factored moments at the bottom are

$$M_{ntx} = M_{nty} = 1.2(0.33 \times 51) + 1.6(0.67 \times 51) = 74.87 \text{ ft-kips}$$

The amplification factor B_1 can be estimated as 1.0 for purposes of making a trial selection. For the two axes,

$$M_{ux} = B_{1x} M_{ntx} = 1.0(149.7) = 149.7 \text{ ft-kips}$$

$$M_{uy} = B_{1y} M_{nty} = 1.0(149.7) = 149.7 \text{ ft-kips}$$

Check the W10s. Try a W10 ×112. From Table 6-1, with $KL = L_b = 16$ ft,

$$p = 0.982 \times 10^{-3},\ b_x = 1.69 \times 10^{-3},\ b_y = 3.43 \times 10^{-3}$$

$$pP_u = (0.982 \times 10^{-3})(139.5) = 0.1370 < 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry}) = 0.5pP_u + \frac{9}{8}(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(0.982 \times 10^{-3})(139.5) + \frac{9}{8}((1.69 \times 10^{-3})(149.7) + (3.43 \times 10^{-3})(149.7))$$

$$= 0.931 < 1.0 \qquad \text{(OK)}$$

Check the W12s. Try a W12 × 96. From Table 6-1,

$$p = 1.05 \times 10^{-3},\ b_x = 1.70 \times 10^{-3},\ b_y = 3.51 \times 10^{-3}$$

$$pP_u = (1.05 \times 10^{-3})(139.5) = 0.1465 < 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$0.5pP_u + \frac{9}{8}(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(1.05 \times 10^{-3})(139.5) + \frac{9}{8}((1.70 \times 10^{-3})(149.7) + (3.51 \times 10^{-3})(149.7))$$

$$= 0.951 < 1.0 \qquad \text{(OK)}$$

Check the W14s. Try a W14 × 90. From Table 6-1,

$$p = 1.02 \times 10^{-3},\ b_x = 1.57 \times 10^{-3},\ b_y = 3.26 \times 10^{-3}$$

$$pP_u = (1.02 \times 10^{-3})(139.5) = 0.1423 < 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$0.5pP_u + \frac{9}{8}(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(1.02 \times 10^{-3})(139.5) + \frac{9}{8}((1.57 \times 10^{-3})(149.7) + (3.26 \times 10^{-3})(149.7))$$

$$= 0.885 < 1.0 \qquad \text{(OK)}$$

Try a W14 × 90 (the lightest)

Calculate B_1 for each axis:

$$C_{mx} = C_{my} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{74.87}{149.7}\right) = 0.3999$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29,000)(999)}{(16 \times 12)^2} = 7756 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \frac{P_u}{P_{e1x}}} = \frac{0.3999}{1 - \frac{139.5}{7756}} = 0.407 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

$$P_{e1y} = \frac{\pi^2 EI_y}{(K_yL)^2} = \frac{\pi^2(29,000)(362)}{(16 \times 12)^2} = 2811 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1 - \frac{P_u}{P_{e1y}}} = \frac{0.3999}{1 - \frac{139.5}{2811}} = 0.421 < 1.0 \qquad \therefore B_{1y} = 1.0 \text{ as assumed}$$

(Since this shape is adequate for $C_b = 1.0$, the steps shown below, computation of the actual value of C_b and an adjustment of b_x, are not necessary.)

Compute C_b and modify b_x to account for C_b.

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(149.7)}{2.5(149.7) + 3(18.73) + 4(37.42) + 3(93.56)} = 2.174$$

$$C_b \times \phi_b M_{nx} = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 2.174 \times \frac{8}{9} \times \frac{1}{1.57 \times 10^{-3}} = 1231 \text{ ft-kips}$$

From the Z_x table, $\phi_b M_{px} = 573$ ft-kips < 1231 ft-kips $\therefore$ $\phi_b M_{nx} = 573$ ft-kips

$$b_x = \frac{8}{9(\phi_b M_{nx})} = \frac{8}{9(573)} = 1.55 \times 10^{-3}$$

$$0.5pP_u + \frac{9}{8}(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(1.02 \times 10^{-3})(139.5) + \frac{9}{8}((1.55 \times 10^{-3})(149.7) + (3.26 \times 10^{-3})(149.7))$$

$= 0.881 < 1.0$ (OK) Use a W14 × 90

(b) ASD Solution

The axial service load is $P_a = 95$ kips

The service-load moments at the top are $M_{ntx} = M_{nty} = 102$ ft-kips

The service-load moments at the bottom are $M_{ntx} = M_{nty} = 51$ ft-kips

The amplification factor B_1 can be estimated as 1.0 for purposes of making a trial selection. For the two axes,

$$M_{ax} = B_{1x}M_{ntx} = 1.0(102) = 102 \text{ ft-kips}$$

$$M_{ay} = B_{1y}M_{nty} = 1.0(102) = 102 \text{ ft-kips}$$

Check the W10s. Try a W10 ×112. From Table 6-1, with $KL = L_b = 16$ ft,

$$p = 1.48 \times 10^{-3},\ b_x = 2.54 \times 10^{-3},\ b_y = 5.15 \times 10^{-3}$$

Determine which interaction equation to use:

$$\frac{P_a}{P_n/\Omega_c} = pP_a = (1.48 \times 10^{-3})(95) = 0.1406 < 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry})$$

$$= 0.5(1.48 \times 10^{-3})(95) + \frac{9}{8}((2.54 \times 10^{-3})(102) + (5.15 \times 10^{-3})(102))$$

$$= 0.953 < 1.0 \qquad \text{(OK)}$$

Check the W12s. Try a W12 × 96. From Table 6-1,

$$p = 1.57 \times 10^{-3},\ b_x = 2.56 \times 10^{-3},\ b_y = 5.28 \times 10^{-3}$$

Determine which interaction equation to use:

$$\frac{P_a}{P_n/\Omega_c} = pP_a = (1.57 \times 10^{-3})(95) = 0.1492 < 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry})$$

$$= 0.5(1.57 \times 10^{-3})(95) + \frac{9}{8}((2.56 \times 10^{-3})(102) + (5.28 \times 10^{-3})(102))$$

$$= 0.974 < 1.0 \qquad \text{(OK)}$$

Check the W14s. Try a W14 × 90. From Table 6-1,

$$p = 1.54 \times 10^{-3},\ b_x = 2.35 \times 10^{-3},\ b_y = 4.90 \times 10^{-3}$$

Determine which interaction equation to use:

$$\frac{P_a}{P_n/\Omega_c} = pP_a = (1.54 \times 10^{-3})(95) = 0.1463 < 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry})$$

$$= 0.5(1.54 \times 10^{-3})(95) + \frac{9}{8}((2.35 \times 10^{-3})(102) + (4.90 \times 10^{-3})(102))$$

$$= 0.905 < 1.0 \qquad \text{(OK)}$$

Try a W14 × 90 (the lightest). Calculate B_1 for each axis:

$$C_{mx} = C_{my} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{51}{102}\right) = 0.4$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(999)}{(16 \times 12)^2} = 7756 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \frac{1.60P_a}{P_{e1x}}} = \frac{0.4}{1 - \frac{1.60(95)}{7756}} = 0.408 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

$$P_{e1y} = \frac{\pi^2 EI_y}{(K_yL)^2} = \frac{\pi^2(29{,}000)(362)}{(16 \times 12)^2} = 2811 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1 - \frac{1.60P_a}{P_{e1y}}} = \frac{0.6}{1 - \frac{1.60(95)}{2811}} = 0.634 < 1.0 \qquad \therefore B_{1y} = 1.0 \text{ as assumed}$$

(Since this shape is adequate for $C_b = 1.0$, the steps shown below, computation of the actual value of C_b and an adjustment of b_x, are not necessary.)

Compute C_b and modify b_x to account for C_b.

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(102)}{2.5(102) + 3(12.75) + 4(25.5) + 3(63.75)} = 2.174$$

$$C_b \times \frac{M_{nx}}{\Omega_b} = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 2.174 \times \frac{8}{9} \times \frac{1}{2.35 \times 10^{-3}} = 822.3 \text{ ft-kips}$$

$$\frac{M_{px}}{\Omega_b} = 382 \text{ ft-kips} < 822.3 \text{ ft-kips} \quad \therefore \text{use } \frac{M_{nx}}{\Omega_b} = 382 \text{ ft-kips}$$

$$b_x = \frac{8}{9(M_{nx}/\Omega_b)} = \frac{8}{9(382)} = 2.33 \times 10^{-3}$$

Check Equation 6.9: $p = 1.54 \times 10^{-3}$, $b_x = 2.33 \times 10^{-3}$, $b_y = 4.90 \times 10^{-3}$

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry})$$

$$= 0.5(1.54 \times 10^{-3})(95) + \frac{9}{8}((2.33 \times 10^{-3})(102) + (4.90 \times 10^{-3})(102))$$

$$= 0.903 < 1.0 \quad \text{(OK)}$$

Use a W14 × 90

6.7-5

(a) LRFD Solution

$$P_u = 1.2(135) + 1.6(415) = 826.0 \text{ kips}$$

The factored moments are

$$M_{ntx}\text{ (top)} = 1.2(90) + 1.6(270) = 540.0 \text{ ft-kips}$$

$$M_{ntx}\text{ (bot)} = 1.2(30) + 1.6(90) = 180.0 \text{ ft-kips}$$

The amplification factor B_1 can be estimated as 1.0 for purposes of making a trial selection.

$$M_{ux} = B_{1x}M_{ntx} = 1.0(540) = 540 \text{ ft-kips}$$

Try a W14 × 159. From Table 6-1, with $KL = L_b = 20$ ft,

$$p = 0.619 \times 10^{-3},\ b_x = 0.863 \times 10^{-3}$$

$$pP_u = (0.619 \times 10^{-3})(826) = 0.5113 > 0.2 \ \therefore \text{ Equation 6.8 controls.}$$

As a preliminary check,

$$\begin{aligned} pP_r + b_x M_{rx} + b_y M_{ry} &= pP_u + b_x M_{ux} + b_y M_{uy} \\ &= (0.619 \times 10^{-3})(826) + (0.863 \times 10^{-3})(540) + 0 \\ &= 0.977 < 1.0 \qquad \text{(OK)} \end{aligned}$$

Calculate B_1 for the axis of bending:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{180}{540}\right) = 0.4667$$

$$P_{e1x} = \frac{\pi^2 E I_x}{(K_x L)^2} = \frac{\pi^2(29{,}000)(1900)}{(0.8 \times 20 \times 12)^2} = 1.475 \times 10^4 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{P_u}{P_{e1x}}} = \frac{0.4667}{1 - \dfrac{826}{14750}} = 0.4944 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

(Since this shape is adequate for $C_b = 1.0$, the steps shown below, computation of the actual value of C_b and an adjustment of b_x, are not necessary.)

Compute C_b and modify b_x to account for C_b.

$$\begin{aligned} C_b &= \frac{12.5 M_{\max}}{2.5 M_{\max} + 3M_A + 4M_B + 3M_C} \\ &= \frac{12.5(540)}{2.5(540) + 3(0) + 4(180) + 3(360)} = 2.143 \end{aligned}$$

$$C_b \times \phi_b M_{nx} = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 2.143 \times \frac{8}{9} \times \frac{1}{0.863 \times 10^{-3}} = 2207 \text{ ft-kips}$$

From the Z_x table, $\phi_b M_{px} = 1080$ ft-kips < 2207 ft-kips $\therefore \phi_b M_{nx} = 1080$ ft-kips

$$b_x = \frac{8}{9(\phi_b M_{nx})} = \frac{8}{9(1080)} = 8.23 \times 10^{-4}$$

$$\begin{aligned} pP_u + b_x M_{ux} + b_y M_{uy} &= (0.619 \times 10^{-3})(826) + (8.23 \times 10^{-4})(540) + 0 \\ &= 0.956 < 1.0 \qquad \text{(OK)} \end{aligned}$$

Use a W14 × 159

(a) ASD Solution

The axial service load is $P_a = P_D + P_L = 135 + 415 = 550$ kips

The service-load moments are

$$M_{ntx}\text{ (top)} = M_D + M_L = 90 + 270 = 360 \text{ ft-kips}$$

$$M_{ntx}\text{ (bot)} = M_D + M_L = 30 + 90 = 120 \text{ ft-kips}$$

The amplification factor B_1 can be estimated as 1.0 for purposes of making a trial selection.

$$M_{ax} = B_{1x} M_{ntx} = 1.0(360) = 360 \text{ ft-kips}$$

Try a W14 × 159. From Table 6-1, with $KL = L_b = 20$ ft,

$$p = 0.930 \times 10^{-3},\ b_x = 1.30 \times 10^{-3}$$

$$pP_u = (0.930 \times 10^{-3})(550) = 0.5115 > 0.2 \ \therefore \text{ Equation 6.8 controls.}$$

As a preliminary check,

$$\begin{aligned} pP_r + b_x M_{rx} + b_y M_{ry} &= pP_a + b_x M_{ax} + b_y M_{ay} \\ &= (0.930 \times 10^{-3})(550) + (1.30 \times 10^{-3})(360) + 0 \\ &= 0.980 < 1.0 \end{aligned}$$

Calculate B_1:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{120}{360}\right) = 0.4667$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29,000)(1900)}{(0.8 \times 20 \times 12)^2} = 1.475 \times 10^4 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{1.60P_a}{P_{e1x}}} = \frac{0.4667}{1 - \dfrac{1.60(550)}{14750}} = 0.4963 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

(Since this shape is adequate for $C_b = 1.0$, the steps shown below, computation of the actual value of C_b and an adjustment of b_x, are not necessary.)

Compute C_b and modify b_x to account for C_b.

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(360)}{2.5(360) + 3(0) + 4(120) + 3(240)} = 2.143$$

$$C_b \times \frac{M_{nx}}{\Omega_b} = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 2.143 \times \frac{8}{9} \times \frac{1}{1.30 \times 10^{-3}} = 1465 \text{ ft-kips}$$

$$\frac{M_{px}}{\Omega_b} = 716 \text{ ft-kips} < 1465 \text{ ft-kips} \ \therefore \text{use } \frac{M_{nx}}{\Omega_b} = 716 \text{ ft-kips}$$

$$b_x = \frac{8}{9(M_{nx}/\Omega_b)} = \frac{8}{9(716)} = 1.24 \times 10^{-3}$$

Check Equation 6.8: $p = 0.930 \times 10^{-3},\ b_x = 1.24 \times 10^{-3}$

$$\begin{aligned} pP_a + b_x M_{ax} + b_y M_{ay} &= (0.930 \times 10^{-3})(550) + (1.24 \times 10^{-3})(360) \\ &= 0.958 \qquad \text{(OK)} \end{aligned}$$

Use a W14 × 159

6.7-6

(a) LRFD Solution

$$P_u = 1.2(92/2) + 1.6(92/2) = 128.8 \text{ kips}$$

The factored moments at the top are

$$M_{ntx} = 1.2(160/2) + 1.6(160/2) = 224.0 \text{ ft-kips}$$

$$M_{nty} = 1.2(24/2) + 1.6(24/2) = 33.6 \text{ ft-kips}$$

The factored moments at the bottom are

$$M_{ntx} = 1.2(214/2) + 1.6(214/2) = 299.6 \text{ ft-kips}$$

$$M_{nty} = 1.2(31/2) + 1.6(31/2) = 43.4 \text{ ft-kips}$$

The amplification factor B_1 can be estimated as 1.0 for purposes of making a trial selection. For the two axes,

$$M_{ux} = B_{1x}M_{ntx} = 1.0(299.6) = 299.6 \text{ ft-kips}$$

$$M_{uy} = B_{1y}M_{nty} = 1.0(43.4) = 43.4 \text{ ft-kips}$$

Check the W10s. Try a W10 ×100. From Table 6-1, with $KL = L_b = 16$ ft,

$$p = 1.11 \times 10^{-3},\ b_x = 1.93 \times 10^{-3},\ \ b_y = 3.89 \times 10^{-3}$$

$$pP_u = (1.11 \times 10^{-3})(128.8) = 0.1430 < 0.2\ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry}) = 0.5pP_u + \frac{9}{8}(b_xM_{ux} + b_yM_{uy})$$

$$= 0.5(1.11 \times 10^{-3})(128.8) + \frac{9}{8}((1.93 \times 10^{-3})(299.6) + (3.89 \times 10^{-3})(43.4))$$

$$= 0.912 < 1.0 \qquad \text{(OK)}$$

Check the W12s. Try a W12 × 79. From Table 6-1,

$$p = 1.28 \times 10^{-3},\ b_x = 2.13 \times 10^{-3},\ \ b_y = 4.37 \times 10^{-3}$$

$$pP_u = (1.28 \times 10^{-3})(128.8) = 0.1649 < 0.2\ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$0.5pP_u + \frac{9}{8}(b_xM_{ux} + b_yM_{uy})$$

$$= 0.5(1.28 \times 10^{-3})(128.8) + \frac{9}{8}((2.13 \times 10^{-3})(299.6) + (4.37 \times 10^{-3})(43.4))$$

$$= 1.01 > 1.0 \qquad \text{(but close; retain this possibility)}$$

Check the W14s. Try a W14 × 82. From Table 6-1,

$$p = 1.43 \times 10^{-3},\ b_x = 1.92 \times 10^{-3},\ \ b_y = 5.29 \times 10^{-3}$$

$$pP_u = (1.43 \times 10^{-3})(128.8) = 0.1842 < 0.2\ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$0.5pP_u + \frac{9}{8}(b_xM_{ux} + b_yM_{uy})$$

$$= 0.5(1.43 \times 10^{-3})(128.8) + \frac{9}{8}((1.92 \times 10^{-3})(299.6) + (5.29 \times 10^{-3})(43.4))$$

$$= 0.998 < 1.0 \qquad \text{(OK)}$$

Try a W12 × 79 (the lightest), $p = 1.28 \times 10^{-3}$, $b_x = 2.13 \times 10^{-3}$, $b_y = 4.37 \times 10^{-3}$

Calculate B_1 for each axis:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{224.0}{299.6}\right) = 0.3009$$

$$C_{my} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{33.6}{43.4}\right) = 0.2903$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29,000)(662)}{(0.8 \times 16 \times 12)^2} = 8031 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \frac{P_u}{P_{e1x}}} = \frac{0.3009}{1 - \frac{128.8}{8031}} = 0.3058 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

$$P_{e1y} = \frac{\pi^2 EI_y}{(K_yL)^2} = \frac{\pi^2(29,000)(216)}{(16 \times 12)^2} = 1677 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1 - \frac{P_u}{P_{e1y}}} = \frac{0.2903}{1 - \frac{128.8}{1677}} = 0.3145 < 1.0 \qquad \therefore B_{1y} = 1.0 \text{ as assumed}$$

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(299.6)}{2.5(299.6) + 3(93.1) + 4(37.8) + 3(168.7)} = 2.222$$

$$C_b \times \phi_b M_{nx} = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 2.222 \times \frac{8}{9} \times \frac{1}{2.13 \times 10^{-3}} = 927.3 \text{ ft-kips}$$

From the Z_x table, $\phi_b M_{px} = 446$ ft-kips < 927.3 ft-kips $\therefore$ $\phi_b M_{nx} = 446$ ft-kips

$$b_x = \frac{8}{9(\phi_b M_{nx})} = \frac{8}{9(446)} = 1.99 \times 10^{-3}$$

$$0.5pP_u + \frac{9}{8}(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(1.28 \times 10^{-3})(128.8) + \frac{9}{8}((1.99 \times 10^{-3})(299.6) + (4.37 \times 10^{-3})(43.4))$$

$$= 0.967 < 1.0 \qquad \text{(OK)}$$

Use a W12 × 79

(a) ASD Solution

The axial service load is $P_a = 92$ kips

The service-load moments at the top are

$M_{ntx} = 160$ ft-kips, $M_{nty} = 24$ ft-kips

The service-load moments at the bottom are

$$M_{ntx} = 214 \text{ ft-kips}, \qquad M_{nty} = 31 \text{ ft-kips}$$

The amplification factor B_1 can be estimated as 1.0 for purposes of making a trial selection. For the two axes,

$$M_{ax} = B_{1x}M_{ntx} = 1.0(214) = 214 \text{ ft-kips}$$

$$M_{ay} = B_{1y}M_{nty} = 1.0(31) = 31 \text{ ft-kips}$$

Check the W10s. Try a W10 ×100. From Table 6-1, with $KL = L_b = 16$ ft,

$$p = 1.67 \times 10^{-3},\ b_x = 2.90 \times 10^{-3},\ b_y = 5.84 \times 10^{-3}$$

Determine which interaction equation to use:

$$\frac{P_a}{P_n/\Omega_c} = pP_a = (1.67 \times 10^{-3})(92) = 0.1536 < 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry})$$

$$= 0.5(1.67 \times 10^{-3})(92) + \frac{9}{8}((2.90 \times 10^{-3})(214) + (5.84 \times 10^{-3})(31))$$

$$= 0.979 < 1.0 \qquad \text{(OK)}$$

Check the W12s. Try a W12 × 79. From Table 6-1,

$$p = 1.92 \times 10^{-3},\ b_x = 3.21 \times 10^{-3},\ b_y = 6.56 \times 10^{-3}$$

Determine which interaction equation to use:

$$\frac{P_a}{P_n/\Omega_c} = pP_a = (1.92 \times 10^{-3})(92) = 0.1766 < 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry})$$

$$= 0.5(1.92 \times 10^{-3})(92) + \frac{9}{8}((3.21 \times 10^{-3})(214) + (6.56 \times 10^{-3})(31))$$

$$= 1.09 > 1.0 \qquad \text{(N.G.)}$$

Check the W14s. Try a W14 × 82. From Table 6-1,

$$p = 1.94 \times 10^{-3},\ b_x = 2.79 \times 10^{-3},\ b_y = 7.95 \times 10^{-3}$$

Determine which interaction equation to use:

$$\frac{P_a}{P_n/\Omega_c} = pP_a = (1.94 \times 10^{-3})(92) = 0.1785 < 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry})$$

$$= 0.5(1.94 \times 10^{-3})(92) + \frac{9}{8}((2.79 \times 10^{-3})(214) + (7.95 \times 10^{-3})(31))$$

$$= 1.04 > 1.0 \qquad \text{(but close; retain this possibility.)}$$

Try a W14 × 82. Calculate B_1 for each axis:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{160}{214}\right) = 0.3009$$

$$C_{my} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{24}{31}\right) = 0.2903$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(881)}{(0.8 \times 16 \times 12)^2} = 1.069 \times 10^4 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{1.60P_a}{P_{e1x}}} = \frac{0.3009}{1 - \dfrac{1.60(92)}{10690}} = 0.3051 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

$$P_{e1y} = \frac{\pi^2 EI_y}{(K_yL)^2} = \frac{\pi^2(29{,}000)(148)}{(16 \times 12)^2} = 1149 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1 - \dfrac{1.60P_a}{P_{e1y}}} = \frac{0.2903}{1 - \dfrac{1.60(92)}{1149}} = 0.3330 < 1.0 \qquad \therefore B_{1y} = 1.0 \text{ as assumed}$$

Compute C_b and modify b_x to account for C_b.

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(31)}{2.5(31) + 3(10.25) + 4(3.5) + 3(17.25)} = 2.227$$

$$C_b \times \frac{M_{nx}}{\Omega_b} = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 2.227 \times \frac{8}{9} \times \frac{1}{2.79 \times 10^{-3}} = 709.5 \text{ ft-kips}$$

$$\frac{M_{px}}{\Omega_b} = 347 \text{ ft-kips} < 709.5 \text{ ft-kips} \quad \therefore \text{use } \frac{M_{nx}}{\Omega_b} = 347 \text{ ft-kips}$$

$$b_x = \frac{8}{9(M_{nx}/\Omega_b)} = \frac{8}{9(347)} = 2.56 \times 10^{-3}$$

Check Equation 6.9: $p = 1.94 \times 10^{-3}$, $b_x = 2.56 \times 10^{-3}$, $b_y = 7.95 \times 10^{-3}$

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry})$$

$$= 0.5(1.94 \times 10^{-3})(92) + \frac{9}{8}((2.56 \times 10^{-3})(214) + (7.95 \times 10^{-3})(31))$$

$= 0.983 < 1.0$ (OK) Use a W12 × 79

6.7-7

(a) LRFD Solution

$$P_u = 1.2(0.25 \times 80) + 1.6(0.75 \times 80) = 120.0 \text{ kips}$$

The factored moments at the top are

$$M_{ntx} = 1.2(0.25 \times 133) + 1.6(0.75 \times 133) = 199.5 \text{ ft-kips}$$

$$M_{nty} = 1.2(0.25 \times 43) + 1.6(0.75 \times 43) = 64.5 \text{ ft-kips}$$

The factored moments at the bottom are

$$M_{ntx} = 1.2(0.25 \times 27) + 1.6(0.75 \times 27) = 40.5 \text{ ft-kips}$$

$$M_{nty} = 1.2(0.25 \times 9) + 1.6(0.75 \times 9) = 13.5 \text{ ft-kips}$$

The amplification factor B_1 can be estimated as 1.0 for purposes of making a trial selection. For the two axes,

$$M_{ux} = B_{1x}M_{ntx} = 1.0(199.5) = 199.5 \text{ ft-kips}$$

$$M_{uy} = B_{1y}M_{nty} = 1.0(64.5) = 64.5 \text{ ft-kips}$$

Check the W10s. Try a W10 ×77. From Table 6-1, with $KL = L_b = 16$ ft,

$$p = 1.46 \times 10^{-3},\ b_x = 2.62 \times 10^{-3},\ \ b_y = 5.16 \times 10^{-3}$$

$$pP_u = (1.46 \times 10^{-3})(120) = 0.1752 < 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry}) = 0.5pP_u + \frac{9}{8}(b_xM_{ux} + b_yM_{uy})$$

$$= 0.5(1.46 \times 10^{-3})(120) + \frac{9}{8}((2.62 \times 10^{-3})(199.5) + (5.16 \times 10^{-3})(64.5))$$

$$= 1.05 > 1.0 \qquad \text{(but close; retain this as a possibility.)}$$

Check the W12s. Try a W12 × 72. From Table 6-1,

$$p = 1.41 \times 10^{-3},\ b_x = 2.37 \times 10^{-3},\ \ b_y = 4.82 \times 10^{-3}$$

$$pP_u = (1.41 \times 10^{-3})(120) = 0.1692 < 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

As a preliminary check,

$$0.5pP_u + \frac{9}{8}(b_xM_{ux} + b_yM_{uy})$$

$$= 0.5(1.41 \times 10^{-3})(120) + \frac{9}{8}((2.37 \times 10^{-3})(199.5) + (4.82 \times 10^{-3})(64.5))$$

$$= 0.966 < 1.0 \qquad \text{(OK)}$$

Check the W14s. Try a W14 × 68. From Table 6-1,

$$p = 1.74 \times 10^{-3},\ b_x = 2.38 \times 10^{-3},\ \ b_y = 6.42 \times 10^{-3}$$

$$pP_u = (1.74 \times 10^{-3})(120) = 0.2088 > 0.2 \ \therefore \text{ Equation 6.8 controls.}$$

As a preliminary check,

$$pP_u + b_xM_{ux} + b_yM_{uy}$$

$$= (1.74 \times 10^{-3})(120) + (2.38 \times 10^{-3})(199.5) + (6.42 \times 10^{-3})(64.5) = 1.098$$

$$= 1.10 > 1.0 \qquad \text{(N.G.)}$$

Try a W12 × 72, $p = 1.41 \times 10^{-3}$, $b_x = 2.37 \times 10^{-3}$, $b_y = 4.82 \times 10^{-3}$

Calculate B_1 for each axis:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{40.5}{199.5}\right) = 0.6812$$

$$C_{my} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{13.5}{64.5}\right) = 0.6837$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29,000)(597)}{(16 \times 12)^2} = 4635 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{P_u}{P_{e1x}}} = \frac{0.6812}{1 - \dfrac{120}{4635}} = 0.6993 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

$$P_{e1y} = \frac{\pi^2 EI_y}{(K_yL)^2} = \frac{\pi^2(29,000)(195)}{(16 \times 12)^2} = 1514 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1 - \dfrac{P_u}{P_{e1y}}} = \frac{0.6837}{1 - \dfrac{120}{1514}} = 0.7426 < 1.0 \qquad \therefore B_{1y} = 1.0 \text{ as assumed}$$

Since the amplification factors are as assumed, and this shape is adequate for $C_b = 1.0$, computation of the actual value of C_b and an adjustment of b_x are not necessary. The preliminary evaluation is sufficient.

Use a W12 × 72

(a) ASD Solution (abbreviated version; rejected trials not shown.)

The axial service load is $P_a = 80$ kips

The service-load moments at the top are

$M_{ntx} = 133$ ft-kips, $M_{nty} = 43$ ft-kips

The service-load moments at the bottom are

$M_{ntx} = 27$ ft-kips, $M_{nty} = 9$ ft-kips

The amplification factor B_1 can be estimated as 1.0 for purposes of making a trial selection. For the two axes,

$$M_{ax} = B_{1x}M_{ntx} = 1.0(133) = 133 \text{ ft-kips}$$

$$M_{ay} = B_{1y}M_{nty} = 1.0(43) = 43 \text{ ft-kips}$$

Try a W12 × 72. From Table 6-1,

$$p = 2.12 \times 10^{-3},\ b_x = 3.56 \times 10^{-3},\ b_y = 7.24 \times 10^{-3}$$

$$pP_u = (2.12 \times 10^{-3})(80) = 0.1696 < 0.2 \ \therefore \text{Equation 6.9 controls.}$$

As a preliminary check,

$$0.5pP_r + \frac{9}{8}(b_xM_{rx} + b_yM_{ry})$$

$$= 0.5(2.12 \times 10^{-3})(80) + \frac{9}{8}((3.56 \times 10^{-3})(133) + (7.24 \times 10^{-3})(43))$$

$$= 0.968 < 1.0 \qquad \text{(OK)}$$

Calculate B_1 for each axis:

$$C_{mx} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{27}{133}\right) = 0.6812$$

$$C_{my} = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{9}{43}\right) = 0.6837$$

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(597)}{(16 \times 12)^2} = 4635 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \frac{1.60P_a}{P_{e1x}}} = \frac{0.6812}{1 - \frac{1.60(80)}{4635}} = 0.7005 < 1.0 \qquad \therefore B_{1x} = 1.0 \text{ as assumed}$$

$$P_{e1y} = \frac{\pi^2 EI_y}{(K_yL)^2} = \frac{\pi^2(29{,}000)(195)}{(16 \times 12)^2} = 1514 \text{ kips}$$

$$B_{1y} = \frac{C_{my}}{1 - \frac{1.60P_a}{P_{e1y}}} = \frac{0.6837}{1 - \frac{1.60(80)}{1514}} = 0.7468 < 1.0 \qquad \therefore B_{1y} = 1.0 \text{ as assumed}$$

Since the amplification factors are as assumed, and this shape is adequate for $C_b = 1.0$, computation of the actual value of C_b and an adjustment of b_x are not necessary. The preliminary evaluation is sufficient.

Use a W12 × 72

6.7-8

Assume that $B_1 = B_2 = 1.0$ for purposes of making a trial selection.

$$M_u = B_1M_{nt} + B_2M_{\ell t} = 1.0(270) + 1.0(30) = 300.0 \text{ ft-kips}$$

Try a W12 × 65. From Table 6-1, For $KL = L_b = 16$ ft,

$$p = 1.57 \times 10^{-3},\ b_x = 2.66 \times 10^{-3}$$

Determine the critical effective length for compression. Use the sway condition.

$$\frac{K_xL}{r_x/r_y} = \frac{2.0(16)}{1.75} = 18.29 \text{ ft} > K_yL = 16 \text{ ft.}$$

$\therefore$ use $KL = 18.29$ ft for p (use $L_b = 16$ ft for b_x). From Table 6-1, $p = 1.71 \times 10^{-3}$ (by interpolation).

$$pP_u = (1.71 \times 10^{-3})(75) = 0.1283 < 0.2 \ \therefore \text{Equation 6.9 controls.}$$

Consider the braced condition first.

For the axis of bending,

$$P_{e1x} = \frac{\pi^2 EI_x}{(K_{1x}L)^2} = \frac{\pi^2(29,000)(533)}{(16 \times 12)^2} = 4138 \text{ kips}$$

$$B_{1x} = \frac{C_{mx}}{1 - \dfrac{P_u}{P_{e1x}}} = \frac{0.6}{1 - \dfrac{75}{4138}} = 0.6111 < 1.0 \qquad \therefore\ B_{1x} = 1.0 \text{ as assumed}$$

Sway condition:

For the axis of bending,

$$P_{e2x} = \frac{\pi^2 EI_x}{(K_{2x}L)^2} = \frac{\pi^2(29,000)(533)}{(2.0 \times 16 \times 12)^2} = 1035 \text{ kips}$$

Assume that $\sum P_u = P_u$ and $\sum P_{e2} = P_{e2}$:

$$B_2 = \frac{1}{1 - \dfrac{\alpha \Sigma P_{nt}}{\Sigma P_{e2}}} = \frac{1}{1 - \dfrac{1.00(75)}{1035}} = 1.078$$

The total amplified moment is

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(270) + 1.078(30) = 302.3 \text{ ft-kips}$$

Modify b_x to account for C_b.

$$C_b \times \phi_b M_n = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 1.67 \times \frac{8}{9} \times \frac{1}{2.66 \times 10^{-3}} = 558.1 \text{ ft-kip}$$

From the Z_x table, $\phi_b M_{px} = 356$ ft-kips < 558.1 ft-kips $\therefore$ $\phi_b M_{nx} = 356$ ft-kips

$$b_x = \frac{8}{9(\phi_b M_{nx})} = \frac{8}{9(356)} = 2.50 \times 10^{-3}$$

Check Equation 6.9: $p = 1.71 \times 10^{-3}$, $b_x = 2.50 \times 10^{-3}$

$$0.5pP_u + \left(\frac{9}{8}\right)(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(1.71 \times 10^{-3})(75) + \frac{9}{8}((2.50 \times 10^{-3})(302.3) + 0) = 0.914 < 1 \qquad \text{(OK)}$$

Use a W12 × 65

6.7-9

(a) LRFD Solution

Load combinations involving dead load (D), roof live load (L_r), and wind load (W) are as follows:

Load Combination 2: $1.2D + 0.5L_r$

$$P_u = 1.2(19) + 0.5(33) = 39.3 \text{ kips}$$

$$M_{nt} = 1.2(79) + 0.5(130) = 159.8 \text{ ft-kips}$$

$$M_{\ell t} = 0$$

(Since the frame and loading are symmetrical, there are no sidesway moments for this load combination.)

Load Combination 3: $1.2D + 1.6L_r + 0.8W$

$$P_u = 1.2(19) + 1.6(33) + 0.8(-11 + 1.4) = 67.92 \text{ kips}$$

$$M_{nt} = 1.2(79) + 1.6(130) + 0.8(-46) = 266.0 \text{ ft-kips}$$

$$M_{\ell t} = 0.8(32) = 25.6 \text{ ft-kips}$$

Load Combination 4: $1.2D + 1.6W + 0.5L_r$

$$P_u = 1.2(19) + 1.6(-11 + 1.4) + 0.5(33) = 23.94 \text{ kips}$$

$$M_{nt} = 1.2(79) + 1.6(-46) + 0.5(130) = 86.2 \text{ ft-kips}$$

$$M_{\ell t} = 1.6(32) = 51.2 \text{ ft-kips}$$

Load combination 3 will govern. For purposes of selecting a trial shape, assume $B_1 = 1.0$.

$$\Sigma P_{e2} = R_M \frac{\Sigma HL}{\Delta_H} = R_M \frac{\Sigma H}{\Delta_H/L} = 0.85\left(\frac{3.6}{1/400}\right) = 1224$$

(The unfactored horizontal load $\Sigma H = 3.6$ kips is used because the drift index is based on the maximum drift caused by *service* loads.)

$$B_2 = \frac{1}{1 - \dfrac{\alpha \Sigma P_{nt}}{\Sigma P_{e2}}} = \frac{1}{1 - \dfrac{1.00(2 \times 67.92)}{1224}} = 1.125$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(266) + 1.125(25.6) = 294.8 \text{ ft-kips}$$

Try a W14 × 61. From Table 6-1, using $KL = L_b = 18$ ft, $p = 2.19 \times 10^{-3}$, $b_x = 2.84 \times 10^{-3}$

Determine the critical effective length for compression. Use the sway condition. Estimate the effective length factor as $K_x = 2.0$

$$\frac{K_x L}{r_x/r_y} = \frac{2.0(18)}{2.44} = 14.75 \text{ ft} < K_y L = 18 \text{ ft.} \quad \therefore KL = 18 \text{ ft as assumed}$$

$$pP_u = (2.19 \times 10^{-3})(67.92) = 0.1487 < 0.2 \quad \therefore \text{ Equation 6.9 controls.}$$

Consider the braced condition first. Use $K_x = 1.0$.

$$P_{e1} = \frac{\pi^2 E I_x}{(K_x L)^2} = \frac{\pi^2 (29{,}000)(640)}{(1.0 \times 18 \times 12)^2} = 3926 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{0}{M_2}\right) = 0.6$$

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} = \frac{0.6}{1 - \frac{67.92}{3926}} = 0.6106 < 1.0 \qquad \therefore B_1 = 1.0 \text{ as assumed}$$

Since $B_1 = 1.0$ is the value originally assumed, and B_2 will not change, the previously computed value of $M_u = 294.8$ ft-kips is unchanged.

Modify b_x to account for C_b.

$$C_b \times \phi_b M_n = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 1.67 \times \frac{8}{9} \times \frac{1}{2.84 \times 10^{-3}} = 522.7 \text{ ft-kips}$$

From the Z_x table, $\phi_b M_p = 383$ ft-kips < 522.7 ft-kips

$$\therefore \phi_b M_n = \phi_b M_p = 383 \text{ ft-kips}$$

$$b_x = \frac{8}{9(\phi_b M_{nx})} = \frac{8}{9(383)} = 2.32 \times 10^{-3}$$

Check Equation 6.9: $p = 2.19 \times 10^{-3}$, $b_x = 2.32 \times 10^{-3}$

$$0.5pP_u + \left(\frac{9}{8}\right)(b_x M_{ux} + b_y M_{uy})$$

$$= 0.5(2.19 \times 10^{-3})(67.92) + \frac{9}{8}((2.32 \times 10^{-3})(294.8) + 0)$$

$$= 0.844 \qquad \text{(OK)}$$

Use a W14 × 61

(b) ASD Solution

Potential load combinations involving dead load (D), roof live load (L_r), and wind load (W) are as follows:

Load Combination 3: $D + L_r$

$$P_a = 19 + 33 = 52 \text{ kips}$$

$$M_{nt} = 79 + 130 = 209 \text{ ft-kips}$$

$$M_{\ell t} = 0$$

(Since the frame and loading are symmetrical, there are no sidesway moments for this load combination.)

Load Combination 5: $D \pm W$

$$P_a = 19 + (-11 + 1.4) = 9.4 \text{ kips}$$

$$M_{nt} = 79 + (-46) = 33 \text{ ft-kips}$$

$$M_{\ell t} = 32 \text{ ft-kips}$$

Load Combination 6: $D + 0.75W + 0.75L_r$

$$P_a = 19 + 0.75(-11 + 1.4) + 0.75(33) = 36.55 \text{ kips}$$

$$M_{nt} = 79 + 0.75(-46) + 0.75(130) = 142.0 \text{ ft-kips}$$

$$M_{\ell t} = 32 \text{ ft-kips}$$

Assume load combination 3 controls. After a shape is selected, check combination 6. For purposes of selecting a trial shape, assume $B_1 = 1.0$.

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.0(209) + 0 = 209 \text{ ft-kips}$$

Try a W14 × 61. From Table 6-1, using $KL = L_b = 18$ ft,

$$p = 3.29 \times 10^{-3},\ b_x = 4.27 \times 10^{-3}$$

Determine the critical effective length for compression. Use the sway condition. Estimate the effective length factor as $K_x = 2.0$

$$\frac{K_x L}{r_x/r_y} = \frac{2.0(18)}{2.44} = 14.75 \text{ ft} < K_y L = 18 \text{ ft}. \quad \therefore\ KL = 18 \text{ ft as assumed}$$

$$pP_a = (3.29 \times 10^{-3})(52) = 0.1711 < 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

For the braced condition, use $K_x = 1.0$.

$$P_{e1} = \frac{\pi^2 E I_x}{(K_x L)^2} = \frac{\pi^2(29{,}000)(640)}{(1.0 \times 18 \times 12)^2} = 3926 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(\frac{0}{M_2}\right) = 0.6$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.60 P_a/P_{e1})} = \frac{0.6}{1 - 1.60(52/3926)}$$

$$= 0.6130 < 1.0 \qquad B_1 = 1.0 \text{ as assumed}$$

Modify b_x to account for C_b.

$$C_b \times \frac{M_{nx}}{\Omega_b} = C_b \times \frac{8}{9} \times \frac{1}{b_x} = 1.67 \times \frac{8}{9} \times \frac{1}{4.27 \times 10^{-3}} = 347.6 \text{ ft-kips}$$

$$\frac{M_{px}}{\Omega_b} = 254 \text{ ft-kips} < 347.6 \text{ ft-kips} \ \therefore \text{use } \frac{M_{nx}}{\Omega_b} = 254 \text{ ft-kips}$$

$$b_x = \frac{8}{9(M_{nx}/\Omega_b)} = \frac{8}{9(254)} = 3.50 \times 10^{-3}$$

Check Equation 6.9: $\quad p = 3.29 \times 10^{-3},\ b_x = 3.50 \times 10^{-3}$

$$0.5pP_r + \frac{9}{8}(b_x M_{rx} + b_y M_{ry})$$

$$= 0.5(3.29 \times 10^{-3})(52) + \frac{9}{8}((3.50 \times 10^{-3})(209) + 0) = 0.909 < 1.0 \qquad \text{(OK)}$$

Check load combination 6: $\quad P_a = 36.55$ kips, $M_{nt} = 142$ ft-kips, $M_{\ell t} = 32$ ft-kips

$$pP_a = (3.29 \times 10^{-3})(36.55) = 0.1202 < 0.2 \ \therefore \text{ Equation 6.9 controls.}$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r/P_{e1})} = \frac{C_m}{1 - (1.60 P_a/P_{e1})} = \frac{0.6}{1 - 1.60(36.55)/3926}$$

$$= 0.6091 < 1.0 \qquad B_1 = 1.0 \text{ as assumed}$$

$$\Sigma P_{e2} = R_M \frac{\Sigma HL}{\Delta_H} = R_M \frac{\Sigma H}{\Delta_H/L} = 0.85\left(\frac{3.6}{1/400}\right) = 1224$$

$$B_2 = \frac{1}{1 - \frac{\alpha \Sigma P_{nt}}{\Sigma P_{e2}}} = \frac{1}{1 - \frac{1.60(2 \times 36.55)}{1224}} = 1.106$$

$$M_a = B_1 M_{nt} + B_2 M_{lt} = 1.0(142) + 1.106(32) = 177.4 \text{ ft-kips}$$

Check Equation 6.9: $p = 3.29 \times 10^{-3}$, $b_x = 3.50 \times 10^{-3}$

$$0.5pP_a + \left(\frac{9}{8}\right)(b_x M_{ax} + b_y M_{ay})$$

$$= 0.5(3.29 \times 10^{-3})(36.55) + \frac{9}{8}((3.50 \times 10^{-3})(177.4) + 0)$$

$$= 0.759 < 1.0 \quad \text{(OK)}$$

Use a W14 × 61

6.7-10

(a) LRFD Solution

Determine the total vertical load to be stabilized by the bracing.

Load combination 2: $1.2D + 0.5L_r$

$$\Sigma P_u = [1.2(0.860)(45) + 0.5(1.45)(45)] \times 3 = 237.2 \text{ kips}$$

Load Combination 3: $1.2D + 1.6L_r + 0.8W$

$$\Sigma P_u = [1.2(0.860)(45) + 1.6(1.45)(45) + 0.8(-0.510)(45)] \times 3 = 397.4 \text{ kips}$$

Combination 3 controls; $P_r = \Sigma P_u = 397.4$ kips

From AISC Equation A-6-1, the lateral shear to be resisted is

$$P_{br} = 0.004P_r = 0.004(397.4) = 1.590 \text{ kips}$$

Design both braces as tension-only members, and let the entire force be resisted by one brace. Since P_{br} is the horizontal component of the brace force (see Figure 6.24a in the textbook), the brace force is

$$F = \frac{P_{br}}{\cos\theta}, \text{ where } \theta = \tan^{-1}(18/45) = 21.80°$$

$$F = \frac{1.590}{\cos(21.80°)} = 1.712 \text{ kips}$$

Based on the limit state of tension yielding, the required area is

$$A = \frac{F}{0.9F_y} = \frac{1.712}{0.9(36)} = 0.0528 \text{ in.}^2$$

From AISC Equation A-6-2, the required lateral stiffness is

$$\beta_{br} = \frac{1}{\phi}\left(\frac{2P_r}{L_b}\right) = \frac{1}{0.75}\left[\frac{2(397.4)}{(18 \times 12)}\right] = 4.906 \text{ kips/in.}$$

The length of the brace is

$$L = (18 \times 12)/\sin(21.80°) = 581.6 \text{ in.}$$

Let

$$\frac{AE}{L}\cos^2\theta = \frac{1}{\phi}\left(\frac{2P_r}{L_b}\right) = 4.906$$

$$A = \frac{4.906L}{E\cos^2\theta} = \frac{4.906(581.6)}{29{,}000\cos^2(21.8°)} = 0.1141 \text{ in.}^2$$

The stiffness requirement controls.

Use a tension brace with a cross-sectional area of at least 0.114 in.^{2}

(b) ASD Solution

Determine the total vertical load to be stabilized by the bracing.

Load combination 3 controls: $D + L_r$

$$P_r = \Sigma P_a = [0.860(45) + 1.45(45)] \times 3 = 311.9 \text{ kips}$$

From AISC Equation A-6-1, the lateral shear to be resisted is

$$P_{br} = 0.004P_r = 0.004(311.9) = 1.248 \text{ kips}$$

Design both braces as tension-only members, and let the entire force be resisted by one brace. Since P_{br} is the horizontal component of the brace force (see Figure 6.24a in the textbook), the brace force is

$$F = \frac{P_{br}}{\cos\theta}, \text{ where } \theta = \tan^{-1}(18/45) = 21.80°$$

$$F = \frac{1.248}{\cos(21.80°)} = 1.344 \text{ kips}$$

Based on the limit state of tension yielding, the required area is

$$A = \frac{F}{F_y/\Omega} = \frac{1.344}{36/2.00} = 7.467 \times 10^{-2} \text{ in.}^2$$

From AISC Equation A-6-2, the required lateral stiffness is

$$\beta_{br} = \Omega\left(\frac{2P_r}{L_b}\right) = 2.00\left[\frac{2(311.9)}{(18 \times 12)}\right] = 5.776 \text{ kips/in.}$$

The length of the brace is

$$L = (18 \times 12)/\sin(21.80°) = 581.6 \text{ in.}$$

Let

$$\frac{AE}{L}\cos^2\theta = \Omega\left(\frac{2P_r}{L_b}\right) = 5.776$$

$$A = \frac{5.776L}{E\cos^2\theta} = \frac{5.776(581.6)}{29{,}000\cos^2(21.8°)} = 0.1344 \text{ in.}^2$$

The stiffness requirement controls.

Use a tension brace with a cross-sectional area of at least 0.134 in.^{2}

6.8-1

The loads transmitted by the purlins are as follows:

Snow: $20(3)(25) = 1500$ lb = 1.500 kips

Roof: metal deck: 2 psf
roofing: 4 psf
Insulation: 3 psf
Total: 9 psf

$9(3)(25) = 675.0$ lb

Purlins: $8.5(25) = 212.5$ lb

Total dead load = $675 + 212.5 = 887.5$ lb = 0.8875 kips

(a) LRFD Solution

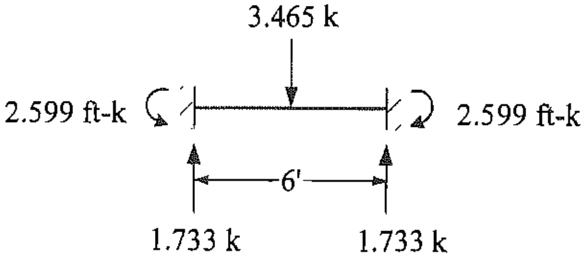
Load combination 3 will control:

$$P_u = 1.2D + 1.6S = 1.2(0.8875) + 1.6(1.500) = 3.465 \text{ kips}$$

The fixed-end moment for each top chord member is

$$M = M_{nt} = \frac{P_u L}{8} = \frac{3.465(6)}{8} = 2.599 \text{ ft-kips}$$

The reaction at each end of the fixed-end top chord member is $\frac{P_u}{2} = \frac{3.465}{2} = 1.733$ kips



Total interior panel point load = $3.465 + 2(1.733) = 6.931$ kips

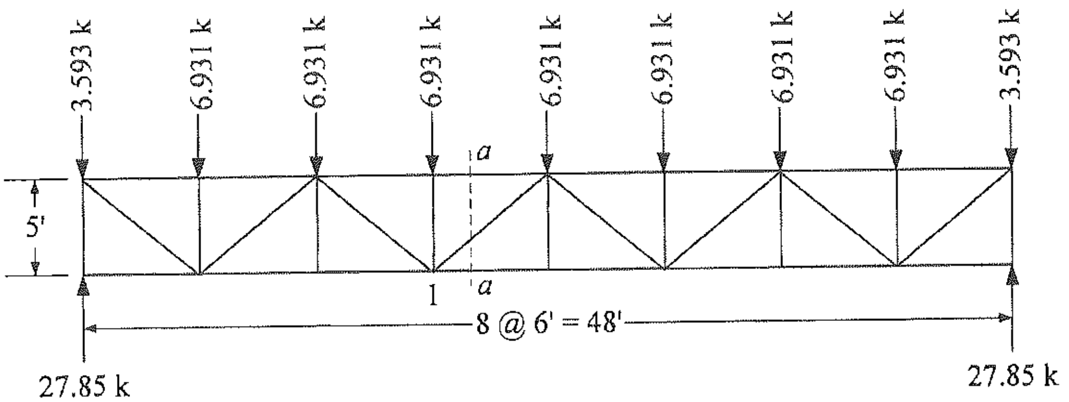
Exterior panel point load:

Snow: $1500/2 = 750.0$ lb
Roof: $675/2 = 337.5$ lb
Purlins: 212.5 lb

$$P_u = 1.2(0.3375 + 0.2125) + 1.6(0.750) = 1.860 \text{ kips}$$

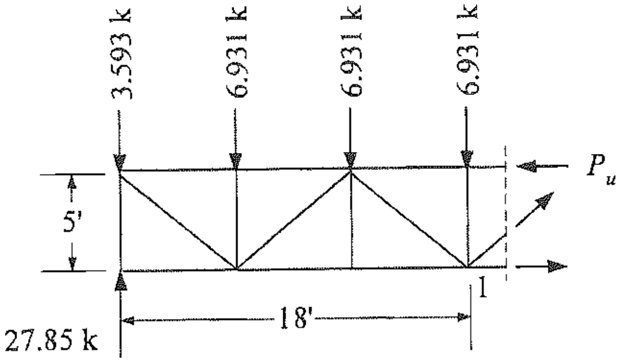
Total exterior panel point load = $1.733 + 1.860 = 3.593$ kips

Truss reaction at each end = $\frac{2(3.593) + 7(6.931)}{2} = 27.85$ kips



Consider a free body of the portion of the truss left of section a-a:

$$\sum M_1 = (27.85 - 3.593)(18) - 6.931(12 + 6) - P_u(5) = 0, \qquad P_u = 62.37 \text{ kips}$$



Design for an axial compressive load of 62.37 kips and a bending moment of 2.599 ft-kips.

Try a WT5 × 15. From the column load tables with $K_xL = 6$ ft and $K_yL = 3$ft, $\phi_cP_n = 166$ kips

For the axis of bending,

$$P_{e1} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29,000)(9.28)}{(6 \times 12)^2} = 512.4 \text{ kips}$$

$$C_m = 1.0 \text{ (transversely-loaded member)}$$

$$B_1 = \frac{C_m}{1 - \dfrac{P_u}{P_{e1}}} = \frac{1.0}{1 - \dfrac{62.37}{512.4}} = 1.139$$

$$M_u = B_1M_{nt} + B_2M_{lt} = 1.139(2.599) + 0 = 2.96 \text{ ft-kips}$$

The shape is compact for flexure. Check the limit state of yielding. Because the maximum moment is a fixed-end moment, the stem will be in compression, assuming that the flange is at the top. For stems in compression,

$$M_n = M_p = F_yZ_x \le M_y \qquad \text{(AISC Equation F9-3)}$$

Since $S_x < Z_x$, the yield moment M_y will control.

$$M_n = M_y = F_y S_x = 50(2.24) = 112.0 \text{ in.-kips}$$

Check lateral-torsional buckling. From AISC Equation F9-5,

$$B = \pm 2.3\left(\frac{d}{L_b}\right)\sqrt{\frac{I_y}{J}} = -2.3\left(\frac{5.24}{6 \times 12}\right)\sqrt{\frac{8.35}{0.310}} = -0.8687$$

(The minus sign is used when the stem is in compression anywhere along the unbraced length.) From AISC Equation F9-4,

$$M_n = M_{cr} = \frac{\pi\sqrt{EI_yGJ}}{L_b}\left[B + \sqrt{1+B^2}\right]$$

$$= \frac{\pi\sqrt{29,000(8.35)(11,200)0.310)}}{6 \times 12}\left[-0.8687 + \sqrt{1+(-0.8687)^2}\right]$$

$$= 576.8 \text{ in.-kips}$$

The limit state of yielding controls.

$$\phi_b M_n = 0.90(112.0) = 100.8 \text{ in.-kips} = 8.4 \text{ ft-kips}$$

Determine which interaction equation to use:

$$\frac{P_u}{\phi_c P_n} = \frac{62.37}{166} = 0.3757 > 0.2 \quad \therefore \text{ use AISC Equation H1-1a.}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.3757 + \frac{8}{9}\left(\frac{2.96}{8.4} + 0\right)$$

$$= 0.689 < 1.0 \quad \text{(OK)}$$

Use a WT5 × 15.

(b) ASD Solution

Load combination 3 will control:

$$P_a = D + S = 0.8875 + 1.500 = 2.388 \text{ kips}$$

The fixed-end moment for each top chord member is

$$M = M_{nt} = \frac{P_a L}{8} = \frac{2.388(6)}{8} = 1.791 \text{ ft-kips}$$

The reaction at each end of the fixed-end top chord member is $\frac{P_a}{2} = \frac{2.388}{2} = 1.194$ kips

Total interior panel point load = $2.388 + 2(1.194) = 4.776$ kips

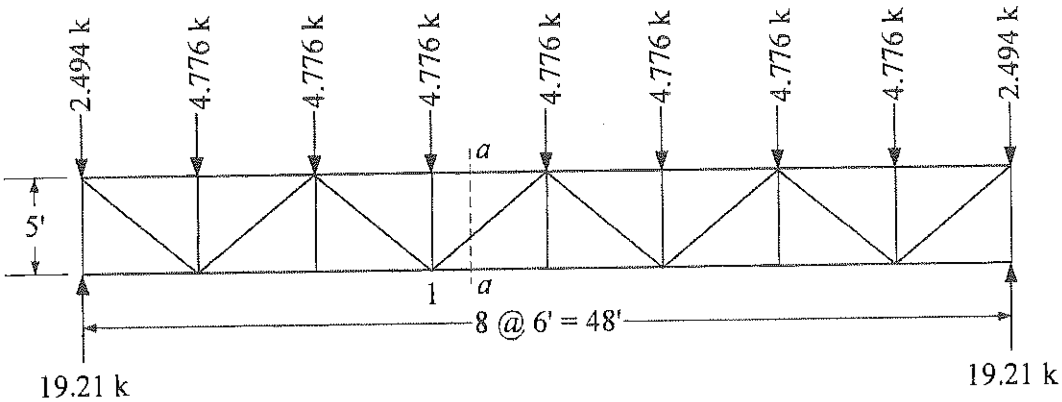
Exterior panel point load:

Snow: 1500/2 = 750.0 lb
Roof: 675/2 = 337.5 lb
Purlins: 212.5 lb

$$P_a = 0.750 + 0.3375 + 0.2125 = 1.3 \text{ kips}$$

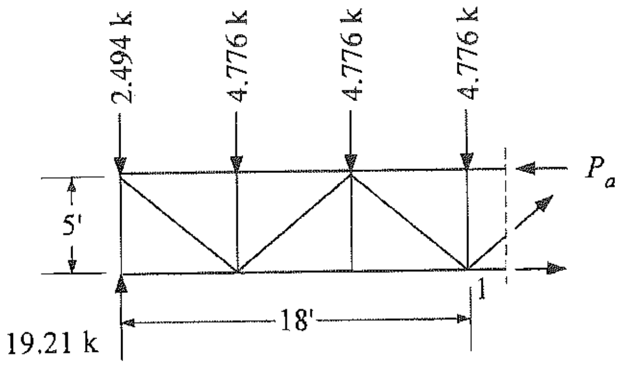
Total exterior panel point load = $1.194 + 1.3 = 2.494$ kips

Truss reaction at each end $= \dfrac{2(2.494)+7(4.776)}{2} = 19.21$ kips



Consider a free body of the portion of the truss left of section a-a:

$$\sum M_1 = (19.21-2.494)(18) - 4.776(12+6) - P_a(5) = 0, \qquad P_a = 42.98 \text{ kips}$$



Design for an axial compressive load of 42.98 kips and a bending moment of 1.791 ft-kips.

Try a WT5 × 15. From the column load tables with $K_xL = 6$ ft and $K_yL = 3$ft, $\dfrac{P_n}{\Omega_c} = 110$ kips

For the axis of bending,

$$P_{e1} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(9.28)}{(6\times 12)^2} = 512.4 \text{ kips}$$

$C_m = 1.0$ (transversely-loaded member)

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{1.0}{1-1.60(42.98)/512.4} = 1.155$$

$$M_a = B_1M_{nt} + B_2M_{\ell t} = 1.155(1.791) + 0 = 2.069 \text{ ft-kips}$$

From part (a), $M_n = 112.0$ in.-kips

$$\frac{M_n}{\Omega_b} = \frac{112.0}{1.67} = 67.07 \text{ in.-kips} = 5.589 \text{ ft-kips}$$

Determine which interaction equation to use:

$$\frac{P_a}{P_n/\Omega_c} = \frac{42.98}{110} = 0.3907 > 0.2 \therefore \text{ use Eq. 6.5 (AISC Eq. H1-1a).}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.3907 + \frac{8}{9}\left(\frac{2.069}{5.589} + 0\right)$$

$$= 0.720 < 1.0 \quad \text{(OK)}$$

Use a WT5 × 15.

6.8-2

For the deck and built-up roof, the tributary width of top chord is $46.6/8 = 5.825$ ft.

$(2 + 5)(5.825)(25) = 1019$ lb

Purlins: $12(25) = 300.0$ lb

For the snow, the tributary width of top chord is $45/8 = 5.625$ ft.

$18(5.625)(25) = 2531$ lb

Assume a truss weight of 10% of the other gravity loads:

$0.10(1019 + 300 + 2531) = 385.0$ lb

Total dead load $= 1019 + 300 + 385 = 1704$ lb

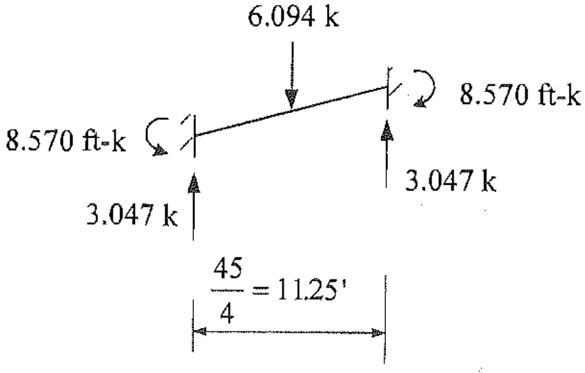
(a) LRFD Solution

Load combination 3 will control:

$$P_u = 1.2D + 1.6S = 1.2(1.704) + 1.6(2.531) = 6.094 \text{ kips}$$

The fixed-end moment for each top chord member is (see figure)

$$M = M_{nt} = \frac{P_u L}{8} = \frac{6.094(11.25)}{8} = 8.570 \text{ ft-kips}$$



Total interior panel point load $= 6.094 + 2(3.047) = 12.19$ kips

Exterior panel point load:

Snow: $2531/2 = 1266$ lb

Deck and roof: $1019/2 = 509.5$ lb

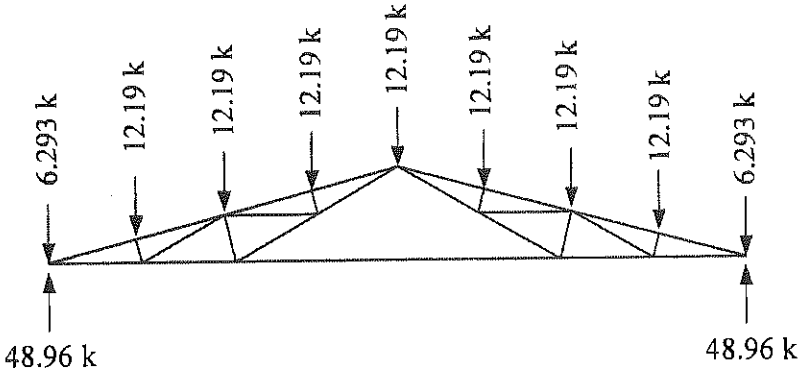
Purlins: 300 lb

Truss weight: $0.10(1266 + 509.5 + 300) = 207.6$ lb

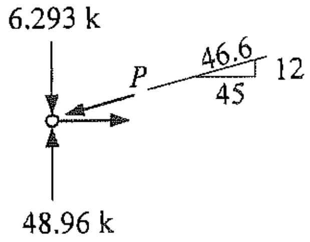
$P_u = 1.2(0.5095 + 0.300 + 0.2076) + 1.6(1.266) = 3.246$ kips

Total exterior panel point load $= 3.047 + 3.246 = 6.293$ kips

Truss reaction at each end $= \dfrac{7(12.19) + 2(6.293)}{2} = 48.96$ kips



The maximum top chord load is in the member adjacent to the support. Consider a free body of the pin at the left support:



$$\sum F_y = 48.96 - 6.293 - \frac{12}{46.6}P = 0, \qquad P = 165.7 \text{ kips compression}$$

Design for an axial compressive load of 165.7 kips and a bending moment of 8.570 ft-kips.

Try a WT7 × 34

From the column load tables with $K_xL = 46.6/4 = 11.65$ ft and $K_yL = 11.65/2 = 5.825$ ft,

$\phi_c P_n = 290.5$ kips

Since bending is about the x axis and the member is braced against sidesway,

$M_{nt} = 8.570$ ft-kips and $M_{\ell t} = 0$

For the axis of bending,

$$P_{e1} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29{,}000)(32.6)}{(11.65 \times 12)^2} = 477.4 \text{ kips}$$

$C_m = 1.0$ (transversely-loaded member)

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} = \frac{1.0}{1 - \frac{165.7}{477.4}} = 1.532$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.532(8.570) + 0 = 13.13 \text{ ft-kips}$$

There is no footnote in the column load tables, so the shape is not slender. Since bending is about the weak axis, there is no lateral-torsional buckling.

Check the limit state of yielding. Because the maximum moment is a fixed-end moment, the stem will be in compression, assuming that the flange is at the top. For stems in compression,

$$M_n = M_p = F_y Z_x \leq M_y \qquad \text{(AISC Equation F9-3)}$$

Since $S_x < Z_x$, the yield moment M_y will control.

$$M_n = M_y = F_y S_x = 50(5.69) = 284.5 \text{ in.-kips}$$

$$\phi_b M_n = 0.90(284.5) = 256.1 \text{ in.-kips} = 21.34 \text{ ft-kips}$$

Determine which interaction equation to use:

$$\frac{P_u}{\phi_c P_n} = \frac{165.7}{290.5} = 0.5704 > 0.2 \therefore \text{ use AISC Equation H1-1a.}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.5704 + \frac{8}{9}\left(\frac{13.13}{21.34} + 0\right) = 1.12 > 1.0 \qquad \text{(N.G.)}$$

Try a WT7 × 37

From the column load tables with $K_xL = 11.65$ ft and $K_yL = 5.825$ ft, $\phi_c P_n = 318.1$ kips

$$P_{e1} = \frac{\pi^2 E I_x}{(K_x L)^2} = \frac{\pi^2(29{,}000)(36.0)}{(11.65 \times 12)^2} = 527.2 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} = \frac{1.0}{1 - \frac{165.7}{527.2}} = 1.458$$

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.458(8.570) + 0 = 12.50 \text{ ft-kips}$$

There is no footnote in the column load tables, so the shape is not slender. Since bending is about the weak axis, there is no lateral-torsional buckling.

$$M_n = M_p = F_y Z_x \leq M_y$$

Since $S_x < Z_x$, the yield moment M_y will control.

$$M_n = M_y = F_y S_x = 50(6.25) = 312.5 \text{ in.-kips}$$

$$\phi_b M_n = 0.90(312.5) = 281.3 \text{ in.-kips} = 23.44 \text{ ft-kips}$$

$$\frac{P_u}{\phi_c P_n} = \frac{165.7}{318.1} = 0.5209 > 0.2 \therefore \text{ use AISC Equation H1-1a.}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.5209 + \frac{8}{9}\left(\frac{12.50}{23.44} + 0\right) = 0.995 < 1.0 \quad \text{(OK)}$$

Use a WT7 × 37

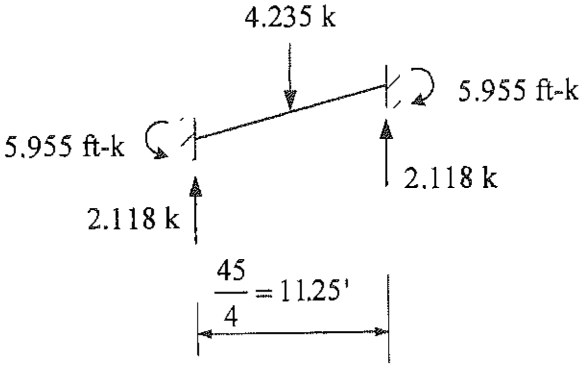
(b) ASD Solution

Load combination 3 will control:

$$P_a = D + S = 1.704 + 2.531 = 4.235 \text{ kips}$$

The fixed-end moment for each top chord member is (see figure)

$$M = M_{nt} = \frac{P_a L}{8} = \frac{4.235(11.25)}{8} = 5.955 \text{ ft-kips}$$



Total interior panel point load = $4.235 + 2(2.118) = 8.471$ kips

Exterior panel point load:

Snow: $2531/2 = 1266$ lb

Deck and roof: $1019/2 = 509.5$ lb

Purlins: 300 lb

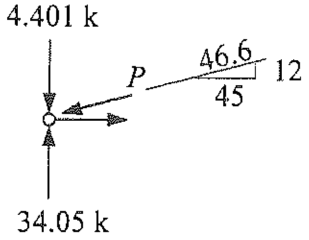
Truss weight: $0.10(1266 + 509.5 + 300) = 207.6$ lb

$$P_a = 1.266 + 0.5095 + 0.300 + 0.2076 = 2.283 \text{ kips}$$

Total exterior panel point load = $2.118 + 2.283 = 4.401$ kips

Truss reaction at each end = $\dfrac{7(8.471) + 2(4.401)}{2} = 34.05$ kips

The maximum top chord load is in the member adjacent to the support. Consider a free body of the pin at the left support:



$$\sum F_y = 34.05 - 4.401 - \frac{12}{46.6}P = 0, \qquad P = 115.1 \text{ kips compression}$$

Design for an axial compressive load of 115.1 kips and a bending moment of 5.955 ft-kips.

Try a WT7 × 37

From the column load tables with $K_xL = 11.65$ ft and $K_yL = 5.825$ ft, $\frac{P_n}{\Omega_c} = 211.4$ kips

$$P_{e1} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29,000)(36.0)}{(11.65 \times 12)^2} = 527.2 \text{ kips}$$

$C_m = 1.0$ (transversely-loaded member)

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{1.0}{1-1.60(115.1)/527.2} = 1.537$$

$$M_a = B_1 M_{nt} + B_2 M_{\ell t} = 1.537(5.955) + 0 = 9.153 \text{ ft-kips}$$

There is no footnote in the column load tables, so the shape is not slender. Since bending is about the weak axis, there is no lateral-torsional buckling.

$$M_n = M_p = F_y Z_x \le M_y$$

Since $S_x < Z_x$, the yield moment M_y will control.

$$M_n = M_y = F_y S_x = 50(6.25) = 312.5 \text{ in.-kips}$$

$$\frac{M_n}{\Omega_b} = \frac{312.5}{1.67} = 187.1 \text{ in.-kips} = 15.59 \text{ ft-kips}$$

Determine which interaction equation to use:

$$\frac{P_a}{P_n/\Omega_c} = \frac{115.1}{211.4} = 0.5445 > 0.2 \therefore \text{ use Eq. 6.5 (AISC Eq. H1-1a).}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.5445 + \frac{8}{9}\left(\frac{9.153}{15.59} + 0\right)$$

$$= 1.07 > 1.0 \qquad \text{(N.G.)}$$

Try a WT7 × 41

From the column load tables with $K_xL = 11.65$ ft and $K_yL = 5.825$ ft, $\frac{P_n}{\Omega_c} = 237.0$ kips

$$P_{e1} = \frac{\pi^2 EI_x}{(K_xL)^2} = \frac{\pi^2(29,000)(41.2)}{(11.65 \times 12)^2} = 603.4 \text{ kips}$$

$$B_1 = \frac{C_m}{1-(1.60P_a/P_{e1})} = \frac{1.0}{1-1.60(115.1)/603.4} = 1.439$$

$$M_a = B_1M_{nt} + B_2M_{\ell t} = 1.439(5.955) + 0 = 8.569 \text{ ft-kips}$$

There is no footnote in the column load tables, so the shape is not slender. Since bending is about the weak axis, there is no lateral-torsional buckling.

$$M_n = M_p = F_yZ_x \leq M_y$$

Since $S_x < Z_x$, the yield moment M_y will control.

$$M_n = M_y = F_yS_x = 50(7.14) = 357.0 \text{ in.-kips}$$

$$\frac{M_n}{\Omega_b} = \frac{357.0}{1.67} = 213.8 \text{ in.-kips} = 17.82 \text{ ft-kips}$$

$$\frac{P_a}{P_n/\Omega_c} = \frac{115.1}{237.0} = 0.4857 > 0.2 \therefore \text{ use Eq. 6.5 (AISC Eq. H1-1a).}$$

$$\frac{P_a}{P_n/\Omega_c} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b}\right) = 0.4857 + \frac{8}{9}\left(\frac{8.569}{17.82} + 0\right)$$

$$= 0.913 < 1.0 \quad \text{(OK)}$$

Use a WT7 × 41

CHAPTER 7 - SIMPLE CONNECTIONS

7.3-1

(a) Minimum spacing $= 2\frac{2}{3}d = 2.667(7/8) = 2.33\text{ in.} < 2.75\text{ in.}$ (OK)

Minimum edge distance from AISC Table J3.4 = 1.5 in.= actual L_e (OK)

(b) Check bearing on gusset plate (it has the smaller thickness; everything else is the same).

$$h = d + \frac{1}{16} = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}\text{ in.}$$

For the holes nearest the edge,

$$L_c = L_e - \frac{h}{2} = 1.5 - \frac{15/16}{2} = 1.031\text{ in.}$$

$$R_n = 1.2L_c tF_u = 1.2(1.031)(3/8)(58) = 26.91\text{ kips}$$

The upper limit is

$$2.4dtF_u = 2.4(7/8)(3/8)(58) = 45.68\text{ kips} > 26.91\text{ kips}$$

$$\therefore \text{ use } R_n = 26.91\text{ kips}$$

For the other bolts,

$$L_c = s - h = 2.75 - \frac{15}{16} = 1.813\text{ in.}$$

$$R_n = 1.2L_c tF_u = 1.2(1.813)(3/8)(58) = 47.32\text{ kips}$$

$$2.4dtF_u = 45.68\text{ kips} < 47.32\text{ kips} \therefore \text{ use } R_n = 45.68\text{ kips}$$

For the connection, the total bearing strength is

$$26.91 + 2(45.68) = 118.3\text{ kips} \qquad \underline{R_n = 118\text{ kips}}$$

7.3-2

(a) Minimum spacing $= 2\frac{2}{3}d = 2.667(3/4) = 2.0\text{ in.} < 2.5\text{ in.}$ (OK)

Minimum edge distance from AISC Table J3.4 = 1.25 in. < 1.5 in. (OK)

(b) Check bearing on gusset plate (it has the smaller thickness; everything else is the same).

$$h = d + \frac{1}{16} = \frac{3}{4} + \frac{1}{16} = \frac{13}{16}\text{ in.}$$

For the holes nearest the edge,

$$L_c = L_e - \frac{h}{2} = 1.5 - \frac{13/16}{2} = 1.094\text{ in.}$$

From Table 2-4 in Part 2 of the *Manual*, $F_u = 70$ ksi for A242 steel of this thickness.

$R_n = 1.2L_ctF_u = 1.2(1.094)(3/8)(70) = 34.46$ kips

The upper limit is

$2.4dtF_u = 2.4(3/4)(3/8)(70) = 47.25$ kips > 34.46 kips

$\therefore$ use $R_n = 34.46$ kips

For the other bolts,

$$L_c = s - h = 3 - \frac{13}{16} = 2.188 \text{ in.}$$

$R_n = 1.2L_ctF_u = 1.2(2.188)(3/8)(70) = 68.92$ kips

$2.4dtF_u = 47.25$ kips < 68.92 kips $\therefore$ use $R_n = 47.25$ kips

For the connection, the total bearing strength is

$2(34.46) + 4(47.25) = 257.9$ kips $\qquad \underline{R_n = 258 \text{ kips}}$

7.4-1

(a) Minimum spacing $= 2\frac{2}{3}d = 2.667(7/8) = 2.33$ in. < 3 in. (OK)

Minimum edge distance from AISC Table J3.4 = 1.5 in. < 2 in. (OK)

(b) Design strengths. Bolt shear: $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

The nominal shear capacity of one bolt is

$R_n = F_{nv}A_b = 24(0.6013) = 14.43$ kips

For 6 bolts, the nominal shear strength is $6(14.43) = 86.58$ kips

Check bearing on gusset plate. $h = d + \frac{1}{16} = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$ in.

For the holes nearest the edge, $L_c = L_e - \frac{h}{2} = 3 - \frac{15/16}{2} = 2.531$ in.

$R_n = 1.2L_ctF_u = 1.2(2.531)(3/8)(58) = 66.06$ kips

The upper limit is

$2.4dtF_u = 2.4(7/8)(3/8)(58) = 45.68$ kips < 66.06 kips

$\therefore$ use $R_n = 45.68$ kips

For the other bolts,

$$L_c = s - h = 3 - \frac{15}{16} = 2.063 \text{ in.}$$

$R_n = 1.2L_ctF_u = 1.2(2.063)(3/8)(58) = 53.84$ kips

$2.4dtF_u = 45.68$ kips < 53.84 kips $\therefore$ use $R_n = 45.68$ kips

For the connection, the total nominal bearing strength is $6(45.68) = 274.1$ kips

Although the bearing strength of the gusset plate will control (its thickness is less than that of the channel, and its ultimate tensile stress is less), the bearing strength of the C8 × 18.7 will be computed for illustration.

$t_w = 0.487$ in. and $F_u = 65$ ksi

For the holes nearest the edge, $L_c = L_e - \frac{h}{2} = 2 - \frac{15/16}{2} = 1.531$ in.

$R_n = 1.2L_c tF_u = 1.2(1.531)(0.487)(65) = 58.16$ kips

The upper limit is

$2.4dtF_u = 2.4(7/8)(0.487)(65) = 66.48 \text{ kips} > 58.16 \text{ kips}$

$\therefore$ use $R_n = 58.16$ kips

For the other bolts,

$L_c = s - h = 3 - \frac{15}{16} = 2.063$ in.

$R_n = 1.2L_c tF_u = 1.2(2.063)(0.487)(65) = 78.37$ kips

$2.4dtF_u = 66.48 \text{ kips} < 78.37 \text{ kips}$ $\therefore$ use $R_n = 66.48$ kips

For the connection, the total nominal bearing strength is

$2(58.16) + 4(66.48) = 382.2$ kips (gusset plate controls for bearing)

For shear, $\phi R_n = 0.75(86.58) = \underline{64.9 \text{ kips}}$

For bearing, $\phi R_n = 0.75(274.1) = \underline{206 \text{ kips}}$

(c) Allowable strengths: From part (b),

nominal shear strength = 86.58 kips, nominal bearing strength = 274.1 kips

For shear, $\frac{R_n}{\Omega} = \frac{86.58}{2.00} = \underline{43.3 \text{ kips}}$

For bearing, $\frac{R_n}{\Omega} = \frac{274.1}{2.00} = \underline{137 \text{ kips}}$

7.4-2

(a) Minimum spacing = $2\frac{2}{3}d = 2.667(7/8) = 2.33 \text{ in.} < 2.75$ in. (OK)

Minimum edge distance from AISC Table J3.4 = 1.5 in. = actual L_e (OK)

(b) Bolt shear: $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Shear capacity of one bolt is

$R_n = F_{nv}A_b = 48(0.6013) \times 2 = 57.72$ kips (double shear)

For 4 bolts (consider each side of the connection separately), $4(57.72) = 231$ kips

For bearing, the steel is the same for the tension member and the splice plates, the edge distances are the same for the tension member and the splice plates, and the combined thickness of the splice plates is the same as the thickness of the tension member. As a consequence, the bearing strength will be the same for both the tension member and the splice plates.

Check bearing on the tension member. $h = d + \frac{1}{16} = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$ in.

For the holes nearest the edge, $L_c = L_e - \frac{h}{2} = 1.5 - \frac{15/16}{2} = 1.031$ in.

$$R_n = 1.2L_ctF_u = 1.2(1.031)(1/2)(58) = 35.88 \text{ kips}$$

The upper limit is

$$2.4dtF_u = 2.4(7/8)(1/2)(58) = 60.9 \text{ kips} > 35.88 \text{ kips}$$

$$\therefore \text{ use } R_n = 35.88 \text{ kips}$$

For the other bolts,

$$L_c = s - h = 2.75 - \frac{15}{16} = 1.813 \text{ in.}$$

$$R_n = 1.2L_ctF_u = 1.2(1.813)(1/2)(58) = 63.09 \text{ kips}$$

$$2.4dtF_u = 60.9 \text{ kips} < 63.09 \text{ kips} \therefore \text{ use } R_n = 60.9 \text{ kips}$$

For the connection, the total nominal bearing strength is

$$2(35.88) + 2(60.9) = 194 \text{ kips}$$

For shear, $R_n = 231$ kips. For bearing, $R_n = 194$ kips

7.4-3

$$A_b = \pi d^2/4 = \pi(3/4)^2/4 = 0.4418 \text{ in.}^2$$

Nominal shear capacity of one bolt in double shear is

$$R_n = F_{nv}A_b = 48(0.4418) \times 2 = 42.41 \text{ kips}$$

The nominal bearing strength of one bolt (gusset plate thickness controls) is

$$2.4dtF_u = 2.4(3/4)(3/8)(58) = 39.15 \text{ kips} < 42.41 \text{ kips for shear}$$

$\therefore$ Bearing strength controls.

(a) LRFD solution: $\phi R_n = 0.75(39.15) = 29.36$ kips/bolt

$$P_u = 1.2D + 1.6L = 1.2(7) + 1.6(14) = 30.8 \text{ kips}$$

Number of bolts required is $\frac{30.8}{29.36} = 1.05$ Use 2 bolts

(b) ASD solution: $\frac{R_n}{\Omega} = \frac{39.15}{2.00} = 19.58$ kips/bolt

$P_a = D + L = 7 + 14 = 21$ kips

Number of bolts required is $\frac{21}{19.58} = 1.07$ Use 2 bolts

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7.4-4

$A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013$ in.^{2}

Nominal shear capacity of one bolt in double shear is

$R_n = F_{nv}A_b = 48(0.6013) \times 2 = 57.72$ kips

Assume that the upper limit of $2.4dtF_u$ controls for bearing strength. Since 3/8 in. < 1/4 + 1/4 = 1/2 in., the bearing strength of one bolt is

$2.4dtF_u = 2.4(7/8)(3/8)(58) = 45.68$ kips < 57.72 kips for shear

∴ Bearing strength controls.

(a) LRFD solution: $\phi R_n = 0.75(45.68) = 34.26$ kips/bolt

$P_u = 1.2D + 1.6L = 1.2(0.25 \times 60) + 1.6(0.75 \times 60) = 90.0$ kips

Number of bolts required is $\frac{90}{34.26} = 2.63$, use 4 bolts for symmetry on each side.

Use 8 bolts

(b) ASD solution: $\frac{R_n}{\Omega} = \frac{45.68}{2.00} = 19.58$ kips/bolt, $P_a = D + L = 60$ kips

Number of bolts required is $\frac{60}{19.58} = 3.06$, use 4 bolts for symmetry on each side.

Use 8 bolts

7.4-5

Bolt shear: $A_b = \pi d^2/4 = \pi(3/4)^2/4 = 0.4418$ in.^{2}

Nominal shear capacity of one bolt is

$R_n = F_{nv}A_b = 48(0.4418) = 21.21$ kips

For 6 bolts, $6(21.21) = 127.3$ kips

Check bearing on tension member (it has the smaller edge distance; everything else is the same):

$$h = \frac{3}{4} + \frac{1}{16} = \frac{13}{16} \text{ in.}$$

For the holes nearest the edge,

$$L_c = L_e - \frac{h}{2} = 1.5 - \frac{13/16}{2} = 1.094 \text{ in.}$$

$$R_n = 1.2L_ctF_u = 1.2(1.094)(5/16)(58) = 23.79 \text{ kips}$$

The upper limit is

$$2.4dtF_u = 2.4(3/4)(5/16)(58) = 32.63 \text{ kips} > 23.79 \text{ kips}$$

$$\therefore \text{ use } R_n = 23.79 \text{ kips}$$

For the other bolts,

$$L_c = s - h = 3.5 - \frac{13}{16} = 2.688 \text{ in.}$$

$$R_n = 1.2L_ctF_u = 1.2(2.688)(5/16)(58) = 58.46 \text{ kips}$$

$$2.4dtF_u = 32.63 \text{ kips} < 58.46 \text{ kips} \therefore \text{ use } R_n = 32.63 \text{ kips}$$

For the connection, the nominal bearing strength is

$$2(23.79) + 4(32.63) = 178.1 \text{ kips}$$

Shear controls. $R_n = 127.3$ kips.

(a) LRFD solution: $\phi R_n = 0.75(127.3) = 95.48$ kips

Let $1.2D + 1.6(2D) = 95.48$

$D = 21.70$ kips, $L = 43.40$ kips, Total = 21.70 + 43.40 = 65.1 kips

(b) ASD solution: $\frac{R_n}{\Omega} = \frac{127.3}{2.00} = 63.65 \text{ kips} = P_a$ Total = 63.7 kips

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7.4-6

Bolt shear (assume that the threads are in shear): $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Nominal shear capacity of one bolt is

$$R_n = F_{nv}A_b \times 2 \text{ (for double shear)} = 48(0.6013) \times 2 = 57.72 \text{ kips/bolt}$$

For 5 bolts, $5(57.72) = 288.6$ kips

Check bearing on the gusset plate (it is thinner than the combined thickness of the angles):

$$h = \frac{7}{8} + \frac{1}{16} = \frac{15}{16} \text{ in.}$$

For the hole nearest the edge,

$$L_c = L_e - \frac{h}{2} = 2 - \frac{15/16}{2} = 1.531 \text{ in.}$$

$$R_n = 1.2L_ctF_u = 1.2(1.531)(3/8)(58) = 39.96 \text{ kips}$$

The upper limit is

$2.4dtF_u = 2.4(7/8)(3/8)(58) = 45.68 \text{ kips} > 39.96 \text{ kips}$

$\therefore$ use $R_n = 39.96$ kips

For the other bolts,

$$L_c = s - h = 3 - \frac{15}{16} = 2.063 \text{ in.}$$

$$R_n = 1.2L_ctF_u = 1.2(2.063)(3/8)(58) = 53.84 \text{ kips}$$

$2.4dtF_u = 45.68 \text{ kips} < 53.84 \text{ kips}$ $\therefore$ use $R_n = 45.68$ kips

For the connection, the nominal bearing strength is

$$39.96 + 4(45.68) = 222.7 \text{ kips}$$

Bearing controls. $R_n = 222.7$ kips.

(a) LRFD Solution

$$\phi R_n = 0.75(222.7) = 167 \text{ kips}$$

$P_u = 1.2D + 1.6L = 1.2(40) + 1.6(100) = 208 \text{ kips} > 167 \text{ kips}$ (N.G.)

connection does not have enough capacity.

(b) ASD Solution

$$\frac{R_n}{\Omega} = \frac{222.7}{2.00} = 111 \text{ kips}$$

$P_a = D + L = 40 + 100 = 140 \text{ kips} > 111 \text{ kips}$ (N.G.)

connection does not have enough capacity.

7.6-1

Bolt shear (assume that the threads are in shear): $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Nominal shear capacity of one bolt is

$$R_n = F_{nv}A_b \times 2 \text{ (for double shear)} = 48(0.6013) \times 2 = 57.72 \text{ kips/bolt}$$

For 8 bolts, $8(57.72) = 461.8$ kips

Check bearing on gusset plate (it is thinner than the combined thickness of the angles, the edge distance is the same as for the angles, and the ultimate tensile stress F_u is smaller):

$$h = \frac{7}{8} + \frac{1}{16} = \frac{15}{16} \text{ in.}$$

For the hole nearest the edge,

$$L_c = L_e - \frac{h}{2} = 2 - \frac{15/16}{2} = 1.531 \text{ in.}$$

$$R_n = 1.2L_ctF_u = 1.2(1.531)(5/8)(58) = 66.60 \text{ kips}$$

The upper limit is

$$2.4dtF_u = 2.4(7/8)(5/8)(58) = 76.13 \text{ kips} > 66.60 \text{ kips}$$

$$\therefore \text{ use } R_n = 66.60 \text{ kips}$$

For the other bolts,

$$L_c = s - h = 3 - \frac{15}{16} = 2.063 \text{ in.}$$

$$R_n = 1.2L_ctF_u = 1.2(2.063)(5/8)(58) = 89.74 \text{ kips}$$

$$2.4dtF_u = 76.13 \text{ kips} < 89.74 \text{ kips} \therefore \text{ use } R_n = 76.13 \text{ kips}$$

For the connection, the bearing strength is

$$2(66.60) + 6(76.13) = 590.0 \text{ kips}$$

Tension on the gross section: $A_g = 2(7.13) = 14.26 \text{ in.}^2$

$$P_n = F_yA_g = 50(14.26) = 713.0 \text{ kips}$$

Net section:

$$A_n = A_g - \Sigma td_h = 2\left[7.13 - 2\left(\frac{5}{8}\right)\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 11.76 \text{ in.}^2$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.72}{9} = 0.8089$$

$$A_e = A_gU = 11.76(0.8089) = 9.513 \text{ in.}^2$$

$$P_n = F_uA_e = 65(9.513) = 618.3 \text{ kips}$$

(a) LRFD solution. Compute the design strength:

For bolt shear, $\phi R_n = 0.75(461.8) = 346.4$ kips

For bearing, $\phi R_n = 0.75(590.0) = 442.5$ kips

For tension on the gross section, $\phi_t P_n = 0.90(713.0) = 641.7$ kips

For tension on the net section, $\phi_t P_n = 0.75(618.3) = 463.7$ kips

Bolt shear controls. $P_u = 346.4$ kips. Since $D > 8L$, load combination 1 controls.

$$1.4D = 1.4(8.5L) = 346.4 \qquad \Rightarrow \qquad L = 29.11 \text{ kips}$$

$$P = D + L = 8.5(29.11) + 29.11 = 277 \text{ kips} \qquad \underline{P = 277 \text{ kips}}$$

(b) ASD solution. Compute the allowable strength:

For bolt shear, $\dfrac{R_n}{\Omega} = \dfrac{461.8}{2.00} = 230.9$ kips

For bearing, $\dfrac{R_n}{\Omega} = \dfrac{590.0}{2.00} = 295.0$ kips

For tension on the gross section, $\dfrac{R_n}{\Omega_t} = \dfrac{713.0}{1.67} = 426.9$ kips

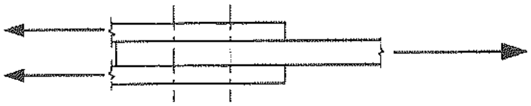
For tension on the net section, $\frac{R_n}{\Omega_t} = \frac{618.3}{2.00} = 309.2$ kips

Bolt shear controls. $P_a = 231$ kips $P = 231$ kips

7.6-2

Determine the nominal shear and bearing strengths per bolt. The shear strength is

$$R_n = F_{nv}A_b = 60(0.6013) \times 2 = 72.16 \text{ kips/bolt}$$



Bearing: $h = d + \frac{1}{16} = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$ in.

Edge bolts: $L_c = L_e - \frac{h}{2} = 1.5 - \frac{15/16}{2} = 1.031$ in.

$$R_n = 1.2L_ctF_u = 1.2(1.031)(0.5)(58) = 35.88 \text{ kips/bolt}$$

Upper limit $= 2.4dtF_u = 2.4(7/8)(0.5)(58) = 60.9$ kips/bolt > 35.88 kips/bolt

$\therefore$ use $R_n = 35.88$ kips/bolt

Other bolts: $L_c = s - h = 3 - \frac{15}{16} = 2.063$ in.

$$R_n = 1.2L_ctF_u = 1.2(2.063)(0.5)(58) = 71.79 \text{ kips/bolt}$$

$2.4dtF_u = 60.9$ kips/bolt < 71.79 kips/bolt $\therefore$ use $\phi R_n = 60.9$ kips/bolt

(a) LRFD Solution

$$P_u = 1.2P_D + 1.6P_L = 1.2(35) + 1.6(86) = 179.6 \text{ kips}$$

Select the number of bolts based on shear, then check bearing. The shear strength is

$$\phi R_n = 0.75(72.16) = 54.12$$

No. bolts required $= \frac{P_u}{\phi R_n} = \frac{179.6}{54.12} = 3.32$, Try 4 on each side.

The total bearing strength is

$\phi R_n = 0.75[2(35.88) + 2(60.9)] = 145.2$ kips < 179.6 kips (N.G.)

Try 2 rows of 3 bolts on each side (for symmetry):

Total $\phi R_n = 0.75[3(35.88) + 3(60.9)] = 217.8$ kips >179.6 kips (OK)

Use 6 bolts each side, 12 total

(b) ASD Solution

$P_a = P_D + P_L = 35 + 86 = 121.0$ kips

Select the number of bolts based on shear, then check bearing. The shear strength is

$$\frac{R_n}{\Omega} = \frac{72.16}{2.00} = 36.08 \text{ kips/bolt}$$

No. bolts required $= \dfrac{121}{36.08} = 3.35$, Try 4 on each side.

The total bearing strength is

$$\frac{R_n}{\Omega} = \frac{1}{2.00}[2(35.88) + 2(60.9)] = 96.8 \text{ kips} < 121 \text{ kips} \qquad \text{(N.G.)}$$

Try 2 rows of 3 bolts on each side (for symmetry):

Total $R_n/\Omega = [3(35.88) + 3(60.9)]/2.00 = 145$ kips >121 kips (OK)

Use 6 bolts each side, 12 total

7.6-3

LRFD Solution

$P_u = 1.2D + 1.6L = 1.2(45) + 1.6(90) = 198.0$ kips

The bearing strength of the gusset plate is smaller than the bearing strength of the member. For one bolt,

$\phi R_n = \phi(2.4dtF_u) = 0.75(2.4)(7/8)(3/8)(65) = 38.39$ kips

(a) For shear strength, assume that threads are in the plane of shear.

$A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

$\phi R_n = 0.75F_{nv}A_b = 0.75(24)(0.6013) = 10.82$ kips/bolt < 38.39 kips/bolt

Number required $= 198/10.82 = 18.3$

Use an even number for symmetry. Use 20 bolts

(b) $\phi R_n = 0.75F_{nv}A_b = 0.75(48)(0.6013) = 21.65$ kips/bolt < 38.39 kips/bolt

Number required $= 198/21.65 = 9.2$ Use 10 bolts

(c) $\phi R_n = 0.75F_{nv}A_b = 0.75(60)(0.6013) = 27.06$ kips/bolt < 38.39 kips/bolt

Number required $= 198/27.06 = 7.3$ Use 8 bolts

(d) A307: $20(1) = 20.0$

A325: $10(1.7) = 17.0$

A490: $8(2.6) = 20.8$

A325 bolts have the smallest relative cost. A325 bolts are the most economical

ASD Solution

$$P_a = D + L = 45 + 90 = 135 \text{ kips}$$

The bearing strength of the gusset plate is smaller than the bearing strength of the member. For one bolt,

$$\frac{R_n}{\Omega} = \frac{2.4dtF_u}{\Omega} = \frac{2.4(7/8)(3/8)(65)}{2.00} = 25.59 \text{ kips}$$

(a) For shear strength, assume that threads are in the plane of shear.

$$A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$$

$$\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{\Omega} = \frac{24(0.6013)}{2.00} = 7.216 \text{ kips/bolt} < 25.59 \text{ kips/bolt}$$

Number required = 135/7.216 = 18.71

Use an even number for symmetry. Use 20 bolts

(b) $\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{\Omega} = \frac{48(0.6013)}{2.00} = 14.43 \text{ kips/bolt} < 25.59 \text{ kips/bolt}$

Number required = 135/14.43 = 9.356 Use 10 bolts

(c) $\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{\Omega} = \frac{60(0.6013)}{2.00} = 18.04 \text{ kips/bolt} < 25.59 \text{ kips/bolt}$

Number required = 135/18.04 = 7.483 Use 8 bolts

(d) A307: 20(1) = 20.0

A325: 10(1.7) = 17.0

A490: 8(2.6) = 20.8

A325 bolts have the smallest relative cost. A325 bolts are the most economical

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7.6-4

(a)

Bolt Diameter (in.)	Single-shear design strength (kips)	Slip-critical design strength, one slip plane (kips)	Single-shear allowable strength (kips)	Slip-critical allowable strength, one slip plane (kips)
1/2	7.069	4.746	4.712	3.164
5/8	11.04	7.515	7.363	5.010
3/4	15.90	11.07	10.60	7.383
7/8	21.65	15.42	14.43	10.28
1	28.27	20.17	18.85	13.45
1 1/8	35.78	22.15	23.86	14.77
1 1/4	44.18	28.08	29.45	18.72
1 3/8	53.46	33.62	35.64	22.41
1 1/2	63.62	40.74	42.41	27.16

(b) Shear never controls in a slip-critical connection; the slip-critical strength is always smaller.

7.6-5

Determine the nominal strengths for all limit states. For shear,

$$A_b = \pi d^2/4 = \pi(1.125)^2/4 = 0.994 \text{ in.}^2$$

$$R_n = F_{nv}A_b = 48(0.994) = 47.71 \text{ kips/bolt}$$

For 4 bolts, $R_n = 4(47.71) = 190.8$ kips

Slip-critical strength: From AISC Table J3-1, the minimum bolt tension is $T_b = 56$ kips. From AISC Equation J3-4,

$$R_n = \mu D_u h_{sc} T_b N_s = 0.35(1.13)(1.0)(56)(1.0) = 22.15 \text{ kips/bolt}$$

For 4 bolts, $R_n = 4(22.15) = 88.6$ kips

Bearing: $h = d + \frac{1}{16} = 1\frac{1}{8} + \frac{1}{16} = 1.188$ in.

Edge bolts: $L_c = L_e - \frac{h}{2} = 2 - \frac{1.188}{2} = 1.406$ in.

$$R_n = 1.2L_c tF_u) = 1.2(1.406)(1/2)(58) = 48.93 \text{ kips/bolt}$$

Upper limit $= 2.4dtF_u) = 2.4(1.125)(1/2)(58)$

$= 78.3$ kips/bolt > 48.93 kips/bolt $\therefore$ use $R_n = 48.93$ kips/bolt

Other bolts: $L_c = s - h = 3 - 1.188 = 1.812$ in.

$$R_n = 1.2L_c tF_u) = 1.2(1.812)(1/2)(58) = 63.06 \text{ kips/bolt}$$

$2.4dtF_u) = 78.3$ kips/bolt > 63.06 kips/bolt $\therefore$ use $R_n = 63.06$ kips/bolt

Total $R_n = 2(48.93) + 2(63.06) = 224.0$ kips

Tension on the gross section: $A_g = 0.5(6.5) = 3.25$ in.^{2}

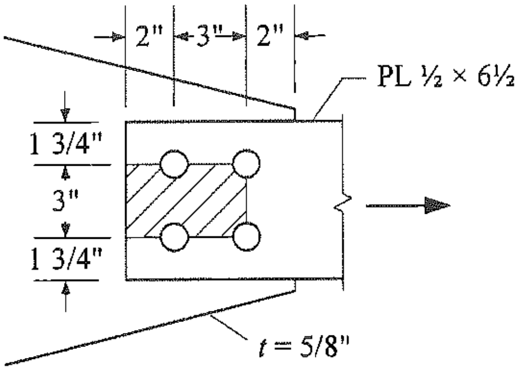
$$P_n = F_y A_g = 36(3.25) = 117.0 \text{ kips}$$

Tension on the net section:

$$A_e = A_n = A_g - \Sigma t d_h = 3.25 - 2\left(\frac{1}{2}\right)\left(1\frac{1}{8} + \frac{1}{8}\right) = 2.0 \text{ in.}^2$$

$$P_n = F_u A_e = 58(2.0) = 116.0 \text{ kips}$$

Check block shear (tension member controls).



$$A_{gv} = 2 \times \frac{1}{2}(3+2) = 5.0 \text{ in.}^2$$

$$A_{nv} = 2 \times \frac{1}{2}\left[3 + 2 - 1.5\left(1\frac{1}{8} + \frac{1}{8}\right)\right] = 3.125 \text{ in.}^2$$

$$A_{nt} = \frac{1}{2}\left(3 - 1\frac{1}{4}\right) = 0.875 \text{ in.}^2$$

For this type of block shear, $U_{bs} = 1.0$. From AISC Equation J4-5,

$$R_n = 0.6F_u A_{nv} + U_{bs} F_u A_{nt}$$

$$= 0.6(58)(3.125) + 1.0(58)(0.875) = 159.5 \text{ kips}$$

with an upper limit of

$$0.6F_y A_{gv} + U_{bs} F_u A_{nt} = 0.6(36)(5.0) + 1.0(58)(0.875) = 158.8 \text{ kips}$$

The nominal block shear strength is therefore 158.8 kips.

(a) LRFD solution. Compute design strengths:

For bolt shear, $\phi R_n = 0.75(190.8) = 143.1$ kips

For bolt slip, $\phi R_n = 1.00(88.6) = 88.6$ kips (Slip is being treated as a serviceability limit state, so $\phi = 1.0$.)

For bearing, $\phi R_n = 0.75(224.0) = 168.0$ kips

For tension on the gross section, $\phi R_n = 0.90(117.0) = 105.3$ kips

For tension on the net section, $\phi R_n = 0.75(116.0) = 87.0$ kips

For block shear, $\phi R_n = 0.75(158.8) = 119.1$ kips

Tension on the net section controls. $\phi R_n = 87.0 = P_u$

$1.2D + 1.6L = 1.2D + 1.6(3D) = 87.0$, Solution is: $\{D = 14.5\}$

$P = D + L = 14.5 + 3(14.5) = 58.0$ kips $\underline{P = 58.0 \text{ kips}}$

(b) ASD solution. Compute allowable strengths:

For bolt shear, $\frac{R_n}{\Omega} = \frac{190.8}{2.00} = 95.4$ kips

For bolt slip, $\frac{R_n}{\Omega} = \frac{88.6}{1.50} = 59.07$ kips (Slip is being treated as a serviceability limit state, so $\Omega = 1.50$.)

For bearing, $\frac{R_n}{\Omega} = \frac{224.0}{2.00} = 112.0$ kips

For tension on the gross section, $\frac{R_n}{\Omega_t} = \frac{117.0}{1.67} = 70.06$ kips

For tension on the net section, $\frac{R_n}{\Omega_t} = \frac{116.0}{2.00} = 58.0$ kips

For block shear, $\frac{R_n}{\Omega} = \frac{158.8}{2.00} = 79.4$ kips

Tension on the net section controls $P_a = 58.0$ kips $\underline{P = 58.0 \text{ kips}}$

7.6-6

Before proceeding to the LRFD and ASD solutions, compute the nominal bolt shear, bearing, and block shear strengths

Check slenderness: $\frac{L}{r_{\min}} = \frac{L}{r_z} = \frac{9(12)}{0.756} = 143 < 300$ (OK)

For shear,

$A_b = \pi d^2/4 = \pi(1.125)^2/4 = 0.994 \text{ in.}^2$

$R_n = F_{nv}A_b = 48(0.994) = 47.71$ kips/bolt

For 6 bolts, $R_n = 6(47.71) = 286.3$ kips

Slip-critical strength: From AISC Table J3-1, the minimum bolt tension is $T_b = 56$ kips. From AISC Equation J3-4,

$R_n = \mu D_u h_{sc} T_b N_s = 0.35(1.13)(1.0)(56)(1.0) = 22.15$ kips/bolt

For 6 bolts, $R_n = 6(22.15) = 132.9$ kips

Bearing: $h = d + \frac{1}{16} = 1\frac{1}{8} + \frac{1}{16} = 1.188$ in.

Edge bolt: $L_c = L_e - \frac{h}{2} = 2 - \frac{1.188}{2} = 1.406$ in.

$R_n = 1.2L_ctF_u) = 1.2(1.406)(3/8)(58) = 36.70$ kips/bolt

Upper limit $= 2.4dtF_u) = 2.4(1.125)(3/8)(58) = 58.73$

$= 58.73$ kips/bolt > 36.70 kips/bolt ∴ use $R_n = 36.70$ kips/bolt

Other bolts: $L_c = s - h = 3.5 - 1.188 = 2.312$ in.

$R_n = 1.2L_ctF_u) = 1.2(2.312)(3/8)(58) = 60.34$ kips/bolt

$2.4dtF_u) = 58.73$ kips/bolt < 60.34 kips/bolt ∴ use $R_n = 58.73$ kips/bolt

Total $R_n = 36.70 + 5(58.73) = 330.4$ kips

Tension on the gross section: $P_n = F_yA_g = 36(4.50) = 162.0$ kips

Tension on the net section: Use a hole diameter of $1\frac{1}{8} + \frac{1}{8} = 1.25$

$A_n = A_g - \Sigma td_h = 4.50 - \left(\frac{1}{2}\right)(1.25) = 3.875 \text{ in.}^2$

$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.829}{17.5} = 0.9526$

$A_e = A_nU = 3.875(0.9526) = 3.691 \text{ in.}^2$

$P_n = F_uA_e = 58(3.691) = 214.1$ kips

Check block shear on the angle.

$A_{gv} = \frac{1}{2}[5(3.5) + 2] = 9.75 \text{ in.}^2$

$A_{nv} = \frac{1}{2}[5(3.5) + 2 - 0.5(1.25)] = 9.438 \text{ in.}^2$

$A_{nt} = \frac{1}{2}[2.5 - 0.5(1.25)] = 0.9375 \text{ in.}^2$

For this type of block shear, $U_{bs} = 1.0$. From AISC Equation J4-5,

$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$

$= 0.6(58)(9.438) + 1.0(58)(0.9375) = 382.8$ kips

with an upper limit of

$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(9.75) + 1.0(58)(0.9375) = 265.0$ kips

The nominal block shear strength is therefore 265.0 kips.

(a) LRFD solution. Compute design strengths:

For bolt shear, $\phi R_n = 0.75(286.3) = 215$ kips

For bolt slip, $\phi R_n = 1.00(132.9) = 133$ kips (Slip is being treated as a serviceability limit state, so $\phi = 1.0.$)

For bearing, $\phi R_n = 0.75(330.4) = 248$ kips

For tension on the gross section, $\phi R_n = 0.90(162.0) = 146$ kips

For tension on the net section, $\phi R_n = 0.75(214.1) = 161$ kips

For block shear, $\phi R_n = 0.75(265.0) = 199$ kips

Slip-critical strength controls. $\phi R_n = 133$ kips

For load combination 2, $P_u = 1.2D + 1.6L = 1.2(20) + 1.6(60) = 120.0$ kips

For load combination 4, $P_u = 1.2D + 1.6W + 0.5L = 1.2(20) + 1.6(20) + 0.5(60)$

$= 86.0$ kips

$\therefore$ combination 2 controls; $P_u = 120$ kips< 133 kips (OK)

member and connection are satisfactory.

(b) ASD Solution. Compute allowable strengths:

For bolt shear, $R_n/\Omega = 286.3/2.00 = 143$ kips

For bolt slip, $R_n/\Omega = 132.9/1.50 = 88.6$ kips (Slip is being treated as a serviceability limit state, so $\Omega = 1.50.$)

For bearing, $R_n/\Omega = 330.4/2.00 = 165$ kips

For tension on the gross section, $R_n/\Omega = 162.0/1.67 = 97.0$ kips

For tension on the net section, $R_n/\Omega = 214.1/2.00 = 107$ kips

For block shear, $R_n/\Omega = 265.0/2.00 = 133$ kips

Slip-critical strength controls. $R_n/\Omega = 88.6$ kips

For load combination 2, $P_a = D + L = 20 + 60 = 80$ kips

For load combination 6, $P_a = D + 0.75W + 0.75L = 20 + 0.75(20) + 0.75(60) = 80$ kips

$\therefore P_u = 80$ kips < 88.6 kips (OK)

member and connection are satisfactory

7.7-1

(a) LRFD Solution

$P_u = 1.2D + 1.6L = 1.2(40) + 1.6(80) = 176.0$ kips

Bolt shear (assume that the threads are in shear): $A_b = \pi d^2/4 = \pi(1.375)^2/4 = 1.485$ in.^{2}

Shear capacity of one bolt is

$\phi R_n = \phi F_{nv} A_b = 0.75(48)(1.485) = 53.46$ kips/bolt

Slip-critical strength. Assuming Class A surfaces,

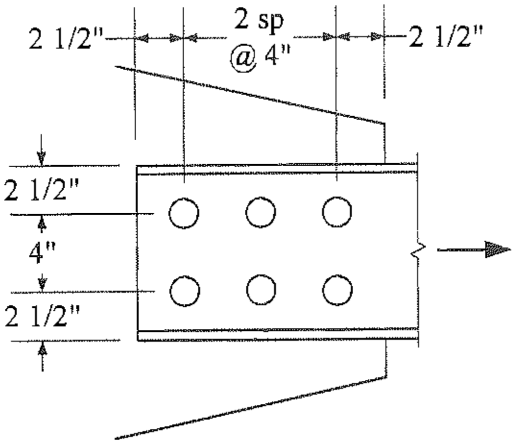
$$\phi R_n = \phi \mu D_u h_{sc} T_b N_s = 1.00[0.35(1.13)(1.0)(85)(1.0)] = 33.62 \text{ kips/bolt}$$

Number of bolts required $= \dfrac{176}{33.62} = 5.24$ try 6 bolts.

Minimum spacing $= 2\frac{2}{3}d = 2.667\left(1\frac{3}{8}\right) = 3.667$ in., try 4 in.

Minimum edge distance from AISC Table J3.4 $= 1\frac{3}{4} \times 1\frac{3}{8} = 2.41$ in., try $2\frac{1}{2}$ in.

Try the following layout:



Check bearing. To determine which component to check, compare the product of the thickness and the ultimate stress (the edge distances and spacings are the same for both components). For the gusset plate,

$$tF_u = 0.5(58) = 29.0 \text{ kips/in.}$$

For the tension member, $F_u = 70$ ksi for A242 steel, and

$$t_w F_u = 0.448(70) = 31.36 \text{ kips/in.} > 29.0 \text{ kip/in}$$

∴ check bearing on the gusset plate. $h = 1\frac{3}{8} + \frac{1}{16} = 1.438$ in.

For the holes nearest the edge, $L_c = L_e - \dfrac{h}{2} = 2.5 - \dfrac{1.438}{2} = 1.781$ in.

$$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(1.781)(1/2)(58) = 46.48 \text{ kips}$$

The upper limit is

$$\phi(2.4dtF_u) = 0.75(2.4)\left(1\tfrac{3}{8}\right)(1/2)(58) = 71.78 \text{ kips} > 46.48 \text{ kips}$$

∴ use $\phi R_n = 46.48$ kips

For the other bolts, $L_c = s - h = 4 - 1.438 = 2.562$ in.

$$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(2.562)(1/2)(58) = 66.87 \text{ kips}$$

$$\phi(2.4dtF_u) = 71.78 \text{ kips} > 66.87 \text{ kips} \therefore \text{ use } \phi R_n = 66.87 \text{ kips}$$

For the connection, the total bearing strength is

$$2(46.48)+4(66.87)=360\text{ kips}>P_u=175\text{ kips}\qquad\text{(OK)}$$

Use 6 bolts as shown.

(b) ASD Solution

$$P_a=D+L=40+80=120\text{ kips}$$

Bolt shear (assume that the threads are in shear): $A_b=\pi d^2/4=\pi(1.375)^2/4=1.485\text{ in.}^2$

Shear capacity of one bolt is

$$\frac{R_n}{\Omega}=\frac{F_{nv}A_b}{\Omega}=\frac{48(1.485)}{2.00}=35.64\text{ kips/bolt}$$

Slip-critical strength. Assuming Class A surfaces,

$$\frac{R_n}{\Omega}=\frac{\mu D_u h_{sc}T_b N_s}{\Omega}=\frac{0.35(1.13)(1.0)(85)(1.0)}{1.50}=22.41\text{ kips/bolt}$$

Number of bolts required $=\dfrac{120}{22.41}=5.36$ try 6 bolts.

Minimum spacing $=2\frac{2}{3}d=2.667\left(1\frac{3}{8}\right)=3.667$ in., try 4 in.

Minimum edge distance from AISC Table J3.4 $=1\frac{3}{4}\times 1\frac{3}{8}=2.41$ in., try 2½ in.

Try the layout shown in the figure above (in part a).

Check bearing. To determine which component to check, compare the product of the thickness and the ultimate stress (the edge distances and spacings are the same for both components). For the gusset plate,

$$tF_u=0.5(58)=29.0\text{ kips/in.}$$

For the tension member, $F_u=70$ ksi for A242 steel, and

$$t_wF_u=0.448(70)=31.36\text{ kips/in.}>29.0\text{ kip/in}$$

$\therefore$ check bearing on the gusset plate. $h=1\frac{3}{8}+\frac{1}{16}=1.438$ in.

For the holes nearest the edge, $L_c=L_e-\dfrac{h}{2}=2.5-\dfrac{1.438}{2}=1.781$ in.

$$\frac{R_n}{\Omega}=\frac{1.2L_ctF_u}{2.00}=\frac{1.2(1.781)(1/2)(58)}{2.00}=30.99\text{ kips}$$

The upper limit is

$$\frac{2.4dtF_u}{\Omega}=\frac{2.4\left(1\frac{3}{8}\right)(1/2)(58)}{2.00}=47.85\text{ kips}>30.99\text{ kips}$$

$\therefore$ use $R_n=30.99$ kips

For the other bolts, $L_c=s-h=4-1.438=2.562$ in.

$$\frac{R_n}{\Omega} = \frac{1.2L_c tF_u}{2.00} = \frac{1.2(2.562)(1/2)(58)}{2.00} = 44.58 \text{ kips}$$

$$\frac{2.4dtF_u}{\Omega} = 47.85 \text{ kips} > 44.58 \text{ kips} \therefore \text{ use } \phi R_n = 44.58 \text{ kips}$$

For the connection, the total bearing strength is

$$2(30.99) + 4(44.58) = 240 \text{ kips} > P_a = 120 \text{ kips} \qquad \text{(OK)}$$

Use 6 bolts as shown.

7.7-2

Load Combination 2: $1.2D + 1.6L = 1.2(50) + 1.6(100) = 220.0$ kips

Load Combination 4: $1.2D + 1.6W + 0.5L = 1.2(50) + 1.6(45) + 0.5(100) = 182.0$ kips

Combination 2 controls; use $P_u = 220$ kips. Slip-critical strength will control over shear. Use the following spreadsheet table to help select the number of bolts.

Diameter (in.)	T_b (kips)	ϕR_n / bolt (kips)	No. bolts required
1/2	12	4.746	46.4
5/8	19	7.515	29.3
3/4	28	11.07	19.9
7/8	39	15.42	14.3
1	51	20.17	10.9
1 1/8	56	22.15	9.9
1 1/4	71	28.08	7.8
1 3/8	85	33.62	6.5
1 1/2	103	40.74	5.4

Try ten $1\frac{1}{4}$-inch diameter bolts in two lines. $\phi R_n = 10(28.08) = 281 \text{ kips} > 220 \text{ kips}$ (OK)

Select a tension member.

$$\text{Required } A_g = \frac{P_u}{0.9F_y} = \frac{220}{0.9(36)} = 6.79 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{0.75F_u} = \frac{220}{0.75(58)} = 5.06 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{20 \times 12}{300} = 0.80 \text{ in.}$$

Try $L8 \times 6 \times \frac{5}{8}$

$$A_g = 8.36 \text{ in.}^2 > 6.79 \text{ in.}^2 \qquad \text{(OK)} \qquad r_{\min} = r_z = 1.29 \text{ in.} > 0.80 \text{ in.} \qquad \text{(OK)}$$

$$A_n = 8.36 - 2\left(1\frac{1}{4} + \frac{1}{8}\right)\left(\frac{5}{8}\right) = 6.641 \text{ in.}^2$$

Use the alternative value of U from AISC Table D3.1, Case 8:

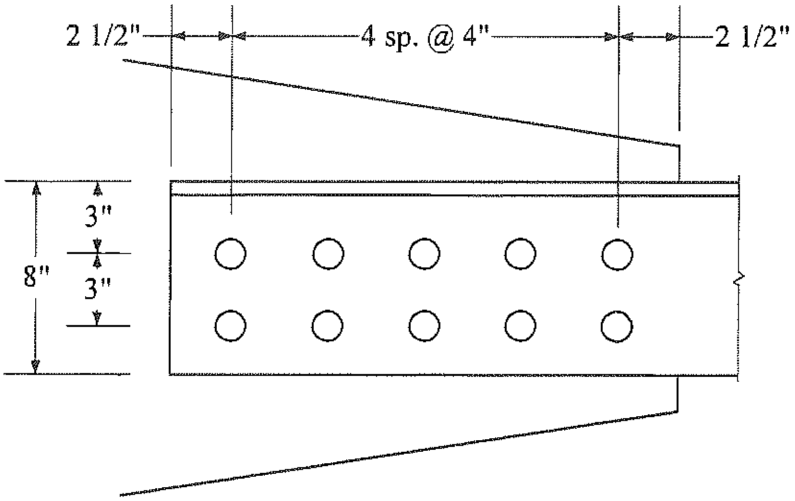
$$A_e = A_n U = 6.641(0.80) = 5.31 \text{ in.}^2 > 5.06 \text{ in.}^2 \text{ (OK)}$$

Determine the bolt layout. In the transverse direction, use the usual gage distances. For the transverse direction,

$$\text{Minimum spacing} = 2\tfrac{2}{3}d = 2.667\left(1\tfrac{1}{4}\right) = 3.334 \text{ in. Use 4 in.}$$

Minimum edge distance from AISC Table J3.4 = $2\frac{1}{4}$ in. Use $2\frac{1}{2}$ in.

Try the following layout.



Check bearing (gusset plate controls). $h = 1\frac{1}{4} + \frac{1}{16} = 1.313$ in.

For the hole nearest the edge, $L_c = L_e - \frac{h}{2} = 2.5 - \frac{1.313}{2} = 1.844$ in.

$$R_n = 1.2L_c tF_u = 1.2(1.844)(3/8)(58) = 48.13 \text{ kips}$$

$$2.4dtF_u = 2.4\left(1\tfrac{1}{4}\right)(3/8)(58) = 65.25 \text{ kips} > 48.13 \text{ kips}$$

$$\therefore \text{ use } R_n = 48.13 \text{ kips}$$

For the other bolts, $L_c = s - h = 4 - 1.313 = 2.687$ in.

$$R_n = 1.2L_c tF_u = 1.2(2.687)(3/8)(58) = 70.13 \text{ kips}$$

$$2.4dtF_u = 65.25 \text{ kips} < 70.13 \text{ kips} \therefore \text{ use } R_n = 65.25 \text{ kips}$$

Total *nominal* bearing strength = 2(48.13) + 8(65.25) = 618.3 kips

Total *design* bearing strength = 0.75(618.3) = 464 kips > P_u = 220 kips (OK)

Check block shear on the gusset plate. For hole diameters, use $1\frac{1}{4} + \frac{1}{8} = 1.375$ in. The shear areas are

$$A_{gv} = \frac{3}{8}[4(4) + 2.5] \times 2 = 13.88 \text{ in.}^2$$

$$A_{nv} = \frac{3}{8}[18.5 - 4.5(1.375)] \times 2 = 9.234 \text{ in.}^2$$

The tension area is $A_{nt} = \frac{3}{8}[3 - 1(1.375)] = 0.6094 \text{ in.}^2$

From AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$
$$= 0.6(58)(9.234) + 1.0(58)(0.6094) = 356.7 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(13.88) + 1.0(58)(0.6094) = 335.2 \text{ kips}$$

The *nominal* block shear strength of the gusset plate is therefore 335.2 kips

The *design* block shear strength is $\phi R_n = 0.75(335.2) = 251 \text{ kips} > P_u = 220 \text{ kips}$ (OK)

Use an L8 × 6 × $\frac{5}{8}$ and ten $1\frac{1}{4}$-inch diameter A325 bolts in two lines as shown.

(b) ASD Solution

Load Combination 2: $D + L = 50 + 100 = 150 \text{ kips}$

Load Combination 6: $D + 0.75W + 0.75L = 50 + 0.75(45) + 0.75(100) = 158.8 \text{ kips}$

Combination 6 controls; use $P_a = 158.8$ kips. Slip-critical strength will control over shear. Use the following spreadsheet table to help select the number of bolts.

Diameter (in.)	T_b (kips)	R_n/Ω per bolt (kips)	No. bolts required
1/2	12	3.164	50.2
5/8	19	5.010	31.7
3/4	28	7.383	21.5
7/8	39	10.28	15.4
1	51	13.45	11.8
1 1/8	56	14.77	10.8
1 1/4	71	18.72	8.5
1 3/8	85	22.41	7.1
1 1/2	103	27.16	5.8

Try ten $1\frac{1}{4}$-inch diameter bolts in two lines. $\frac{R_n}{\Omega} = 10(18.72) = 187 \text{ kips} > 158.8 \text{ kips}$ (OK)

Select a tension member.

$$\text{Required } A_g = \frac{P_a}{0.6F_y} = \frac{158.8}{0.6(36)} = 7.35 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_a}{0.5F_u} = \frac{158.8}{0.5(58)} = 5.48 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{20 \times 12}{300} = 0.80 \text{ in.}$$

Try L8 × 6 × $\frac{3}{4}$

$A_g = 9.94 \text{ in.}^2 > 7.35 \text{ in.}^2$ (OK) $r_{\min} = r_z = 1.29 \text{ in.} > 0.80 \text{ in.}$ (OK)

$$A_n = 9.94 - 2\left(1\frac{1}{4} + \frac{1}{8}\right)\left(\frac{3}{4}\right) = 7.878 \text{ in.}^2$$

Use the alternative value of U from AISC Table D3.1, Case 8:

$$A_e = A_n U = 7.878(0.80) = 6.30 \text{ in.}^2 > 5.48 \text{ in.}^2 \text{ (OK)}$$

Determine the bolt layout. In the transverse direction, use the usual gage distances. For the transverse direction,

$$\text{Minimum spacing} = 2\tfrac{2}{3}d = 2.667\left(1\frac{1}{4}\right) = 3.334 \text{ in. Use 4 in.}$$

Minimum edge distance from AISC Table J3.4 = $2\frac{1}{4}$ in. Use $2\frac{1}{2}$ in.

Try the layout shown in the figure above, in part a.

Check bearing (gusset plate controls). From part a,

The total *nominal* bearing strength = 618.3 kips

The total *allowable* bearing strength is

$$\frac{618.3}{2.00} = 309 \text{ kips} > P_a = 158.8 \text{ kips} \quad \text{(OK)}$$

Check block shear on the gusset plate. From part a,

The *nominal* block shear strength of the gusset plate is 335.2 kips

The *allowable* block shear strength is

$$\frac{335.2}{2.00} = 168 \text{ kips} > P_a = 158.8 \text{ kips} \quad \text{(OK)}$$

Use an L8 × 6 × $\frac{3}{4}$ and ten $1\frac{1}{4}$-inch diameter A325 bolts in two lines as shown.

7.7-3

(a) LRFD Solution: $P_u = 1.2D + 1.6L = 1.2(45) + 1.6(105) = 222.0 \text{ kips}$

Try 1-in. diameter bolts. Slip-critical strength will control over shear:

$$R_n = \mu D_u h_{sc} T_b N_s = 0.35(1.13)(1.0)(51)(2) = 40.34 \text{ kips/bolt}$$

$$\phi R_n = 0.75(40.34) = 30.26 \text{ kips/bolt}$$

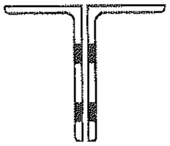
$$\text{Number of bolts required} = \frac{222}{30.26} = 7.34, \text{ try 8 bolts in two rows.}$$

$$\text{Select a tension member. Required } A_g = \frac{P_u}{0.9F_y} = \frac{222}{0.9(36)} = 6.85 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{0.75F_u} = \frac{222}{0.75(58)} = 5.10 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{15 \times 12}{300} = 0.6 \text{ in.}$$

Try 2L7 × 4 × $\frac{7}{16}$ LLBB



$$A_g = 9.27 \text{ in.}^2 > 6.85 \text{ in.}^2 \qquad \text{(OK)}$$

$$r_{\min} = r_y = 1.55 \text{ in.} > 0.6 \text{ in.} \qquad \text{(OK)}$$

$$A_n = 9.27 - 4\left(1 + \frac{1}{8}\right)\left(\frac{7}{16}\right) = 7.301 \text{ in.}^2$$

Use the alternative value of U from AISC Table D3.1, Case 8:

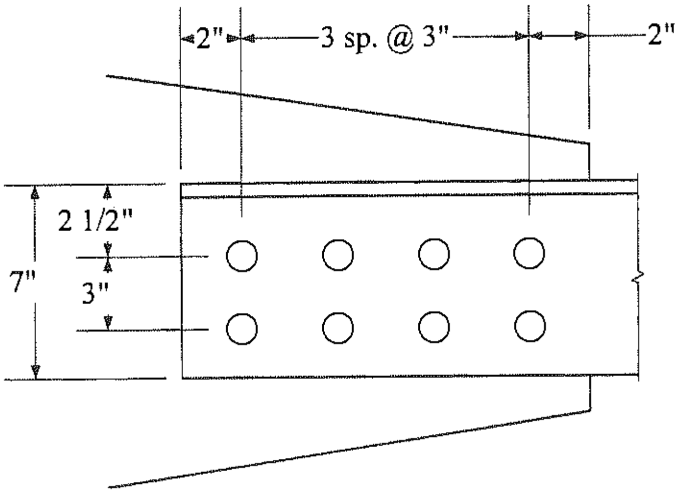
$$A_e = A_nU = 7.301(0.80) = 5.84 \text{ in.}^2 > 5.10 \text{ in.}^2 \text{ (OK)}$$

Determine the bolt layout. In the transverse direction, use the usual gage distances. For the longitudinal direction,

Minimum spacing = $2\frac{2}{3}d = 2.667(1) = 2.67$ in. Use 3 in.

Minimum edge distance from AISC Table J3.4 = $1\frac{3}{4}$ in. Use 2 in.

Try the following layout.



Check bearing (gusset plate controls). $h = 1 + \frac{1}{16} = 1.063$ in.

For the hole nearest the edge, $L_c = L_e - \frac{h}{2} = 2 - \frac{1.063}{2} = 1.469$ in.

$$R_n = 1.2L_ctF_u = 1.2(1.469)(3/8)(58) = 38.34 \text{ kips}$$

$2.4dtF_u = 2.4(1)(3/8)(58) = 52.2$ kips > 38.34 kips

$\therefore$ use $R_n = 38.34$ kips

For the other bolts, $L_c = s - h = 3 - 1.063 = 1.937$ in.

$R_n = 1.2L_c tF_u = 1.2(1.937)(3/8)(58) = 50.56$ kips

$2.4dtF_u = 52.2$ kips > 50.56 kips $\therefore$ use $R_n = 50.56$ kips

Total *nominal* bearing strength = $2(38.34) + 6(50.56) = 380.0$ kips

Total *design* bearing strength = $0.75(380.0) = 285$ kips > $P_u = 220$ kips (OK)

Check block shear on the gusset plate. For hole diameters, use $1 + \frac{1}{8} = 1.125$ in. The shear areas are

$$A_{gv} = \frac{3}{8}[3(3) + 2] \times 2 = 8.25 \text{ in.}^2$$

$$A_{nv} = \frac{3}{8}[11 - 3.5(1.125)] \times 2 = 5.297 \text{ in.}^2$$

The tension area is $A_{nt} = \frac{3}{8}[3 - 1(1.125)] = 0.7031 \text{ in.}^2$

From AISC Equation J4-5,

$$R_n = 0.6F_u A_{nv} + U_{bs}F_u A_{nt}$$

$$= 0.6(58)(5.297) + 1.0(58)(0.7031) = 225.1 \text{ kips}$$

with an upper limit of

$$0.6F_y A_{gv} + U_{bs}F_u A_{nt} = 0.6(36)(8.25) + 1.0(58)(0.7031) = 219.0 \text{ kips}$$

The *nominal* block shear strength of the gusset plate is therefore 219.0 kips

The *design* block shear strength is

$\phi R_n = 0.75(219.0) = 164.3$ kips < $P_u = 220$ kips (N.G.)

Increase the bolt spacing s :

$$A_{gv} = \frac{3}{8}[3s + 2] \times 2 = 2.25s + 1.5 \text{ in.}^2$$

$$A_{nv} = \frac{3}{8}[3s + 2 - 3.5(1.125)] \times 2 = 2.25s - 1.453 \text{ in.}^2$$

$$0.6F_u A_{nv} = 0.6(58)[2.25s - 1.453] = [78.3s - 50.56]$$

Let

$$\phi[0.6F_u A_{nv} + U_{bs}F_u A_{nt}] = 0.75[(78.3s - 50.56) + 1.0(58)(0.7031)] = 222$$

Solution is: $\{s = 3.905\}$ Try $s = 4$ in.

Check upper limit:

$$A_{gv} = 2.25s + 1.5 \text{ in.}^2 = 2.25(4) + 1.5 = 10.5 \text{ in.}^2$$

$$A_{nv} = 2.25s - 1.453 = 2.25(4) - 1.453 = 7.547 \text{ in.}^2$$

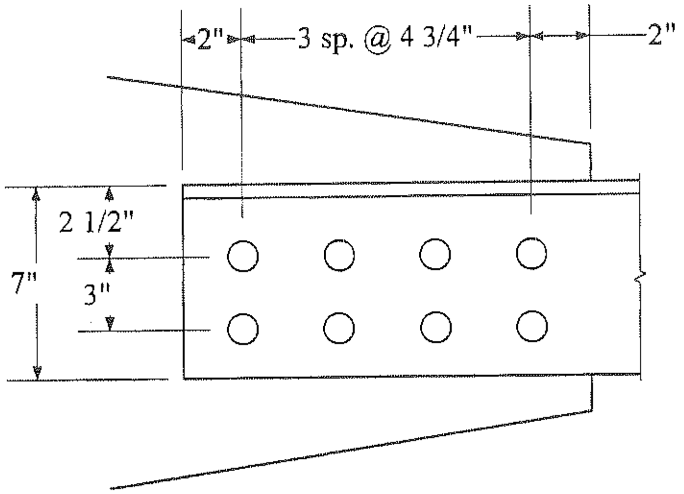
$$R_n = 0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(10.5) + 1.0(58)(0.7031) = 267.6 \text{ kips}$$

$$\phi R_n = 0.75(267.6) = 200.7 \text{ kips} < 220 \text{ kips}$$

This means that the upper limit controls. Let

$$\phi(0.6F_yA_{gv} + U_{bs}F_uA_{nt}) = 0.75[0.6(36)(2.25s + 1.5) + 1.0(58)(0.7031)] = 222$$

Solution is: $\{s = 4.585\}$ Use $s = 4\frac{3}{4}$ in.



Use 2L7 × 4 × $\frac{7}{16}$ LLBB and eight 1-inch diameter A325 bolts in two lines as shown.

(b) ASD Solution: $P_a = D + L = 45 + 105 = 150$ kips

Try 1-in. diameter bolts. Slip-critical strength will control over shear:

$$R_n = \mu D_u h_{sc} T_b N_s = 0.35(1.13)(1.0)(51)(2) = 40.34 \text{ kips/bolt}$$

$$\frac{R_n}{\Omega} = \frac{40.34}{2.00} = 20.17 \text{ kips/bolt}$$

Number of bolts required $= \dfrac{150}{20.17} = 7.44$, try 8 bolts in two rows.

Select a tension member. Required $A_g = \dfrac{P_a}{0.6F_y} = \dfrac{150}{0.6(36)} = 6.94 \text{ in.}^2$

$$\text{Required } A_e = \frac{P_a}{0.5F_u} = \frac{150}{0.5(58)} = 5.17 \text{ in.}^2$$

$$\text{Required } r_{\min} = \frac{L}{300} = \frac{15 \times 12}{300} = 0.6 \text{ in.}$$

Try 2L7 × 4 × $\frac{7}{16}$ LLBB

$A_g = 9.27 \text{ in.}^2 > 6.94 \text{ in.}^2$ (OK)

$r_{\min} = r_y = 1.55 \text{ in.} > 0.6 \text{ in.}$ (OK)

$$A_n = 9.27 - 4\left(1 + \frac{1}{8}\right)\left(\frac{7}{16}\right) = 7.301 \text{ in.}^2$$

Use the alternative value of U from AISC Table D3.1, Case 8:

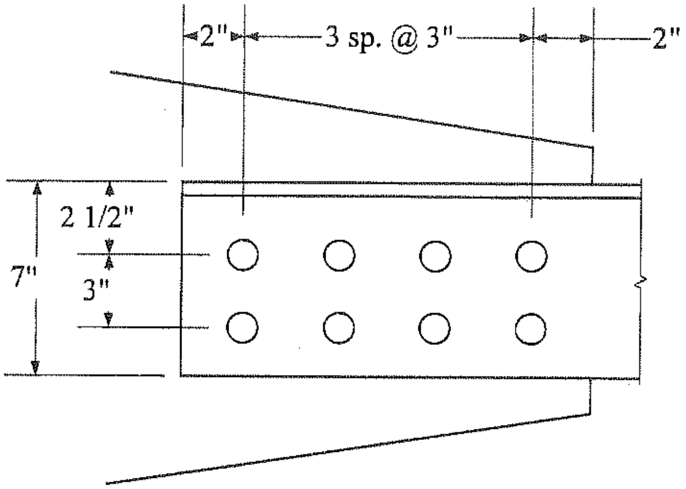
$$A_e = A_n U = 7.301(0.80) = 5.84 \text{ in.}^2 > 5.17 \text{ in.}^2 \text{ (OK)}$$

Determine the bolt layout. In the transverse direction, use the usual gage distances. For the longitudinal direction,

$$\text{Minimum spacing} = 2\tfrac{2}{3}d = 2.667(1) = 2.67 \text{ in. Use 3 in.}$$

$$\text{Minimum edge distance from AISC Table J3.4} = 1\tfrac{3}{4} \text{ in. Use 2 in.}$$

Try the following layout.



From part a, for the same layout, the total *nominal* bearing strength is

$$R_n = 380.0 \text{ kips}$$

$$\text{Total } allowable \text{ bearing strength} = \frac{R_n}{\Omega} = \frac{380.0}{2.00} = 190 \text{ kips} > P_a = 150 \text{ kips} \quad \text{(OK)}$$

Check block shear on the gusset plate. From part a, for the same layout, the *nominal* block shear strength of the gusset plate is

$$R_n = 219.0 \text{ kips}$$

The *allowable* block shear strength is

$$\frac{R_n}{\Omega} = \frac{219.0}{2.00} = 110 \text{ kips} < P_a = 150 \text{ kips (N.G.)}$$

Increase the bolt spacing s :

$$A_{gv} = \frac{3}{8}[3s + 2] \times 2 = 2.25s + 1.5 \text{ in.}^2$$

$$A_{nv} = \frac{3}{8}[3s + 2 - 3.5(1.125)] \times 2 = 2.25s - 1.453 \text{ in.}^2$$

$$0.6F_u A_{nv} = 0.6(58)[2.25s - 1.453] = [78.3s - 50.56]$$

Let

$$\frac{1}{\Omega}[0.6F_uA_{nv} + U_{bs}F_uA_{nt}]$$

$$= \frac{1}{2.00}[(78.3s - 50.56) + 1.0(58)(0.7031)] = 150, \text{ Solution is: } \{s = 3.956\}$$

Try $s = 4$ in. Check the upper limit:

$$A_{gv} = 2.25s + 1.5 \text{ in.}^2 = 2.25(4) + 1.5 = 10.5 \text{ in.}^2$$

$$A_{nv} = 2.25s - 1.453 = 2.25(4) - 1.453 = 7.547 \text{ in.}^2$$

$$R_n = 0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(10.5) + 1.0(58)(0.7031) = 267.6 \text{ kips}$$

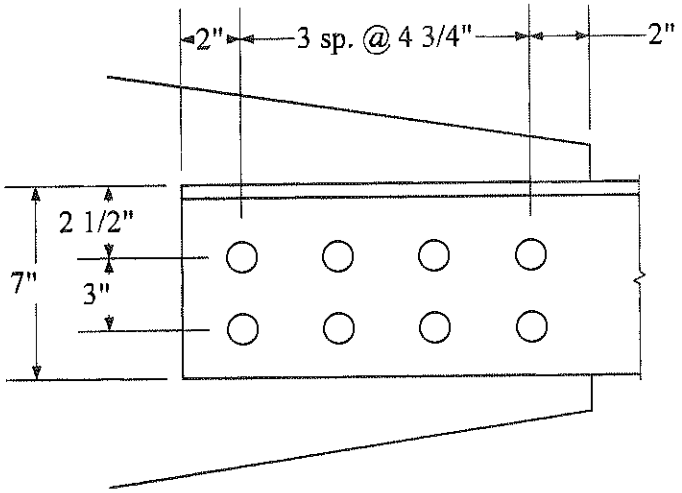
$$\frac{R_n}{\Omega} = \frac{267.6}{2.00} = 134 \text{ kips} < 150 \text{ kips}$$

This means that the upper limit controls. Let

$$\frac{1}{\Omega}(0.6F_yA_{gv} + U_{bs}F_uA_{nt})$$

$$= \frac{1}{2.00}[0.6(36)(2.25s + 1.5) + 1.0(58)(0.7031)] = 150, \text{ Solution is: } \{s = 4.667\}$$

Use $s = 4\frac{3}{4}$ in.



Use 2L7 × 4 × $\frac{7}{16}$ LLBB and eight 1-inch diameter A325 bolts in two lines as shown.

7.8-1

$$b = \frac{5.5 - t_w}{2} = \frac{5.5 - 0.585}{2} = 2.458 \text{ in}$$

$$a = \frac{b_f - 5.5}{2} = \frac{10.4 - 5.5}{2} = 2.45 \text{ in.}$$

$1.25b = 1.25(2.458) = 3.073$ in. > 2.45 in. $\therefore$ use $a = 2.45$ in.

$$b' = b - \frac{d}{2} = 2.458 - \frac{3/4}{2} = 2.083 \text{ in.}$$

$$a' = a + \frac{d}{2} = 2.45 + \frac{3/4}{2} = 2.825 \text{ in.}$$

$$d' = d + \frac{1}{8} = \frac{3}{4} + \frac{1}{8} = \frac{7}{8} \text{ in.}, \qquad p = 10/3 = 3.333 \text{ in.}$$

$$\delta = 1 - d'/p = 1 - (7/8)/3.333 = 0.7375 \text{ in.}$$

$$A_b = \frac{\pi(3/4)^2}{4} = 0.4418 \text{ in.}^2$$

(a) LRFD solution.

$$B = \phi R_n = \phi F_{nt} A_b = 0.75(90)(0.4418) = 29.82 \text{ kips}$$

Total factored load $= 1.2D + 1.6L = 1.2(25) + 1.6(75) = 150.0$ kips

$$T = \frac{150}{6} = 25.0 \text{ kips/bolt}$$

$$\frac{B}{T} - 1 = \frac{29.82}{25} - 1 = 0.1928, \qquad \frac{a'}{b'} = \frac{2.825}{2.083} = 1.356$$

$$\alpha = \frac{\left(\frac{B}{T} - 1\right)\frac{a'}{b'}}{\delta\left[1 - \left(\frac{B}{T} - 1\right)\frac{a'}{b'}\right]} = \frac{0.1928(1.356)}{0.7375[1 - 0.1928(1.356)]} = 0.4799 < 1.0$$

$$\text{Required } t_f = \sqrt{\frac{4.44Tb'}{pF_u(1+\delta\alpha)}} = \sqrt{\frac{4.44(25)(2.083)}{3.333(65)[1 + 0.7375(0.4799)]}} = 0.888 \text{ in.}$$

Actual thickness = 0.985 in. > 0.888 in. (OK)

Compute the total bolt force, including prying force (this step not required):

$$\alpha = \frac{1}{\delta}\left(\frac{4.44Tb'}{pt_f^2F_u} - 1\right) = \frac{1}{0.7375}\left[\frac{4.44(25)(2.083)}{3.333(0.985)^2(65)} - 1\right] = 0.1356$$

$$\delta\alpha = 0.7375(0.1356) = 0.1000$$

$$B_c = T\left[1 + \frac{\delta\alpha}{(1+\delta\alpha)}\frac{b'}{a'}\right] = 25\left[1 + \frac{0.1}{(1+0.1)}\frac{2.083}{2.825}\right]$$

$$= 26.68 \text{ kips} < \phi R_n = 29.82 \text{ kips} \qquad \text{(OK)}$$

Connection is adequate. Required $t_f = 0.888$ in. < 0.985 in.

(b) ASD solution

The allowable tensile strength of one bolt is

$$B = \frac{R_n}{\Omega} = \frac{F_{nt}A_b}{\Omega} = \frac{90.0(0.4418)}{2.00} = 19.88 \text{ kips}$$

The total applied load is

$$D + L = 25 + 75 = 100 \text{ kips}$$

The external load per bolt, excluding prying force, is $T = 100/6 = 16.67$ kips.

$$\frac{B}{T} - 1 = \frac{19.88}{16.67} - 1 = 0.1926, \qquad \frac{a'}{b'} = \frac{2.825}{2.083} = 1.356$$

$$\alpha = \frac{[(B/T - 1)](a'/b')}{\delta\{1 - [(B/T) - 1](a'/b')\}} = \frac{0.1926(1.356)}{0.7375[1 - 0.1926(1.356)]} = 0.4794$$

From Equation 7.19,

$$\text{Required } t_f = \sqrt{\frac{6.68Tb'}{pF_u(1+\delta\alpha)}} = \sqrt{\frac{6.68(16.67)(2.083)}{3.333(65)[1 + 0.7375(0.4794)]}}$$

$$= 0.889 \text{ in.} < 0.985 \text{ in.} \qquad \text{(OK)}$$

Determine the total bolt force, including prying force (this step not required):

$$\alpha = \frac{1}{\delta}\left(\frac{6.68Tb'}{pt_f^2F_u} - 1\right) = \frac{1}{0.7375}\left(\frac{6.68(16.67)(2.083)}{3.333(0.985)^2(65)} - 1\right) = 0.1404$$

$$B_c = T\left[1 + \frac{\delta\alpha}{(1+\delta\alpha)}\frac{b'}{a'}\right] = 16.67\left[1 + \frac{0.7375(0.1404)}{(1 + 0.7375 \times 0.1404)}\frac{2.083}{2.825}\right]$$

$$= 17.82 \text{ kips} < 19.88 \text{ kips} \qquad \text{(OK)}$$

Connection is adequate. Required $t_f = 0.889$ in. < 0.985 in.

7.8-2

For b, use the distance from the bolt centerline to the mid-thickness of the angle leg.

$$b = \frac{5.375 - 3/8 - 5/8}{2} = 2.188 \text{ in}$$

$$a = \frac{4 + 4 + 3/8 - 5.375}{2} = 1.5 \text{ in.}$$

$1.25b = 1.25(2.188) = 2.735$ in. > 1.5 in. $\therefore$ use $a = 1.5$ in.

$$b' = b - \frac{d}{2} = 2.188 - \frac{1/2}{2} = 1.938 \text{ in.}$$

$$a' = a + \frac{d}{2} = 1.5 + \frac{1/2}{2} = 1.75 \text{ in.}$$

$$d' = d + \frac{1}{8} = \frac{1}{2} + \frac{1}{8} = \frac{5}{8} \text{ in.}, \qquad p = \frac{7}{2} = 3.5 \text{ in.}$$

$$\delta = 1 - d'/p = 1 - (5/8)/3.5 = 0.8214 \text{ in.}$$

$$A_b = \frac{\pi(1/2)^2}{4} = 0.1963 \text{ in.}^2$$

(a) LRFD solution.

$$B = \phi R_n = \phi F_{nt}A_b = 0.75(90)(0.1963) = 13.25 \text{ kips}$$

Total factored load $= 1.2D + 1.6L = 1.2(6) + 1.6(15) = 31.2$ kips

$$T = \frac{31.2}{4} = 7.8 \text{ kips/bolt}$$

$$\frac{B}{T} - 1 = \frac{13.25}{7.8} - 1 = 0.6987, \qquad \frac{a'}{b'} = \frac{1.75}{1.938} = 0.9030$$

$$\alpha = \frac{\left(\frac{B}{T} - 1\right)\frac{a'}{b'}}{\delta\left[1 - \left(\frac{B}{T} - 1\right)\frac{a'}{b'}\right]} = \frac{0.6987(0.9030)}{0.8214[1 - 0.6987(0.9030)]} = 2.081 > 1.0$$

$\therefore$ use $\alpha = 1.0$

$$\text{Required } t_f = \sqrt{\frac{4.44Tb'}{pF_u(1 + \delta\alpha)}} = \sqrt{\frac{4.44(7.8)(1.938)}{3.5(58)[1 + 0.8214(1.0)]}} = 0.426 \text{ in.}$$

Actual thickness = 5/8 in. > 0.426 in. (OK)

Compute the total bolt force, including prying force (this step not required):

$$\alpha = \frac{1}{\delta}\left(\frac{4.44Tb'}{pt_f^2F_u} - 1\right) = \frac{1}{0.8214}\left[\frac{4.44(7.8)(1.938)}{3.5(5/8)^2(58)} - 1\right] = -0.187$$

$\therefore$ use $\alpha = 1.0$

$$\delta\alpha = 0.8214(1.0) = 0.8214$$

$$B_c = T\left[1 + \frac{\delta\alpha}{(1 + \delta\alpha)}\frac{b'}{a'}\right] = 7.8\left[1 + \frac{0.8214}{(1 + 0.8214)}\left(\frac{1.938}{1.75}\right)\right]$$

$$= 11.7 \text{ kips} < \phi R_n = 13.25 \text{ kips} \qquad \text{(OK)}$$

Connection is adequate. Required $t_f = 0.426$ in. < 5/8 in.

(b) ASD solution

The allowable tensile strength of one bolt is

$$B = \frac{R_n}{\Omega} = \frac{F_{nt}A_b}{\Omega} = \frac{90.0(0.1963)}{2.00} = 8.834 \text{ kips}$$

The total applied load is

$$D + L = 6 + 15 = 21 \text{ kips}$$

The external load per bolt, excluding prying force, is $T = 21/4 = 5.25$ kips.

$$\frac{B}{T} - 1 = \frac{8.834}{5.25} - 1 = 0.6827, \qquad \frac{a'}{b'} = \frac{1.75}{1.938} = 0.9030$$

$$\alpha = \frac{[(B/T - 1)](a'/b')}{\delta\{1 - [(B/T) - 1](a'/b')\}} = \frac{0.6827(0.9030)}{0.8214[1 - 0.6827(0.9030)]} = 1.96 > 1.0$$

$\therefore$ use $\alpha = 1.0$

From Equation 7.19,

$$\text{Required } t_f = \sqrt{\frac{6.68Tb'}{pF_u(1 + \delta\alpha)}} = \sqrt{\frac{6.68(5.25)(1.938)}{3.5(58)[1 + 0.8214(1.0)]}}$$

$$= 0.429 \text{ in.} < 5/8 \text{ in.} \qquad \text{(OK)}$$

Determine the total bolt force, including prying force (this step not required):

$$\alpha = \frac{1}{\delta}\left(\frac{6.68Tb'}{pt_f^2F_u} - 1\right) = \frac{1}{0.8214}\left(\frac{6.68(5.25)(1.938)}{3.5(5/8)^2(58)} - 1\right) = -0.1740$$

$\therefore$ use $\alpha = 1.0$

$$B_c = T\left[1 + \frac{\delta\alpha}{(1+\delta\alpha)}\frac{b'}{a'}\right] = 5.25\left[1 + \frac{0.8214(1.0)}{(1+0.8214)}\frac{1.938}{1.75}\right]$$

$$= 7.87 \text{ kips} < 8.834 \text{ kips} \qquad \text{(OK)}$$

Connection is adequate. Required $t_f = 0.429$ in. < 5/8 in.

7.9-1

Nominal shear strength (assume that the threads are in shear):

$$A_b = \frac{\pi d^2}{4} = \frac{\pi(7/8)^2}{4} = 0.6013 \text{ in.}^2$$

$$R_n = F_{nv}A_b = 48(0.6013) = 28.86 \text{ kips/bolt}$$

Bearing strength (the WT flange controls): $h = \frac{7}{8} + \frac{1}{16} = 0.9375$ in.

For the hole nearest the edge, $L_c = L_e - \frac{h}{2} = 2 - \frac{0.9375}{2} = 1.531$ in.

$$R_n = 1.2L_ctF_u = 1.2(1.531)(0.640)(65) = 76.43 \text{ kips}$$

$$2.4dtF_u = 2.4(7/8)(0.640)(65) = 87.36 \text{ kips}$$

$\therefore$ use $R_n = 76.43$ kips

For the other bolts, $L_c = s - h = 3 - 0.9375 = 2.063$ in.

$$R_n = 1.2L_ctF_u = 1.2(2.063)(0.640)(65) = 103.0 \text{ kips}$$

$2.4dtF_u = 87.36$ kips < 103.0 kips $\therefore$ use $R_n = 87.36$ kips

(a) LRFD solution

$$P_u = 1.2D + 1.6L = 1.2(84) + 1.6(66) = 206.4 \text{ kips}$$

T_u = Tensile force = $206.4\cos 30° = 178.7$ kips, or $\frac{178.7}{8} = 22.34$ kips/bolt

V_u = Shear/bearing force = $206.4\sin 30° = 103.2$ kips, or $\frac{103.2}{8} = 12.9$ kips/bolt

The design shear strength is $\phi R_n = 0.75(28.86) = 21.6$ kips > 12.9 kips (OK)

For the design bearing strength, conservatively use the smaller of the two strengths computed: $\phi R_n = 0.75(76.43) = 57.3$ kips > 12.9 kips (OK)

Tensile strength:

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}}f_v \le F_{nt}, \qquad f_v = \frac{12.9}{0.6013} = 21.45 \text{ ksi}$$

$$F'_{nt} = 1.3(90) - \frac{90}{0.75(48)}(21.45) = 63.38 \text{ ksi} < 90 \text{ ksi}$$

$$\phi R_n = \phi F'_{nt}A_b = 0.75(63.38)(0.6013) = 28.6 \text{ kips/bolt} > 22.3 \text{ kips/bolt (OK)}$$

The connection is adequate.

(b) ASD solution

$$P_a = D + L = 84 + 66 = 150 \text{ kips}$$

$$T_a = \text{Tensile force} = 150\cos 30° = 129.9 \text{ kips, or } \frac{129.9}{8} = 16.24 \text{ kips/bolt}$$

$$V_a = \text{Shear/bearing force} = 150\sin 30° = 75.00 \text{ kips, or } \frac{75.00}{8} = 9.375 \text{ kips/bolt}$$

The allowable shear strength is $\frac{R_n}{\Omega} = \frac{28.86}{2.00} = 14.4 \text{ kips} > 9.38 \text{ kips}$ (OK)

For the allowable bearing strength, conservatively use the smaller of the two strengths computed: $\frac{R_n}{\Omega} = \frac{76.43}{2.00} = 38.2 \text{ kips} > 9.375 \text{ kips}$ (OK)

Tensile strength:

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}}f_v \le F_{nt}, \qquad f_v = \frac{9.375}{0.6013} = 15.59 \text{ ksi}$$

$$F'_{nt} = 1.3(90) - \frac{2.00(90)}{48}(15.59) = 58.54 \text{ ksi} < 90 \text{ ksi}$$

$$\frac{R_n}{\Omega} = \frac{F'_{nt}A_b}{\Omega} = \frac{58.54(0.6013)}{2.00} = 17.6 \text{ kips/bolt} > 16.2 \text{ kips/bolt (OK)}$$

The connection is adequate.

7.9-2

Nominal shear strength:

$$A_b = \frac{\pi d^2}{4} = \frac{\pi(3/4)^2}{4} = 0.4418 \text{ in.}^2$$

$$R_n = F_{nv}A_b = 48(0.4418) = 21.21 \text{ kips/bolt}$$

Bearing strength ($t_f = 0.605$ in. for both W and WT):

$$R_n = 2.4dtF_u = 2.4(3/4)(0.605)(65) = 70.79 \text{ kips}$$

(a) LRFD solution

$$P_u = 1.2D + 1.6L = 1.2(30) + 1.6(70) = 148.0 \text{ kips}$$

$$T_u = \text{Tensile force} = 148\left(\frac{2}{\sqrt{5}}\right) = 132.4 \text{ kips, or } \frac{132.4}{6} = 22.1 \text{ kips/bolt}$$

$$V_u = \text{Shear/bearing force} = 148\left(\frac{1}{\sqrt{5}}\right) = 66.19 \text{ kips, or } \frac{66.19}{6} = 11.03 \text{ kips/bolt}$$

The design shear strength is $\phi R_n = 0.75(21.21) = 15.9 \text{ kips} > 11.0 \text{ kips}$ (OK)

The design bearing strength is $\phi R_n = 0.75(70.79) = 53.1 \text{ kips} > 11.0 \text{ kips}$ (OK)

Tensile strength:

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_v \le F_{nt}, \qquad f_v = \frac{11.03}{0.4418} = 24.97 \text{ ksi}$$

$$F'_{nt} = 1.3(90) - \frac{90}{0.75(48)}(24.97) = 54.58 \text{ ksi} < 90 \text{ ksi}$$

$$\phi R_n = \phi F'_{nt} A_b = 0.75(54.58)(0.4418) = 18.1 \text{ kips/bolt} < 22.1 \text{ kips/bolt (N.G.)}$$

The connection is not adequate.

(b) ASD solution

$$P_a = D + L = 30 + 70 = 100 \text{ kips}$$

$$T_a = \text{Tensile force} = 100\left(\frac{2}{\sqrt{5}}\right) = 89.44 \text{ kips, or } \frac{89.44}{6} = 14.9 \text{ kips/bolt}$$

$$V_a = \text{Shear/bearing force} = 100\left(\frac{1}{\sqrt{5}}\right) = 44.72 \text{ kips, or } \frac{44.72}{6} = 7.453 \text{ kips/bolt}$$

The allowable shear strength is $\frac{R_n}{\Omega} = \frac{21.21}{2.00} = 10.6 \text{ kips} > 7.453 \text{ kips}$ (OK)

The allowable bearing strength is $\frac{R_n}{\Omega} = \frac{70.79}{2.00} = 35.4 \text{ kips} > 7.453 \text{ kips}$ (OK)

Tensile strength:

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_v \le F_{nt}, \qquad f_v = \frac{7.453}{0.4418} = 16.87 \text{ ksi}$$

$$F'_{nt} = 1.3(90) - \frac{2.00(90)}{48}(16.87) = 53.74 \text{ ksi} < 90 \text{ ksi}$$

$$\frac{R_n}{\Omega} = \frac{F'_{nt} A_b}{\Omega} = \frac{53.74(0.4418)}{2.00} = 11.9 \text{ kips/bolt} < 14.9 \text{ kips/bolt (N.G.)}$$

The connection is not adequate.

7.9-3

$$P_u = 1.2D + 1.6L = 1.2(20) + 1.6(60) = 120.0 \text{ kips}$$

$$T_u = \frac{3}{5}(120) = 72.0 \text{ kips}, \qquad V_u = \frac{4}{5}(120) = 96.0 \text{ kips}$$

Assume that the tension strength will control:

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_v \le F_{nt}$$

$$= 1.3(90) - \frac{90}{0.75(48)} f_v \le 90 = 117 - 2.5f_v \le 90$$

$$\phi F'_{nt} = 0.75(117 - 2.5f_v) \le 0.75(90) = 87.75 - 1.875f_v \le 67.7$$

Let $\phi F'_{nt} = \dfrac{72}{\Sigma A_b}$ and $f_v = \dfrac{96}{\Sigma A_b}$

$$\frac{72}{\Sigma A_b} = 87.75 - 1.875\left(\frac{96}{\Sigma A_b}\right)$$

$$72 = 87.75\Sigma A_b - 180.0 \qquad \Rightarrow \qquad \Sigma A_b = 2.872 \text{ in.}^2$$

The area of one bolt is $A_b = \dfrac{\pi(7/8)^2}{4} = 0.6013 \text{ in.}^2$

Required $N_b = \dfrac{\Sigma A_b}{A_b} = \dfrac{2.872}{0.6013} = 4.78$ Try 6 bolts for symmetry.

First, check the upper limit on F'_{nt}:

$$f_v = \frac{96}{\Sigma A_b} = \frac{96}{6(0.6013)} = 26.61 \text{ ksi}$$

$$F'_{nt} = 117 - 2.5f_v = 117 - 2.5(26.61) = 50.48 \text{ ksi} < 90 \text{ ksi (OK)}$$

Check shear: V_u/bolt = 96/6 = 16.0 kips/bolt

$$\phi R_n = \phi F_{nv} A_b = 0.75(48)(0.6013) = 21.6 \text{ kips/bolt} > 16.0 \text{ kips/bolt} \qquad \text{(OK)}$$

Bearing strength (the WT flange thickness controls):

$$\phi(2.4dtF_u) = 0.75(2.4)(7/8)(0.345)(65) = 35.3 \text{ kips/bolt} > 16.0 \text{ kips/bolt (OK)}$$

Use 6 bolts.

(b) ASD solution

$$T_a = \frac{3}{5}(80) = 48 \text{ kips}, \qquad V_a = \frac{4}{5}(80) = 64 \text{ kips}$$

Assume that tension controls:

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_v \le F_{nt}$$

$$= 1.3(90) - \frac{2.00(90)}{48} f_v \le 90 = 117 - 3.75f_v \le 90$$

$$\frac{F'_{nt}}{\Omega} = \frac{(117 - 3.75f_v)}{2.00} \le \frac{90}{2.00} = 58.5 - 1.875f_v \le 45$$

Let $\dfrac{F'_{nt}}{\Omega} = \dfrac{48}{\Sigma A_b}$ and $f_v = \dfrac{64}{\Sigma A_b}$

where ΣA_b is the total bolt area. Substituting and solving for ΣA_b, we get

$$\frac{48}{\Sigma A_b} = 58.5 - 1.875\left(\frac{64}{\Sigma A_b}\right)$$

$48 = 58.5\Sigma A_b - 120 \quad \Rightarrow \quad \Sigma A_b = 2.872 \text{ in.}^2$

The area of one bolt is $A_b = \dfrac{\pi(7/8)^2}{4} = 0.6013 \text{ in.}^2$

And the number of bolts required is $N_b = \dfrac{\Sigma A_b}{A_b} = \dfrac{2.872}{0.6013} = 4.78$

Try six bolts for symmetry. First, check the upper limit on F'_{nt} :

$$f_v = \frac{64}{\Sigma A_b} = \frac{64}{6(0.6013)} = 17.74 \text{ ksi}$$

$$F'_{nt} = 117 - 3.75f_v = 117 - 3.75(17.74) = 50.48 \text{ ksi} < 90 \text{ ksi} \qquad \text{(OK)}$$

Check shear.

$$V_a/\text{bolt} = 64/6 = 10.7 \text{ kips}$$

$$\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{2.00} = \frac{48(0.6013)}{2.00} = 14.4 \text{ kips/bolt} > 10.7 \text{ kips/bolt} \qquad \text{(OK)}$$

Bearing strength (the WT flange thickness controls):

$$\frac{2.4dtF_u}{\Omega} = \frac{2.4(7/8)(0.345)(65)}{2.00} = 23.6 \text{ kips/bolt} > 10.7 \text{ kips/bolt} \quad \text{(OK)}$$

Use 6 bolts.

7.9-4

(a) LRFD solution

Tension member-to-gusset plate connection:

$$P_u = 1.2D + 1.6L = 1.2(0.25 \times 120) + 1.6(0.75 \times 120) = 180 \text{ kips}$$

Bolt shear strength:

$$A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$$

$$\phi R_n = \phi F_{nv}A_b = 0.75(60)(0.6013) \times 2 \text{ shear planes} = 54.1 \text{ kips/bolt}$$

Check bearing on the gusset plate assuming upper limit controls.

$$\phi R_n = \phi(2.4dtF_u) = 0.75(2.4)(7/8)(7/8)(58) = 79.9 \text{ kips/bolt}$$

Slip-critical strength: From AISC Table J3.1, $T_b = 49$ kips

$$R_n = \mu D_u h_{sc} T_b N_s = 0.35(1.13)(1.0)(49)(2) = 38.76 \text{ kips/bolt}$$

$$\phi R_n = 1.0(38.76) = 38.76 \text{ kips/bolt} \quad \text{(controls)}$$

Number required = $180/38.76 = 4.64$ bolts. Use 5.

Gusset plate-to-column angles connection:

Bearing on the angles will control, because $2 \times \frac{3}{8}$ in. < $\frac{7}{8}$ in.

$$\phi R_n = \phi(2.4dtF_u) = 0.75(2.4)(7/8)(2 \times \frac{3}{8})(58) = 68.5 \text{ kips/bolt}$$

Shear strength and slip-critical strength are same as for tension member-to-gusset plate connection, $\therefore$ use 5 bolts.

Connection angles-to-column flange connection: assume that the slip-critical strength will control. Select the number of bolts based on slip-critical strength, then check tension.

$$T_u = V_u = \frac{1}{\sqrt{2}}(180) = 127.3 \text{ kips}$$

$$\text{Reduction factor} = k_s = 1 - \frac{T_u}{D_u T_b N_b} = 1 - \frac{127.3}{1.13(49)N_b}$$

Let $127.3 = N_b\left[38.76\left(1 - \frac{127.3}{1.13(49)N_b}\right)\right]$, Solution is: $\{N_b = 5.58\}$

Use 5 rows of 2 bolts for symmetry and to match the arrangement of gusset plate-to-connection angle bolts. Check tension.

Shear/tension load per bolt $= 127.3/10 = 12.7$ kips

$\phi R_n = \phi F_{nt} A_b = 0.75(90)(0.6013) = 40.6$ kips/bolt > 12.7 kips/bolt (OK)

Minimum spacing $= 2\frac{2}{3}d = 2.667(7/8) = 2.33$ in.

Minimum edge distance from AISC Table J3.4 = 1.5 in.

Try $L_e = 1\frac{1}{2}$ in. and $s = 2\frac{1}{2}$ in. for all connection elements and check bearing.

For the angles-to-column connection, the angle thickness will control (3/8 in. < 0.695 in.):

$$h = \frac{7}{8} + \frac{1}{16} = 0.9375 \text{ in.}$$

For the hole nearest the edge, $L_c = L_e - \frac{h}{2} = 1.5 - \frac{0.9375}{2} = 1.031$ in.

$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(1.031)(3/8)(58) = 20.2$ kips > 12.7 kips (OK)

$\phi(2.4dtF_u) = 0.75(2.4)(7/8)(3/8)(58) = 34.3$ kips/bolt

For the other bolts, $L_c = s - h = 2.5 - 0.9375 = 1.563$ in.

$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(1.563)(3/8)(58) = 30.6$ kips > 12.7 kips (OK)

For the tension member connection, the gusset plate thickness will control:

For the hole nearest the edge,

$$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(1.031)(7/8)(58)$$
$$= 47.1 \text{ kips} > 180/5 = 36 \text{ kips (OK)}$$

$\phi(2.4dtF_u) = 0.75(2.4)(7/8)(7/8)(58) = 79.9$ kips/bolt

For the other bolts,

$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(1.563)(7/8)(58) = 71.4$ kips > 36.0 kips (OK)

For the gusset plate-to-column connection angles,

$$\phi R_n = \frac{2(3/8)}{7/8} \times 47.1 = 40.4 \text{ kips/bolt} > 36.0 \text{ kips/bolt (OK)}$$

Summary:

Use 5 bolts for tension member-to-gusset plate connection;
use 5 bolts for gusset-plate-to connection angles connection;
use 10 bolts for connection angles to column flange connection;
use edge distances of $1\frac{1}{2}$ in. and bolt spacings of $2\frac{1}{2}$ in. throughout.

(b) ASD solution

Tension member-to-gusset plate connection:

$$P_a = D + L = 120 \text{ kips}$$

Bolt shear strength:

$$A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$$

$$\frac{R_n}{\Omega} = \frac{1}{\Omega} F_{nv} A_b = \frac{1}{2.00}(60)(0.6013) \times 2 \text{ shear planes} = 36.1 \text{ kips/bolt}$$

Check bearing on the gusset plate assuming upper limit controls.

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(2.4dtF_u) = \frac{1}{2.00}(2.4)(7/8)(7/8)(58) = 53.3 \text{ kips/bolt}$$

Slip-critical strength: From AISC Table J3.1, $T_b = 49$ kips

$$R_n = \mu D_u h_{sc} T_b N_s = 0.35(1.13)(1.0)(49)(2) = 38.76 \text{ kips/bolt}$$

$$\frac{R_n}{\Omega} = \frac{38.76}{1.50} = 25.84 \text{ kips/bolt (controls)}$$

Number required = $120/25.84 = 4.64$ bolts. Use 5.

Gusset plate-to-column angles connection:

Bearing on the angles will control, because $2 \times \frac{3}{8}$ in. $< \frac{7}{8}$ in.

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(2.4dtF_u) = \frac{1}{2.00}(2.4)(7/8)(2 \times \frac{3}{8})(58) = 45.7 \text{ kips/bolt}$$

Shear strength and slip-critical strength are same as for tension member-to-gusset plate connection, ∴ use 5 bolts.

Connection angles-to-column flange connection: assume that the slip-critical strength will control. Select the number of bolts based on slip-critical strength, then check tension.

$$T_a = V_a = \frac{1}{\sqrt{2}}(120) = 84.85 \text{ kips}$$

$$\text{Reduction factor} = k_s = 1 - \frac{1.5T_a}{D_u T_b N_b} = 1 - \frac{1.5(84.85)}{1.13(49)N_b}$$

Let $84.85 = N_b\left[25.84\left(1 - \frac{1.5(84.85)}{1.13(49)N_b}\right)\right]$, Solution is: $\{N_b = 5.58\}$

Use 5 rows of 2 bolts for symmetry and to match the arrangement of gusset plate-to-connection angle bolts. Check tension.

Shear/tension load per bolt $= 84.85/10 = 8.49$ kips

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}F_{nt}A_b = \frac{1}{2.00}(90)(0.6013) = 27.1 \text{ kips/bolt} > 8.49 \text{ kips/bolt (OK)}$$

Minimum spacing $= 2\frac{2}{3}d = 2.667(7/8) = 2.33$ in.

Minimum edge distance from AISC Table J3.4 = 1.5 in.

Try $L_e = 1\frac{1}{2}$ in. and $s = 2\frac{1}{2}$ in. for all connection elements and check bearing.

For the angles-to-column connection, the angle thickness will control (3/8 in. < 0.695 in.):

$$h = \frac{7}{8} + \frac{1}{16} = 0.9375 \text{ in.}$$

For the hole nearest the edge, $L_c = L_e - \frac{h}{2} = 1.5 - \frac{0.9375}{2} = 1.031$ in.

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2L_ctF_u) = \frac{1}{2.00}(1.2)(1.031)(3/8)(58) = 13.5 \text{ kips} > 8.49 \text{ kips (OK)}$$

$$\frac{1}{\Omega}(2.4dtF_u) = \frac{1}{2.00}(2.4)(7/8)(3/8)(58) = 22.8 \text{ kips/bolt}$$

For the other bolts, $L_c = s - h = 2.5 - 0.9375 = 1.563$ in.

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2L_ctF_u) = \frac{1}{2.00}(1.2)(1.563)(3/8)(58) = 20.4 \text{ kips} > 8.49 \text{ kips (OK)}$$

For the tension member connection, the gusset plate thickness will control:

For the hole nearest the edge,

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2L_ctF_u) = \frac{1}{2.00}(1.2)(1.031)(7/8)(58) = 31.39$$

$$= 31.4 \text{ kips} > 120/5 = 24 \text{ kips (OK)}$$

$$\frac{1}{\Omega}(2.4dtF_u) = \frac{1}{2.00}(2.4)(7/8)(7/8)(58) = 53.3 \text{ kips/bolt}$$

For the other bolts,

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2L_ctF_u) = \frac{1}{2.00}(1.2)(1.563)(7/8)(58) = 47.6 \text{ kips} > 24 \text{ kips (OK)}$$

For the gusset plate-to-column connection angles,

$$\frac{R_n}{\Omega} = \frac{2(3/8)}{7/8} \times 31.4 = 26.9 \text{ kips/bolt} > 24 \text{ kips/bolt (OK)}$$

Summary:

Use 5 bolts for tension member-to-gusset plate connection;
use 5 bolts for gusset-plate-to connection angles connection;
use 10 bolts for connection angles to column flange connection;
use edge distances of $1\frac{1}{2}$ in. and bolt spacings of $2\frac{1}{2}$ in. throughout.

7.9-5

Let θ = angle that load makes with the horizontal = $\arctan(12.5/13) = 43.88°$

(a) LRFD solution

$$P_u = 1.2D + 1.6L = 1.2(55) + 1.6(145) = 298.0 \text{ kips}$$

$$T_u = 298\cos(43.88°) = 214.8 \text{ kips}, \qquad V_u = 298\sin(43.88°) = 206.6 \text{ kips}$$

Assume that tension controls:

$$F'_{nt} = 1.3(90) - \frac{90}{0.75(48)}f_v \le 90$$

$$= 117 - 2.5f_v \le 90$$

$$\phi F'_{nt} = 0.75(117 - 2.5f_v) \le 0.75(90)$$

$$= 87.75 - 1.875f_v \le 67.5$$

Let $\phi F'_{nt} = \dfrac{214.8}{\Sigma A_b}$ and $f_v = \dfrac{206.6}{\Sigma A_b}$

$$\frac{214.8}{\Sigma A_b} = 87.75 - 1.875\left(\frac{206.6}{\Sigma A_b}\right)$$

$$214.8 = 87.75\Sigma A_b - 387.4 \qquad \Rightarrow \qquad \Sigma A_b = 6.863 \text{ in.}^2$$

$$\text{Required } A_b = \frac{\Sigma A_b}{12} = \frac{6.863}{12} = 0.5719 \text{ in.}^2$$

$$\text{Required diameter} = d_b = \sqrt{\frac{4A_b}{\pi}} = \sqrt{\frac{4(0.5719)}{\pi}} = 0.853 \text{ in.}$$

Try $\frac{7}{8}$-in. diameter bolts, with $A_b = \pi(7/8)^2/4 = 0.6013$ in.^{2}

Check the upper limit on F_t:

$$f_v = \frac{206.6}{\Sigma A_b} = \frac{206.6}{12(0.6013)} = 28.63 \text{ ksi}$$

$$F'_{nt} = 117 - 2.5f_v = 117 - 2.5(28.63) = 45.43 \text{ ksi} < 90 \text{ ksi} \qquad \text{(OK)}$$

Check shear: V_u/bolt = 206.6/12 = 17.2 kips/bolt

$$\phi R_n = \phi F_{nv}A_b = 0.75(48)(0.6013) = 21.6 \text{ kips/bolt} > 17.2 \text{ kips/bolt} \qquad \text{(OK)}$$

Check bearing. $h = \frac{7}{8} + \frac{1}{16} = 0.9375$ in.

For the holes nearest the edge, $L_c = L_e - \frac{h}{2} = 2.5 - \frac{0.9375}{2} = 2.031$ in.

$$\phi R_n = \phi(1.2L_c tF_u) = 0.75(1.2)(2.031)(1.11)(65) = 132 \text{ kips} > 17.2 \text{ kips (OK)}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(7/8)(1.11)(65) = 114 \text{ kips/bolt} > 17.2 \text{ kips (OK)}$$

For the other holes, $L_c = s - h = 4 - 0.9375 = 3.063$ in. Since this is larger than L_c for the edge bolts, no further check is necessary.

Minimum spacing $= 2\frac{2}{3}d = 2.667(7/8) = 2.33$ in. < 4 in. (OK)

Minimum edge distance from AISC Table J3.4 = 1.5 in. < 2.5 in. (OK)

Required $d = 0.853$ in.; use $\frac{7}{8}$-in.-diameter bolts

(b) ASD solution

$$P_a = D + L = 55 + 145 = 200 \text{ kips}$$

$$T_a = 200\cos(43.88°) = 144.2 \text{ kips}, \qquad V_a = 200\sin(43.88°) = 138.6 \text{ kips}$$

Assume that tension controls:

$$F'_{nt} = 1.3(90) - \frac{2.00(90)}{48}f_v \le 90$$

$$= 117 - 3.75f_v \le 90$$

$$\frac{F'_{nt}}{\Omega} = \frac{117 - 3.75f_v}{2.00} \le \frac{90}{2.00}$$

$$= 58.5 - 1.875f_v \le 45$$

Let $\frac{F'_{nt}}{\Omega} = \frac{144.2}{\Sigma A_b}$ and $f_v = \frac{138.6}{\Sigma A_b}$

$$\frac{144.2}{\Sigma A_b} = 58.5 - 1.875\left(\frac{138.6}{\Sigma A_b}\right)$$

$$144.2 = 58.5\Sigma A_b - 259.9 \qquad \Rightarrow \qquad \Sigma A_b = 6.908 \text{ in.}^2$$

$$\text{Required } A_b = \frac{\Sigma A_b}{12} = \frac{6.908}{12} = 0.5757 \text{ in.}^2$$

$$\text{Required diameter} = d_b = \sqrt{\frac{4A_b}{\pi}} = \sqrt{\frac{4(0.5757)}{\pi}} = 0.856 \text{ in.}$$

Try $\frac{7}{8}$-in. diameter bolts, with $A_b = \pi(7/8)^2/4 = 0.6013$ in.^{2}

Check the upper limit on F_t:

$$f_v = \frac{138.6}{\Sigma A_b} = \frac{138.6}{12(0.6013)} = 19.21 \text{ ksi}$$

$$F'_{nt} = 117 - 3.75f_v = 117 - 3.75(19.21) = 44.96 \text{ ksi} < 90 \text{ ksi} \quad \text{(OK)}$$

Check shear: V_a/bolt = 138.6/12 = 11.6 kips/bolt

$$\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{\Omega} = \frac{48(0.6013)}{2.00} = 14.4 \text{ kips/bolt} > 11.6 \text{ kips/bolt} \qquad \text{(OK)}$$

Check bearing. $h = \frac{7}{8} + \frac{1}{16} = 0.9375$ in.

For the holes nearest the edge, $L_c = L_e - \frac{h}{2} = 2.5 - \frac{0.9375}{2} = 2.031$ in.

$$\frac{R_n}{\Omega} = \frac{1.2L_c tF_u}{\Omega} = \frac{1.2(2.031)(1.11)(65)}{2.00} = 87.9 \text{ kips} > 11.6 \text{ kips} \quad \text{(OK)}$$

$$\frac{2.4dtF_u}{\Omega} = \frac{2.4(7/8)(1.11)(65)}{2.00} = 75.8 \text{ kips/bolt} > 11.6 \text{ kips} \quad \text{(OK)}$$

For the other holes, $L_c = s - h = 4 - 0.9375 = 3.063$ in. Since this is larger than L_c for the edge bolts, no further check is necessary.

Minimum spacing $= 2\frac{2}{3}d = 2.667(7/8) = 2.33$ in. < 4 in. (OK)

Minimum edge distance from AISC Table J3.4 = 1.5 in. < 2.5 in. (OK)

Required $d = 0.856$ in.; use $\frac{7}{8}$-in.-diameter bolts

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7.11-1

(a) Tension member gross section strength

$P_n = F_y A_g = 50(2.50) = 125.0$ kips

$\phi_t P_n = 0.90(125) = 112.5$ kips

Net section strength: $U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.428}{13} = 0.9671$

$A_e = A_g U = 2.50(0.9671) = 2.418 \text{ in.}^2$

$P_n = F_u A_e = 65(2.418) = 157.2$ kips

$\phi_t P_n = 0.75(157.2) = 117.9$ kips

Weld strength is

$R_n = 0.707w(0.6F_{EXX}) = 0.707(3/16)(0.6 \times 70) = 5.568$ kips/in.

$\phi R_n = 0.75(5.568) = 4.176$ kips/in.

Base metal shear strength of the plate:

Yielding: $\phi R_n = \phi(0.6F_y t) = 1.00(0.6)(36)(3/8) = 8.1$ kips/in. > 4.176 kips/in.

Rupture: $\phi R_n = \phi(0.6F_u t) = 0.75(0.6)(58)(3/8) = 9.788$ kips/in. > 4.176 kips/in.

Base metal shear strength of the tension member:

Yielding: $\phi R_n = \phi(0.6F_y t) = 1.00(0.6)(50)(0.179) = 5.37$ kips/in. > 4.176 kips/in.

Rupture: $\phi R_n = \phi(0.6F_u t) = 0.75(0.6)(65)(0.179) = 5.236$ kips/in. > 4.176 kips/in.

The weld strength is smaller than the base metal strength.

Total weld strength $= 4.176(13+13) = 108.6$ kips

Weld strength controls overall: $P_u = 108.6$ kips. Let

$1.2D + 1.6(2.5D) = 108.6$, Solution is: $\{D = 20.88\}$

$P = D + L = 20.88 + 2.5(20.88) = 73.1$ kips $\underline{P = 73.1 \text{ kips}}$

(b) ASD solution

Tension member gross section strength

$$P_n = F_y A_g = 50(2.50) = 125.0 \text{ kips}$$

$$\frac{P_n}{\Omega_t} = \frac{125}{1.67} = 74.85 \text{ kips}$$

Net section strength: $U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.428}{13} = 0.9671$

$$A_e = A_g U = 2.50(0.9671) = 2.418 \text{ in.}^2$$

$$P_n = F_u A_e = 65(2.418) = 157.2 \text{ kips}$$

$$\frac{P_n}{\Omega_t} = \frac{157.2}{2.00} = 78.6 \text{ kips}$$

Weld strength:

$$R_n = 0.707w(0.6F_{EXX}) = 0.707(3/16)(0.6 \times 70) = 5.568 \text{ kips/in.}$$

$$\frac{R_n}{\Omega} = \frac{5.568}{2.00} = 2.784 \text{ kips/in.}$$

Base metal shear strength of the plate:

Yielding: $\frac{R_n}{\Omega} = \frac{0.6F_y t}{1.50} = \frac{0.6(36)(3/8)}{1.50} = 5.4 \text{ kips/in} > 2.784 \text{ kips/in.}$

Rupture: $\frac{R_n}{\Omega} = \frac{0.6F_u t}{2.00} = \frac{0.6(58)(3/8)}{2.00} = 6.525 \text{ kips/in.} > 2.784 \text{ kips/in.}$

Base metal shear strength of the tension member:

Yielding: $\frac{R_n}{\Omega} = \frac{0.6F_y t}{1.50} = \frac{0.6(50)(0.179)}{1.50} = 3.58 \text{ kips/in} > 2.784 \text{ kips/in.}$

Rupture: $\frac{R_n}{\Omega} = \frac{0.6F_u t}{2.00} = \frac{0.6(65)(0.179)}{2.00} = 3.491 \text{ kips/in.} > 2.784 \text{ kips/in.}$

The weld strength is smaller than the base metal strength.

Total weld strength $= 2.784(13+13) = 72.38$ kips

Weld strength controls overall: $P_a = 72.38$ kips. $\underline{P = 72.4 \text{ kips}}$

7.11-2

(a) LRFD solution. Gross section: $A_g = 5.12 \text{ in.}^2$

$$\phi_t P_n = 0.90F_yA_g = 0.90(36)(5.12) = 165.9 \text{ kips}$$

Net section: $U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.829}{5} = 0.8342$

$$A_e = A_gU = 5.12(0.8342) = 4.271 \text{ in.}^2$$

$$\phi_t P_n = 0.75F_uA_e = 0.75(58)(4.271) = 185.8 \text{ kips}$$

Weld:

$$\phi R_n = \phi[0.707w(0.6F_{EXX})] = 0.75(0.707)(1/4)(0.6 \times 70) = 5.568 \text{ kips/in.}$$

(Alternate: $\phi R_n = 1.392 \times 4$ sixteenths $= 5.568$ kips/in.)

For the strength of the connection, investigate the two options given in AISC J2.4(c).

1. Use the basic weld strength for both the longitudinal and transverse welds.

$$\phi R_n = 5.568(5 + 5 + 5) = 83.52 \text{ kips (for one angle)}$$

2. Use 0.85 times the basic weld strength for the longitudinal welds and 1.5 times the basic weld strength for the transverse weld.

$$\phi R_n = 0.85(5.568)(5 + 5) + 1.5(5.568)(5) = 89.09 \text{ kips (for one angle)}$$

The larger value controls. For two angles, the total weld strength = $89.09 \times 2 = 178.2$ kips

Check block shear on the gusset plate in lieu of base metal shear strength.

$$A_{gv} = A_{nv} = \frac{3}{8}(5) \times 2 = 3.750 \text{ in.}^2$$

$$A_{nt} = \frac{3}{8}(5) = 1.875 \text{ in.}^2$$

From AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$
$$= 0.6(58)(3.750) + 1.0(58)(1.875) = 239.3 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(3.750) + 1.0(58)(1.875) = 189.8 \text{ kips}$$

Block shear design strength = $\phi R_n = 0.75(189.8) = 142.4$ kips

Block shear controls, and $P_u = 142.4$ kips. Let

$1.2D + 1.6(3D) = 142.4$, Solution is: $\{D = 23.73\}$

$P = D + L = 23.73 + 3(23.73) = 94.92$ kips $P = 94.9$ kips

(b) ASD solution

$$\frac{P_n}{\Omega_t} = \frac{F_y A_g}{\Omega_t} = \frac{36(5.12)}{1.67} = 110.4 \text{ kips}$$

Net section: $U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.829}{5} = 0.8342$

$$A_e = A_g U = 5.12(0.8342) = 4.271 \text{ in.}^2$$

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{\Omega_t} = \frac{1}{2.00}(58)(4.271) = 123.9 \text{ kips}$$

Weld:

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}[0.707w(0.6F_{EXX})] = \frac{1}{2.00}(0.707)(1/4)(0.6 \times 70) = 3.712 \text{ kips/in.}$$

(Alternate: $\frac{R_n}{\Omega} = 0.9279 \times 4$ sixteenths = 3.712 kips/in.)

For the strength of the connection, investigate the two options given in AISC J2.4(c).

1. Use the basic weld strength for both the longitudinal and transverse welds.

$$\frac{R_n}{\Omega} = 3.712(5 + 5 + 5) = 55.68 \text{ kips (for one angle)}$$

2. Use 0.85 times the basic weld strength for the longitudinal welds and 1.5 times the basic weld strength for the transverse weld.

$$\frac{R_n}{\Omega} = 0.85(3.712)(5 + 5) + 1.5(3.712)(5) = 59.39 \text{ kips (for one angle)}$$

The larger value controls. For two angles, the total weld strength = $59.39 \times 2 = 119$ kips

Check block shear on the gusset plate in lieu of base metal shear strength. From the LRFD solution, the nominal block shear strength is

$$R_n = 189.8 \text{ kips}$$

The allowable block shear strength is

$$\frac{R_n}{\Omega} = \frac{189.8}{2.00} = 94.9 \text{ kips}$$

Block shear controls: $\underline{P_a = 94.9 \text{ kips}}$

7.11-3

From Table 2-4 in Part 2 of the *Manual*, for A242 steel, $F_y = 50$ ksi and $F_u = 70$ ksi

(a) LRFD solution

Tension member gross section: $A_g = 0.75(7) = 5.25 \text{ in.}^2$

$$\phi_t P_n = 0.90 F_y A_g = 0.90(50)(5.25) = 236 \text{ kips}$$

Net section: $\phi_t P_n = 0.75 F_u A_e = 0.75(70)(5.25) = 276$ kips

The weld strength is

$\phi R_n = 1.392 \times 8$ sixteenths = 11.14 kips/in.

There is no base metal shear in this connection.

For a 7-in. length, $\phi R_n = 11.14 \times 7 = 77.98$ kips

Total weld strength = $2(77.98) = 156.0$ kips (controls). Let

$$1.2D + 1.6(2D) = 156.0, \text{ Solution is: } \{D = 35.45\}$$

$$P = D + L = 35.45 + 2(35.45) = 106 \text{ kips} \qquad \underline{P = 106 \text{ kips}}$$

(b) ASD solution

Tension member gross section strength: $A_g = 0.75(7) = 5.25$ in.^{2}

$$\frac{P_n}{\Omega_t} = \frac{F_y A_g}{\Omega_t} = \frac{50(5.25)}{1.67} = 157.2 \text{ kips}$$

Net section strength: $\frac{P_n}{\Omega_t} = \frac{F_u A_e}{\Omega_t} = \frac{70(5.25)}{2.00} = 183.8$ kips

Weld strength: $\frac{R_n}{\Omega} = 0.9279 \times 8$ sixteenths = 7.423 kips/in.

Total weld strength = $7.423(7+7) = 103.9$ kips (controls) $\qquad \underline{P = 104 \text{ kips}}$

7.11-4

(a) LRFD solution. Gross section: For the outer member,

$$A_g = (5/16)(3) \times 2 = 1.875 \text{ in.}^2$$

For the inner member,

$$A_g = (1/2)(6) = 3.0 \text{ in.}^2$$

Outer member controls: $\phi_t P_n = 0.90 F_y A_g = 0.90(36)(1.875) = 60.75$ kips

Net section: $A_e = A_g$

$$\phi_t P_n = 0.75 F_u A_e = 0.75(58)(1.875) = 81.56 \text{ kips}$$

Weld: $\phi R_n = 1.392 \times 4$ sixteenths = 5.568 kips/in

1. Basic weld strength for both the longitudinal and transverse welds:

$$\phi R_n = 5.568(3+3+3) \times 2 = 100.2 \text{ kips}$$

2. 0.85 times the basic weld strength for the longitudinal welds and 1.5 times the basic weld strength for the transverse weld:

$$\phi R_n = [0.85(5.568)(3+3) + 1.5(5.568)(3)] \times 2 = 106.9 \text{ kips}$$

Use the larger value of 106.9 kips.

Check block shear on the inner member in lieu of base metal shear strength.

$$A_{gv} = A_{nv} = \frac{1}{2}(3) \times 2 = 3.0 \text{ in.}^2$$

$$A_{nt} = \frac{1}{2}(3) = 1.5 \text{ in.}^2$$

From AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(3.0) + 1.0(58)(1.5) = 191.4 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(3.0) + 1.0(58)(1.5) = 151.8 \text{ kips}$$

Block shear design strength $= \phi R_n = 0.75(151.8) = 113.9$ kips

Gross section tensile strength controls, and $P_u = 60.75$ kips. Let

$1.2D + 1.6(D) = 60.75$, Solution is: $\{D = 21.70\}$

$P = D + L = 21.70 + 21.70 = 43.4$ kips $\underline{P = 43.4 \text{ kips}}$

(b) ASD solution

Gross section: $\dfrac{P_n}{\Omega_t} = \dfrac{F_yA_g}{\Omega_t} = \dfrac{36(1.875)}{1.67} = 40.4$ kips

Net section:

$$\frac{P_n}{\Omega_t} = \frac{F_uA_e}{\Omega_t} = \frac{58(1.875)}{2.00} = 54.4 \text{ kips}$$

Weld: $\dfrac{R_n}{\Omega} = 0.9279 \times 4$ sixteenths $= 3.712$ kips/in.

1. Basic weld strength for both the longitudinal and transverse welds:

$$\frac{R_n}{\Omega} = 3.712(3 + 3 + 3) \times 2 = 66.82 \text{ kips}$$

2. 0.85 times the basic weld strength for the longitudinal welds and 1.5 times the basic weld strength for the transverse weld:

$$\frac{R_n}{\Omega} = [0.85(3.712)(3 + 3) + 1.5(3.712)(3)] \times 2 = 71.27 \text{ kips}$$

Use the larger value of 71.27 kips.

Check block shear on the gusset plate in lieu of base metal shear strength. From the LRFD solution, the nominal block shear strength is

$$R_n = 151.8 \text{ kips}$$

The allowable block shear strength is

$$\frac{R_n}{\Omega} = \frac{151.8}{2.00} = 75.9 \text{ kips}$$

Gross section tensile strength controls: $\underline{P_a = 40.4 \text{ kips}}$

7.11-5

From AISC Table J2.4, the minimum weld size is 3/16 inch (based on the angle thickness). Maximum size = 5/16 − 1/16 = 1/4 in.

(a) LRFD solution: $P_u = 1.2D + 1.6L = 1.2(30) + 1.6(75) = 156.0$ kips

Try w = ¼ in., $\phi R_n = 1.392 \times 4$ sixteenths = 5.568 kips/in.

The base metal shear yield strength (gusset plate controls) is

$$0.6F_y t = 0.6(36)\left(\frac{3}{8}\right) = 8.1 \text{ kips/in.}$$

Shear rupture strength is $0.45F_u t = 0.45(58)\left(\frac{3}{8}\right) = 9.788$ kips/in.

Base metal shear yield strength for the angles is

$$0.6F_y t = 0.6(50)\left(\frac{5}{16} \times 2\right) = 18.75 \text{ kips/in.}$$

and the shear rupture strength is $0.45F_u t = 0.45(65)\left(\frac{3}{8} \times 2\right) = 21.94$ kips/in.

The weld strength of 5.568 kips/in. governs. Both longitudinal and transverse welds will be used. To determine the required length of the longitudinal welds, investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\text{total required length of weld} = \frac{156}{5.568} = 28.02 \text{ in.}$$

$$\text{length of longitudinal welds} = \frac{28.02 - 5}{2} = 11.51 \text{ in.}$$

For the second option, the strength of the longitudinal welds is

$$0.85(5.568) = 4.733 \text{ kips/in.}$$

and the strength of the transverse weld is

$$1.5(5.568) = 8.352 \text{ kips/in.}$$

The load to be carried by the longitudinal welds is

$$156 - 5(8.352) = 114.2 \text{ kips}$$

so the required length of the longitudinal welds is

$$\frac{114.2}{2(4.733)} = 12.06 \text{ in.}$$

The first option requires shorter longitudinal welds. Try a 5-inch transverse weld and two12-inch longitudinal welds. Check the block shear strength of the gusset plate.

$$A_{gv} = A_{nv} = 2 \times \frac{3}{8}(12) = 9.0 \text{ in.}^2 \qquad A_{nt} = \frac{3}{8}(5) = 1.875 \text{ in.}^2$$

$$R_n = 0.6F_u A_{nv} + U_{bs} F_u A_{nt} = 0.6(58)(9.0) + 1.0(58)(1.875) = 422.0 \text{ kips}$$

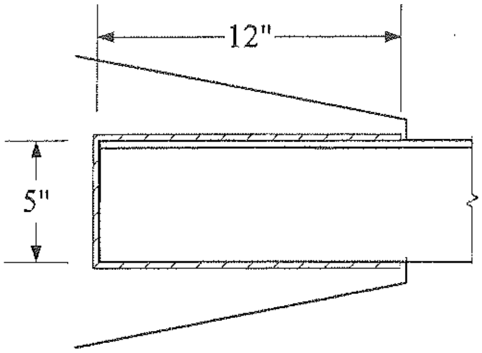
with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(9.0) + 1.0(58)(1.875) = 303.2 \text{ kips (controls)}$$

The design strength is

$$\phi R_n = 0.75(303.2) = 227 \text{ kips} > 156 \text{ kips} \qquad \text{(OK)}$$

Use ¼-in. fillet welds as shown.



(b) ASD solution: $P_a = D + L = 30 + 75 = 105$ kips

Try w = ¼ in., $R_n/\Omega = 0.9279 \times 4$ sixteenths = 3.712 kips/in.

Base metal shear strength (gusset plate controls): The allowable shear yield strength is

$$0.4F_yt = 0.4(36)\left(\frac{3}{8}\right) = 5.4 \text{ kips/in.}$$

and the allowable shear rupture strength is

$$0.3F_ut = 0.3(58)\left(\frac{3}{8}\right) = 6.525 \text{ kips/in.}$$

The weld strength of 3.712 kips/in. governs. Both longitudinal and transverse welds will be used. To determine the required length of the longitudinal welds, investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\text{total required length of weld} = \frac{105}{3.712} = 28.29 \text{ in.}$$

$$\text{length of longitudinal welds} = \frac{28.29 - 5}{2} = 11.65 \text{ in.}$$

For the second option, the strength of the longitudinal welds is

$$0.85(3.712) = 3.155 \text{ kips/in.}$$

and the strength of the transverse weld is

$$1.5(3.712) = 5.568 \text{ kips/in.}$$

The load to be carried by the longitudinal welds is

$$105 - 5(5.568) = 77.16 \text{ kips}$$

so the required length of the longitudinal welds is

$$\frac{77.16}{2(3.155)} = 12.23 \text{ in.}$$

The first option requires shorter longitudinal welds. Try a 5-inch transverse weld and two 12-inch longitudinal welds. Check the block shear strength of the gusset plate. From the LRFD solution, $R_n = 303.2$ kips, and the allowable strength is

$$\frac{R_n}{\Omega} = \frac{303.2}{2.00} = 152 \text{ kips} > 105 \text{ kips} \qquad \text{(OK)}$$

Use ¼-in. fillet welds as shown in the figure above (in LRFD solution).

7.11-6

From AISC Table J2.4, the minimum weld size is 1/8 inch (based on the angle thickness). Maximum size = 1/4 – 1/16 = 3/16 in.

(a) LRFD solution: $P_u = 1.2D + 1.6L = 1.2(7) + 1.6(14) = 30.8$ kips

For one angle, $P_u = 30.8/2 = 15.4$ kips

Try $w = 1/8$ in., $\phi R_n = 1.392 \times 2$ sixteenths = 2.784 kips/in.

Compare the base metal shear strength of the gusset plate with that of the two angles. The plate thickness of 3/8 in. is smaller than 2 × 1/4 = 1/2 inch. Therefore, the weld strength cannot exceed the base metal shear strength for a thickness of 3/8 ÷ 2 = 3/16 in.

The base metal shear yield strength is

$$0.6F_y t = 0.6(36)\left(\frac{3}{16}\right) = 4.05 \text{ kips/in.}$$

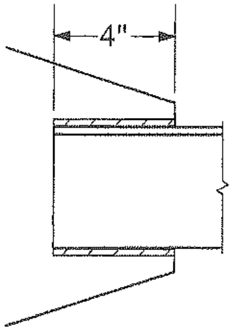
Shear rupture strength is $0.45F_u t = 0.45(58)\left(\frac{3}{16}\right) = 4.894$ kips/in.

The weld strength of 2.784 kips/in. governs. Try two longitudinal welds:

$$\text{total required length of weld} = \frac{30.8}{2.784} = 11.06 \text{ in.}$$

$$\text{length of longitudinal welds} = \frac{11.06 - 4}{2} = 3.53 \text{ in.}$$

Use 1/8-in. fillet welds as shown.



(b) ASD solution: $P_a = D + L = 7 + 14 = 21$ kips

For one angle, $P_a = 21/2 = 10.5$ kips

Try $w = 1/8$ in., $R_n/\Omega = 0.9279 \times 2$ sixteenths = 1.856 kips/in.

Compare the base metal shear strength of the gusset plate with that of the two angles. The plate thickness of 3/8 in. is smaller than 2 × 1/4 = 1/2 inch. Therefore, the weld strength cannot exceed the base metal shear strength for a thickness of 3/8 ÷ 2 = 3/16 in.

The base metal shear yield strength is

$$0.4F_y t = 0.4(36)\left(\frac{3}{16}\right) = 2.7 \text{ kips/in.}$$

and the allowable shear rupture strength is

$$0.3F_u t = 0.3(58)\left(\frac{3}{16}\right) = 3.263 \text{ kips/in.}$$

The weld strength of 1.856 kips/in. governs. Try two longitudinal welds.

$$\text{total required length of weld} = \frac{10.5}{1.856} = 5.657 \text{ in.}$$

$$\text{length of longitudinal welds} = \frac{5.657}{2} = 2.829 \text{ in.}$$

From AISC J2.2b, the length of the welds must be at least equal to the distance between them, therefore, use a length of 4 inches.

Use 1/8-in. fillet welds as shown in the figure above (in LRFD solution).

7.11-7

From AISC Table J2.4, the minimum weld size is 3/16 inch (based on the gusset plate thickness). Maximum size = 0.400 − 1/16 = 0.338 in., or 5/16 in. (to the nearest 1/16 in.)

(a) LRFD solution: $P_u = 1.2D + 1.6L = 1.2(48) + 1.6(120) = 249.6$ kips

Try $w = 3/16$ in., $\phi R_n = 1.392 \times 3$ sixteenths = 4.176 kips/in.

The base metal shear yield strength (gusset plate controls) is

$0.6F_yt = 0.6(36)\left(\frac{3}{8}\right) = 8.1$ kips/in.

Shear rupture strength is $0.45F_ut = 0.45(58)\left(\frac{3}{8}\right) = 9.788$ kips/in.

The weld strength of 4.176 kips/in. governs. Both longitudinal and transverse welds will be used. To determine the required length of the longitudinal welds, investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

total required length of weld = $\frac{249.6}{4.176} = 59.77$ in.

length of longitudinal welds = $\frac{59.77 - 9.00}{2} = 25.39$ in.

For the second option, the strength of the longitudinal welds is

$0.85(4.176) = 3.550$ kips/in.

and the strength of the transverse weld is

$1.5(4.176) = 6.264$ kips/in.

The load to be carried by the longitudinal welds is

$249.6 - 9(6.264) = 193.2$ kips

so the required length of the longitudinal welds is

$$\frac{193.2}{2(3.550)} = 27.21 \text{ in.}$$

To minimize the length of the connection, use the maximum weld size permitted. Use $w = 5/16$ in.

$\phi R_n = 1.392 \times 5$ sixteenths = 6.96 kips/in

First, assuming the same strength for both the longitudinal and transverse welds,

total required length of weld = $\frac{249.6}{6.96} = 35.86$ in.

length of longitudinal welds = $\frac{35.86 - 9.00}{2} = 13.43$ in.

For the second option, the strength of the longitudinal welds is

$0.85(6.96) = 5.916$ kips/in.

and the strength of the transverse weld is

$1.5(6.96) = 10.44$ kips/in.

The load to be carried by the longitudinal welds is

$249.6 - 9(10.44) = 155.6$ kips

so the required length of the longitudinal welds is

$$\frac{155.6}{2(5.916)} = 13.15 \text{ in.}$$

The second option requires slightly shorter longitudinal welds. Try a 9-inch transverse weld and two13½-inch longitudinal welds. Check the block shear strength of the gusset plate.

$$A_{gv} = A_{nv} = 2 \times \frac{3}{8}(13.5) = 10.13 \text{ in.}^2 \qquad A_{nt} = \frac{3}{8}(9) = 3.375 \text{ in.}^2$$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} = 0.6(58)(10.13) + 1.0(58)(3.375) = 548.3 \text{ kips}$$

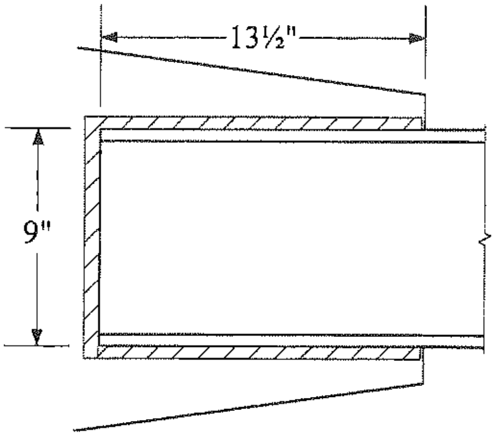
with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(10.13) + 1.0(58)(3.375) = 414.6 \text{ kips (controls)}$$

The design strength is

$$\phi R_n = 0.75(414.6) = 311 \text{ kips} > 249.6 \text{ kips} \qquad \text{(OK)}$$

Use 5/16-in. fillet welds as shown.



(b) ASD solution: $P_a = 48 + 120 = 168$ kips

To minimize the length of the connection, use the maximum weld size permitted. Use $w = 5/16$ in.

$$R_n/\Omega = 0.9279 \times 5 \text{ sixteenths} = 4.640 \text{ kips/in.}$$

Base metal shear strength (gusset plate controls): The allowable shear yield strength is

$$0.4F_yt = 0.4(36)\left(\frac{3}{8}\right) = 5.4 \text{ kips/in.}$$

and the allowable shear rupture strength is

$$0.3F_ut = 0.3(58)\left(\frac{3}{8}\right) = 6.525 \text{ kips/in.}$$

The weld strength of 4.640 kips/in. governs. Both longitudinal and transverse welds will be used. To determine the required length of the longitudinal welds, investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\text{total required length of weld} = \frac{168}{4.640} = 36.21 \text{ in.}$$

$$\text{length of longitudinal welds} = \frac{36.21 - 9}{2} = 13.61 \text{ in.}$$

For the second option, the strength of the longitudinal welds is

$$0.85(4.640) = 3.944 \text{ kips/in.}$$

and the strength of the transverse weld is

$$1.5(4.640) = 6.96 \text{ kips/in.}$$

The load to be carried by the longitudinal welds is

$$168 - 9(6.96) = 105.4 \text{ kips}$$

so the required length of the longitudinal welds is

$$\frac{105.4}{2(3.944)} = 13.36 \text{ in.}$$

The second option requires shorter longitudinal welds. Try a 9-inch transverse weld and two 13½-inch longitudinal welds. Check the block shear strength of the gusset plate. From the LRFD solution, $R_n = 414.6$ kips, and the allowable strength is

$$\frac{R_n}{\Omega} = \frac{414.6}{2.00} = 207 \text{ kips} > 168 \text{ kips} \qquad \text{(OK)}$$

Use 5/16-in. fillet welds as shown in the figure above (in LRFD solution).

7.11-8

(a) LRFD solution

Tension member gross section: $\phi_t P_n = 0.90 F_y A_g = 0.90(36)(1.94) = 62.86$ kips

For the net section, assume $U = 0.80$: $A_e = A_g U = 1.94(0.80) = 1.552 \text{ in.}^2$

$$\phi_t P_n = 0.75 F_u A_e = 0.75(58)(1.552) = 67.51 \text{ kips.} \quad \therefore \text{ use } P_u = 62.86 \text{ kips}$$

For $t = 1/4$ in., min. $w = 1/8$ in., and max $w = 1/4 - 1/16 = 3/16$ in.

Try two longitudinal E70 fillet welds, and to minimize the length of the connection, try $w = 3/16$ inch.

$$\phi R_n = 1.392 \times 3 \text{ sixteenths} = 4.176 \text{ kips/in.}$$

The base metal shear yield strength (angle controls) is

$$0.6 F_y t = 0.6(36)\left(\frac{1}{4}\right) = 5.4 \text{ kips/in.}$$

Shear rupture strength is $0.45 F_u t = 0.45(58)\left(\frac{1}{4}\right) = 6.525$ kips/in.

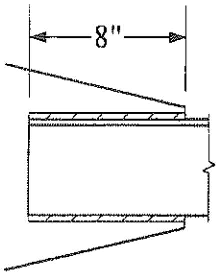
The weld strength of 4.176 kips/in. governs.

$$\text{Required length} = \frac{62.86}{4.176} = 15.1 \text{ in.}$$

Check assumed value of U :

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.08}{8} = 0.865 > \text{assumed value of } 0.80 \qquad \text{(OK)}$$

Use $\frac{3}{16}$-in. E70 fillet welds as shown.



(b) ASD solution

Tension member gross section: $P_n/\Omega_t = F_yA_g/\Omega_t = 36(1.94)/1.67 = 41.82$ kips

For the net section, assume $U = 0.80$: $A_e = A_gU = 1.94(0.80) = 1.552 \text{ in.}^2$

$P_n/\Omega_t = F_uA_e/2.00 = 58(1.552)/2.00 = 45.01$ kips. $\therefore$ use $P_a = 41.82$ kips

For $t = 1/4$ in., min. $w = 1/8$ in., and max $w = 1/4 - 1/16 = 3/16$ in.

Try two longitudinal E70 fillet welds, and to minimize the length of the connection, try $w = 3/16$ inch.

$R_n/\Omega = 0.9279 \times 3$ sixteenths $= 2.784$ kips/in.

The base metal shear yield strength (angle controls) is

$$0.4F_yt = 0.4(36)\left(\frac{1}{4}\right) = 3.6 \text{ kips/in.}$$

Shear rupture strength is $0.3F_ut = 0.3(58)\left(\frac{1}{4}\right) = 4.35$ kips/in.

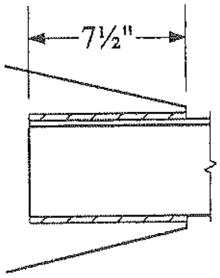
The weld strength of 2.784 kips/in. governs.

$$\text{Required length} = \frac{41.82}{2.784} = 15.0 \text{ in.}$$

Check assumed value of U :

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.08}{7.5} = 0.856 > \text{assumed value of } 0.80 \qquad \text{(OK)}$$

Use $\frac{3}{16}$-in. E70 fillet welds as shown.



7.11-9

(a) LRFD Solution

$P_u = 1.2(12) + 1.6(36) = 72.0$ kips or $72/2 = 36$ kips/angle

$$\text{Req'd } A_g = \frac{P_u}{0.90F_y} = \frac{36}{0.90(36)} = 1.11 \text{ in.}^2$$

$$\text{Req'd } A_e = \frac{P_u}{0.75F_u} = \frac{36}{0.75(58)} = 0.828 \text{ in.}^2$$

$$\text{Min. } r = \frac{L}{300} = \frac{16(12)}{300} = 0.64 \text{ in.}$$

Try 2L2½ × 2½ × ¼, $A_g = 1.19$ in.^{2} (for one angle) > 1.11 in.^{2} (OK)

From the properties table for the double-angle section, $r_{\text{min}} = 0.764$ in.

Net section: Assume $U = 0.80$:

$A_e = A_gU = 1.19(0.80) = 0.952$ in.^{2} > 0.828 in.^{2} (OK)

Weld size: min. $w = 1/8$ in. and max $w = \frac{1}{4} - \frac{1}{16} = \frac{3}{16}$ in.

Try $w = 1/8$ in., $\phi R_n = 1.392 \times 2$ sixteenths = 2.784 kips/in.

The base metal shear yield strength (angle controls) is

$$0.6F_yt = 0.6(36)\left(\frac{1}{4}\right) = 5.4 \text{ kips/in.}$$

Shear rupture strength is $0.45F_ut = 0.45(58)\left(\frac{1}{4}\right) = 6.525$ kips/in.

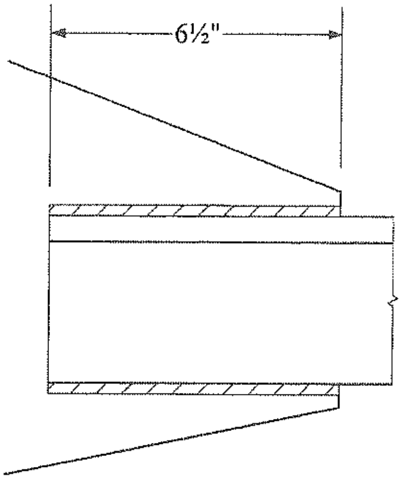
The weld strength of 2.784 kips/in. governs.

Required length = $\frac{36}{2.784} = 12.93$ in., try two 6½-in. longitudinal welds.

Check assumed value of U :

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.711}{6.5} = 0.891 > \text{assumed value of } 0.80 \qquad \text{(OK)}$$

Use 2L2½ × 2½ × ¼, welded with $\frac{1}{8}$-in. E70 fillet welds as shown.



(b) ASD solution

$P_a = 12 + 36 = 48$ kips or $48/2 = 24$ kips/angle

$$\text{Req'd } A_g = \frac{P_a}{0.6F_y} = \frac{24}{0.6(36)} = 1.11 \text{ in.}^2$$

$$\text{Req'd } A_e = \frac{P_a}{0.5F_u} = \frac{24}{0.5(58)} = 0.828 \text{ in.}^2$$

$$\text{Min. } r = \frac{L}{300} = \frac{16(12)}{300} = 0.64 \text{ in.}$$

Try 2L2½ × 2½ × ¼, $A_g = 1.19$ in.^{2} (for one angle) > 1.11 in.^{2} (OK)

From the properties table for the double-angle section, $r_{\min} = 0.764$ in.

Net section: Assume $U = 0.80$:

$$A_e = A_g U = 1.19(0.80) = 0.952 \text{ in.}^2 > 0.828 \text{ in.}^2 \text{ (OK)}$$

Weld size: min. $w = 1/8$ in. and max $w = \frac{1}{4} - \frac{1}{16} = \frac{3}{16}$ in.

Try $w = 1/8$ in., $R_n/\Omega = 0.9279 \times 2$ sixteenths $= 1.856$ kips/in.

The base metal shear yield strength (angle controls) is

$$0.4F_y t = 0.4(36)\left(\frac{1}{4}\right) = 3.6 \text{ kips/in.}$$

Shear rupture strength is $0.3F_u t = 0.3(58)\left(\frac{1}{4}\right) = 4.35$ kips/in.

The weld strength of 1.856 kips/in. governs.

Required length $= \dfrac{24}{1.856} = 12.93$ in., try two 6½-in. longitudinal welds.

Check assumed value of U :

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.711}{6.5} = 0.891 > \text{assumed value of } 0.80 \qquad \text{(OK)}$$

Use 2L2½ × 2½ × ¼, welded with $\frac{1}{8}$-in. E70 fillet welds as shown above (in LRFD solution).

7.11-10

(a) LRFD solution

From load combination 2, $P_u = 1.2D + 1.6L = 1.2(54) + 1.6(80) = 192.8$ kips

From load combination 4,

$$P_u = 1.2D + 1.6W + 0.5L = 1.2(54) + 1.6(75) + 0.5(80) = 224.8 \text{ kips (controls)}$$

$$\text{Req'd } A_g = \frac{P_u}{0.9F_y} = \frac{224.8}{0.9(50)} = 5.00 \text{ in.}^2$$

$$\text{Req'd } A_e = \frac{P_u}{0.75F_u} = \frac{224.8}{0.75(65)} = 4.61 \text{ in.}^2$$

$$\text{Min. } r = \frac{L}{300} = \frac{17.5(12)}{300} = 0.7 \text{ in.}$$

Try C10 × 20, $A_g = 5.87 \text{ in.}^2 > 5.00 \text{ in.}^2$ (OK)

$r_{\min} = 0.690$ in. ≈ 0.7 in. (OK)

Net section: Assume $U = 0.80$:

$$A_e = A_gU = 5.87(0.80) = 4.70 \text{ in.}^2 > 4.61 \text{ in.}^2 \text{ (OK)}$$

Weld size: $t_{PL} = 3/8$ in., $t_w = 0.379$ in., $w_{\min} = \dfrac{3}{16}$ in.

Try a $\frac{3}{16}$-in. fillet weld; use E70 electrodes.

$\phi R_n = 1.392 \times 3$ sixteenths $= 4.176$ kips/in. kips/in.

The base metal shear yield strength (gusset plate controls) is

$$0.6F_yt = 0.6(36)\left(\frac{3}{8}\right) = 8.1 \text{ kips/in.}$$

Shear rupture strength is $0.45F_ut = 0.45(58)\left(\frac{3}{8}\right) = 9.788$ kips/in.

The weld strength of 4.176 kips/in. governs. Both longitudinal and transverse welds will be used. To determine the required length of the longitudinal welds, investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\text{total required length of weld} = \frac{224.8}{4.176} = 53.83 \text{ in.}$$

$$\text{length of longitudinal welds} = \frac{53.83 - 10.0}{2} = 21.9 \text{ in.}$$

For the second option, the strength of the longitudinal welds is

$$0.85(4.176) = 3.550 \text{ kips/in.}$$

and the strength of the transverse weld is

$$1.5(4.176) = 6.264 \text{ kips/in.}$$

The load to be carried by the longitudinal welds is

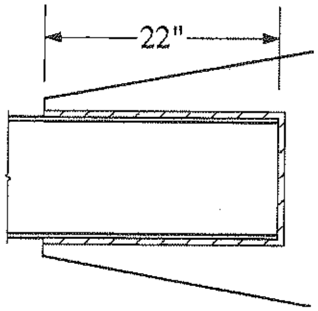
$$224.8 - 10.0(6.264) = 162.2 \text{ kips}$$

so the required length of the longitudinal welds is

$$\frac{162.2}{2(3.550)} = 22.9 \text{ in}$$

The first option results in a shorter connection. Use a transverse weld and two 22-inch longitudinal welds. Check assumed value of U:

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.606}{22} = 0.973 > \text{assumed value of } 0.80 \qquad \text{(OK)}$$



Check block shear on gusset plate.

The shear areas are $A_{gv} = A_{nv} = \frac{3}{8}(22)(2) = 16.5 \text{ in.}^2$

The tension area is $A_{nt} = \frac{3}{8}(10) = 3.75 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(16.5) + 1.0(58)(3.75) = 791.7 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(16.5) + 1.0(58)(3.75) = 573.9 \text{ kips}$$

Use $R_n = 573.9$ kips, $\phi R_n = 0.75(573.9) = 430 \text{ kips} > 224.8 \text{ kips}$ (OK)

Use a C10 × 20, welded with $\frac{3}{16}$-in. E70 fillet welds as shown.

(b) ASD solution

Load combination 6 controls:

$$P_a = D + 0.75W + 0.75L = 54 + 0.75(75) + 0.75(80) = 170.3 \text{ kips}$$

$$\text{Req'd } A_g = \frac{P_a}{0.6F_y} = \frac{170.3}{0.6(50)} = 5.68 \text{ in.}^2$$

$$\text{Req'd } A_e = \frac{P_a}{0.5F_u} = \frac{170.3}{0.5(65)} = 5.24 \text{ in.}^2$$

$$\text{Min. } r = \frac{L}{300} = \frac{17.5(12)}{300} = 0.7 \text{ in.}$$

Try C10 × 20, $A_g = 5.87 \text{ in.}^2 > 5.68 \text{ in.}^2$ (OK)

$r_{\min} = 0.690 \text{ in.} \approx 0.7 \text{ in.}$ (OK)

Net section: Assume $U = 0.90$:

$$A_e = A_g U = 5.87(0.90) = 5.28 \text{ in.}^2 > 5.24 \text{ in.}^2 \text{ (OK)}$$

Weld size: $t_{PL} = 3/8$ in., $t_w = 0.379$ in.

$$w_{\min} = \frac{3}{16} \text{ in.}, \quad w_{\max} = 0.379 - 1/16 = 0.3165 \text{ in. or } 5/16 \text{ in.}$$

Try a ¼-in. fillet weld; use E70 electrodes.

$$\phi R_n = 0.9279 \times 4 \text{ sixteenths} = 3.712 \text{ kips/in.}$$

The base metal shear yield strength (gusset plate controls) is

$$0.4F_y t = 0.4(36)\left(\frac{3}{8}\right) = 5.4 \text{ kips/in.}$$

Shear rupture strength is $0.3F_u t = 0.3(58)\left(\frac{3}{8}\right) = 6.525$ kips/in.

The weld strength of 3.712 kips/in. governs. Both longitudinal and transverse welds will be used. To determine the required length of the longitudinal welds, investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\text{total required length of weld} = \frac{170.3}{3.712} = 45.88 \text{ in.}$$

$$\text{length of longitudinal welds} = \frac{45.88 - 10.0}{2} = 17.9 \text{ in.}$$

For the second option, the strength of the longitudinal welds is

$$0.85(3.712) = 3.155 \text{ kips/in.}$$

and the strength of the transverse weld is

$$1.5(3.712) = 5.568 \text{ kips/in.}$$

The load to be carried by the longitudinal welds is

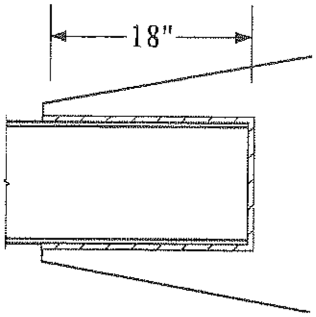
$$170.3 - 10.0(5.568) = 114.6 \text{ kips}$$

so the required length of the longitudinal welds is

$$\frac{114.6}{2(3.155)} = 18.2 \text{ in}$$

The first option results in a shorter connection. Use a transverse weld and two 18-inch longitudinal welds. Check assumed value of U:

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{0.606}{18} = 0.966 > \text{assumed value of } 0.90 \qquad \text{(OK)}$$



Check block shear on gusset plate.

The shear areas are $A_{gv} = A_{nv} = \frac{3}{8}(18)(2) = 13.5 \text{ in.}^2$

The tension area is $A_{nt} = \frac{3}{8}(10) = 3.75 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(13.5) + 1.0(58)(3.75) = 687.3 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(13.5) + 1.0(58)(3.75) = 509.1 \text{ kips}$$

Use $R_n = 509.1$ kips

$$\frac{R_n}{\Omega} = \frac{509.1}{2.00} = 255 \text{ kips} > 170.3 \text{ kips} \qquad \text{(OK)}$$

Use a C10 × 20, welded with ¼-in. E70 fillet welds as shown.

CHAPTER 8 - ECCENTRIC CONNECTIONS

8.2-1

Direct shear components:

$$P_x = \frac{3}{5}(15) = 9 \text{ kips}, \qquad P_y = \frac{4}{5}(15) = 12 \text{ kips}$$

$$p_{cx} = \frac{9}{4} = 2.25 \text{ kips} \rightarrow \qquad p_{cy} = \frac{12}{4} = 3 \text{ kips} \downarrow$$

Eccentricity: $e_x = 12$ in., $e_y = 9 + 1.5 - 4.5 = 6$ in.

$$M = 9(6) + 12(12) = 198 \text{ in.-kips} \curvearrowleft$$

$$\sum(x^2 + y^2) = 2[(4.5)^2 + (1.5)^2] = 45.0 \text{ in.}^2$$

Top bolt is critical. $x = 0$, $y = 9/2 = 4.5$ in.

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{198(4.5)}{45} = 19.8 \text{ kips} \rightarrow$$

$$\sum p_x = 2.25 + 19.8 = 22.05 \text{ kips} \rightarrow \qquad \sum p_y = 3 \text{ kips} \downarrow$$

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(22.05)^2 + (3)^2} = 22.25 \text{ kips} \qquad \underline{p = 22.3 \text{ kips}}$$

8.2-2

Eccentricity: $e_x = 3 + 3 + 4 = 10$ in., $e_y = \dfrac{5.5}{2} = 2.75$ in.

$$\sum(x^2 + y^2) = 2[(6)^2 + (3)^2 + (3)^2 + (6)^2] + 10(2.75)^2 = 255.6 \text{ in.}^2$$

(a) Direct shear components:

$$P_x = P_u \cos 30° = 0.866 P_u, \qquad P_y = P_u \sin 30° = 0.5 P_u$$

$$p_{cx} = \frac{0.866 P_u}{10} = 0.0866 P_u \text{ kips} \rightarrow \qquad p_{cy} = \frac{0.5 P_u}{10} = 0.05 P_u \text{ kips} \downarrow$$

$$M = 0.5 P_u(10) + 0.866 P_u(2.75) = 7.382 P_u \text{ in.-kips} \curvearrowleft$$

Top right bolt is critical. $x = 6$ in., $y = 5.5/2 = 2.75$ in.

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{7.382 P_u(2.75)}{255.6} = 7.942 \times 10^{-2} P_u \text{ kips} \rightarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2 + y^2)} = \frac{7.382 P_u(6)}{255.6} = 0.1733 P_u \text{ kips} \rightarrow$$

$$\sum p_x = 0.0866 P_u + 7.942 \times 10^{-2} P_u = 0.166 P_u \text{ kips} \rightarrow$$

$$\sum p_y = 0.05 P_u + 0.1733 P_u = 0.2233 P_u \text{ kips} \downarrow$$

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(0.166 P_u)^2 + (0.2233 P_u)^2} = 0.2782 P_u \text{ kips}$$

Slip-critical strength will control over shear. Assuming Class A surfaces,

$$R_n = \mu D_u h_{sc} T_b N_s = 0.35(1.13)(1.0)(28)(1) = 11.07 \text{ kips/bolt}$$

$$\phi R_n = 1.0(11.07) = 11.07 \text{ kips/bolt}$$

Let $0.2782P_u = 11.07 \quad \Rightarrow \quad \underline{P_u = 39.8 \text{ kips}}$

8.2-3

Direct shear component: $p_{cy} = \frac{40}{5} = 8 \text{ kips} \downarrow$

Determine location of centroid with respect to lower left bolt:

$$\bar{x} = \frac{3(3)}{5} = 1.8 \text{ in.,} \qquad \bar{y} = \frac{3+2(7)}{5} = 3.4 \text{ in.}$$

Eccentricity: $e_x = 3 + 2 + 8 - 1.8 = 11.2 \text{ in.}$

$$M = 40(11.2) = 448.0 \text{ in.-kips} \curvearrowright$$

$$\sum(x^2+y^2) = (1.8)^2(2) + (3-1.8)^2(3) + (3.4)^2(2) + (3.4-3)^2 + (7-3.4)^2(2)$$
$$= 60.0 \text{ in.}^2$$

Top right bolt is critical. $x = 3 - 1.8 = 1.2 \text{ in.,} \qquad y = 3 + 4 - 3.4 = 3.6 \text{ in.}$

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{448(3.6)}{60} = 26.88 \text{ kips} \rightarrow$$

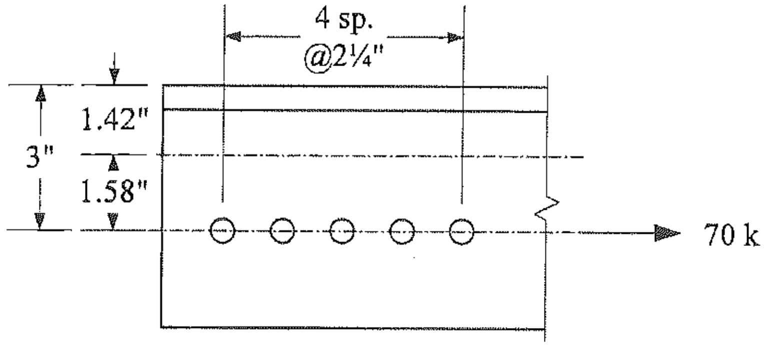
$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{448(1.2)}{60} = 8.96 \downarrow$$

$$\sum p_x = 26.88 \text{ kips} \rightarrow \qquad \sum p_y = 8 + 8.96 = 16.96 \text{ kips} \downarrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(26.88)^2 + (16.96)^2} = 31.8 \text{ kips} \qquad \underline{p = 31.8 \text{ kips}}$$

8.2-4

Direct shear component: $p_{cx} = \frac{70}{5} = 14.0 \text{ kips} \rightarrow$



$$M = 70(1.58) = 110.6 \text{ in.-kips}$$

$$\sum(x^2+y^2) = (2.25)^2(2) + (4.5)^2(2) = 50.63 \text{ in.}^2$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{110.6(4.5)}{50.63} = 9.83 \text{ kips} \uparrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(14.0)^2 + (9.83)^2} = 17.11 \text{ kips}$$

Without considering eccentricity, $p = p_{cx} = 14.0$ kips

Difference = 17.11 − 14.0 = 3.11 kips (22%) Difference = 3.11 kips

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8.2-5

Direct shear components:

$$p_{cx} = \frac{20}{5} = 4 \text{ kips} \leftarrow \qquad p_{cy} = \frac{40}{5} = 8 \text{ kips} \downarrow$$

Determine location of centroid with respect to lower right bolt:

$$\bar{x} = \frac{2(3)}{5} = 1.2 \text{ in.}, \qquad \bar{y} = 3 \text{ in.}$$

Eccentricity: $e_x = 1.2 + 6 = 7.2$ in., $e_y = 3 + 2 = 5$ in.

$$M = 40(7.2) - 20(5) = 188.0 \text{ in.-kips} \curvearrowright$$

$$\sum(x^2+y^2) = 3(1.2)^2 + 2(1.8)^2 + 4(3)^2 = 46.8 \text{ in.}^2$$

Lower right bolt is critical. $x = 1.2$ in., $y = 3$ in.

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{188(3)}{46.8} = 12.05 \text{ kips} \leftarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{188(1.2)}{46.8} = 4.821 \text{ kips} \downarrow$$

$$\sum p_x = 4 + 12.05 = 16.05 \text{ kips} \leftarrow \qquad \sum p_y = 8 + 4.821 = 12.82 \text{ kips} \downarrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(16.05)^2 + (12.82)^2} = 20.5 \text{ kips}$$ $p = 20.5$ kips

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8.2-6

Determine location of centroid with respect to lower left bolt:

$$\bar{x} = \frac{3(1) + 6(1)}{6} = 1.5 \text{ in.}, \qquad \bar{y} = \frac{3(1) + 6(1) + 9(3)}{6} = 6.0 \text{ in.}$$

Eccentricity: $e_x = 12.5 - 1.5 = 11.0$ in., $e_y = 6 + 1.5 = 7.5$ in.

$$\sum(x^2+y^2) = (1.5)^2(4) + (1.5)^2 + (4.5)^2 + (3)^2(3) + (3)^2 + (6)^2 = 103.5 \text{ in.}^2$$

(a) $P_u = 1.2D + 1.6L = 1.2(20) + 1.6(35) = 80.0$ kips

Direct shear components:

$$P_{ux} = \frac{3}{5}(80) = 48 \text{ kips} \rightarrow \qquad P_{uy} = \frac{4}{5}(80) = 64 \text{ kips} \downarrow$$

$$p_{cx} = \frac{48}{6} = 8 \text{ kips} \rightarrow \qquad p_{cy} = \frac{64}{6} = 10.67 \text{ kips} \downarrow$$

$$M = 64(11.0) - 48(7.5) = 344.0 \text{ in.-kips} \curvearrowright$$

Top right bolt is critical. $x = 4.5$ in., $y = 3$ in.

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{344(3)}{103.5} = 9.971 \text{ kips} \rightarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{344(4.5)}{103.5} = 14.96 \text{ kips} \downarrow$$

$$\sum p_x = 8 + 9.971 = 17.97 \text{ kips} \rightarrow \qquad \sum p_y = 10.67 + 14.96 = 25.63 \text{ kips} \downarrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(17.97)^2 + (25.63)^2} = 31.3 \text{ kips}$$

Assuming that the threads are in shear (N bolts),

$$\phi R_n = \phi F_{nv} A_b = 0.75(48)A_b = 31.3 \qquad \Rightarrow \qquad A_b = 0.8694 \text{ in.}^2$$

From $\frac{\pi d^2}{4} = 0.8694$, $d = 1.05$ in. Use $1\frac{1}{8}$-in. diameter bolts

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8.2-7

Determine location of centroid with respect to lower right bolt:

$$\bar{x} = \frac{2(2) + 1(6)}{6} = 1.667 \text{ in.,} \qquad \bar{y} = \frac{2(4) + 1(8)}{6} = 2.667 \text{ in.}$$

Eccentricity: $e_x = 10 + 1.667 = 11.67$ in., $e_y = 10 - 2.667 = 7.333$ in.

$$\sum(x^2+y^2) = (1.667)^2(3) + (0.3333)^2(2) + (4.333)^2$$
$$+ (2.667)^2(3) + (1.333)^2(2) + (5.333)^2 = 80.67 \text{ in.}^2$$

(a) LRFD solution: $P_u = 1.2D + 1.6L = 1.2(18) + 1.6(40) = 85.6$ kips

$$P_{ux} = \frac{4}{5}(85.6) = 68.48 \text{ kips} \leftarrow \qquad P_{uy} = \frac{3}{5}(85.6) = 51.36 \text{ kips} \downarrow$$

Direct shear components: $p_{cx} = \frac{68.48}{6} = 11.41 \text{ kips} \leftarrow \qquad p_{cy} = \frac{51.36}{6} = 8.56 \text{ kips} \downarrow$

$$M = 51.36(11.67) - 68.48(7.333) = 97.21 \text{ in.-kips} \curvearrowright$$

Check top right bolt. $x = 1.667$ in., $y = 8 - 2.667 = 5.333$ in.

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{97.21(5.333)}{80.67} = 6.426 \text{ kips} \rightarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{97.21(1.667)}{80.67} = 2.009 \text{ kips} \downarrow$$

$$\sum p_x = 6.426 - 11.41 = -4.984 \text{ kips } (\leftarrow) \qquad \sum p_y = 8.56 + 2.009 = 10.57 \text{ kips} \downarrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(-4.984)^2 + (10.57)^2} = 11.69 \text{ kips}$$

Check bottom right bolt. $x = 1.667$ in., $y = 2.667$ in.

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{97.21(2.667)}{80.67} = 3.214 \text{ kips} \leftarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{97.21(1.667)}{80.67} = 2.009 \text{ kips} \downarrow$$

$$\sum p_x = 11.41 + 3.214 = 14.62 \text{ kips} \leftarrow \qquad \sum p_y = 8.56 + 2.009 = 10.57 \text{ kips} \downarrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(14.62)^2 + (10.57)^2} = 18.04 \text{ kips (controls)}$$

Assuming that the threads are in shear (N bolts),

$$\phi R_n = \phi F_{nv} A_b = 0.75(48) A_b = 18.04 \qquad \Rightarrow \qquad A_b = 0.5011 \text{ in.}^2$$

From $\frac{\pi d^2}{4} = 0.5011$, $d = 0.799$ in. Required $d = 0.799$ in.; use $\frac{7}{8}$-in. diameter bolts

(b) ASD solution: $P_a = D + L = 18 + 40 = 58$ kips

$$P_{ax} = \frac{4}{5}(58) = 46.4 \text{ kips} \leftarrow \qquad P_{ay} = \frac{3}{5}(58) = 34.8 \text{ kips} \downarrow$$

Direct shear components: $p_{cx} = \frac{46.4}{6} = 7.733 \text{ kips} \leftarrow$ $\quad p_{cy} = \frac{34.8}{6} = 5.800 \text{ kips} \downarrow$

$$M = 34.8(11.67) - 46.4(7.333) = 65.86 \text{ in.-kips} \curvearrowright$$

Bottom right bolt is critical. $x = 1.667$ in., $y = 2.667$ in.

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{65.86(2.667)}{80.67} = 2.177 \text{ kips} \leftarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{65.86(1.667)}{80.67} = 1.361 \text{ kips} \downarrow$$

$$\sum p_x = 7.733 + 2.177 = 9.91 \text{ kips} \leftarrow \qquad \sum p_y = 5.800 + 1.361 = 7.161 \text{ kips} \downarrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(9.91)^2 + (7.161)^2} = 12.23 \text{ kips (controls)}$$

Assuming that the threads are in shear (N bolts),

$$\frac{R_n}{\Omega} = \frac{F_{nv} A_b}{\Omega} = \frac{48 A_b}{2.00} = 12.23 \qquad \Rightarrow \qquad A_b = 0.5096 \text{ in.}^2$$

From $\frac{\pi d^2}{4} = 0.5096$, $d = 0.806$ in. Required $d = 0.806$ in.; use $\frac{7}{8}$-in. diameter bolts

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8.2-8

LRFD solution

Determine location of centroid with respect to lower left bolt:

$$\bar{x} = \frac{3(5.5)}{7} = 2.357 \text{ in.}, \qquad \bar{y} = \frac{2(3.5)+1(7)+2(10.5)}{7} = 5.0 \text{ in.}$$

$$M = 31.18(5.5+1.5+6.5-2.357) - 18.0(10.5+2-5) = 212.4 \text{ in.-kips} \curvearrowleft$$

$$\sum(x^2+y^2) = (2.357)^2(4) + (5.5-2.357)^2(3) + (5)^2(2)$$
$$+ (5-3.5)^2(2) + (7-5)^2(1) + (10.5-5)^2(2) = 170.9 \text{ in.}^2$$

$$P_u = 1.2D + 1.6L = 1.2(10) + 1.6(15) = 36.0 \text{ kips}$$

Direct shear components:

$$P_{ux} = 36\cos 60° = 18.0 \text{ kips} \leftarrow \qquad P_{uy} = 36\sin 60° = 31.18 \text{ kips} \downarrow$$

$$p_{cx} = \frac{18}{7} = 2.571 \text{ kips} \leftarrow \qquad p_{cy} = \frac{31.18}{7} = 4.454 \text{ kips} \downarrow$$

Eccentric shear components: Check lower right bolt.

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{212.4(5)}{170.9} = 6.214 \text{ kips} \leftarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2+y^2)} = \frac{212.4(5.5-2.357)}{170.9} = 3.906 \text{ kips} \downarrow$$

$$\sum p_x = 2.571 + 6.214 = 8.785 \text{ kips} \leftarrow \qquad \sum p_y = 4.454 + 3.906 = 8.36 \text{ kips} \downarrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(8.785)^2 + (8.36)^2} = 12.13 \text{ kips}$$

Slip-critical strength will control over shear. Assuming Class A surfaces,

$$R_n = \mu D_u h_{sc} T_b N_s = 0.35(1.13)(1.0)T_b(1.0) = 0.3955T_b \text{ kips/bolt}$$

$$\phi R_n = 1.0(0.3955T_b) = 0.3955T_b$$

For $0.3955T_b = 12.13$, $T_b = 30.7$ kips.

From AISC Table J3.1, $T_b = 39$ kips for a $\frac{7}{8}$-inch diameter bolt.

Use $\frac{7}{8}$-inch diameter A325 bolts.

8.2-9

Location of centroid with respect to lower row of bolts: $\bar{y} = \frac{3(6)+3(1)}{6} = 3.5$ in.

$$\sum(x^2+y^2) = 4(3)^2 + 2(3.5)^2 + (0.5)^2 + 3(2.5)^2 = 79.5 \text{ in.}^2$$

(a) LRFD solution: $P_u = 1.2D + 1.6L = 1.2(2.5) + 1.6(7.5) = 15.0$ kips

Direct shear components: $p_{cx} = \frac{15}{6} = 2.5 \text{ kips} \rightarrow$

Lower right bolt controls.

$$M = 15(10 + 3.5) = 202.5 \text{ in.-kips} \curvearrowleft$$

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{202.5(3.5)}{79.5} = 8.915 \text{ kips} \rightarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2 + y^2)} = \frac{202.5(3)}{79.5} = 7.642 \text{ kips} \downarrow$$

$$\sum p_x = 2.5 + 8.915 = 11.42 \text{ kips} \rightarrow \qquad \sum p_y = 7.642 \text{ kips} \downarrow$$

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(11.42)^2 + (7.642)^2} = 13.74 \text{ kips}$$

Assuming that the threads are in shear (N bolts),

$$\phi R_n = \phi F_{nv} A_b = 0.75(48)A_b = 13.74 \qquad \Rightarrow \qquad A_b = 0.3817 \text{ in.}^2$$

From $\frac{\pi d^2}{4} = 0.3817$, $d = 0.697$ in., Try $d = \frac{3}{4}$ inch.

Check bearing. $h = d + \frac{1}{16} = \frac{3}{4} + \frac{1}{16} = \frac{13}{16}$ in. For the holes nearest the edge,

$$L_c = L_e - \frac{h}{2} = 2 - \frac{13/16}{2} = 1.594 \text{ in.}$$

$$\phi R_n = \phi(1.2 L_c t F_u) = 0.75(1.2)(1.594)(3/8)(58) = 31.2 \text{ kips}$$

$$\phi(2.4 d t F_u) = 0.75(2.4)(3/4)(3/8)(58) = 29.4 \text{ kips} < 31.2 \text{ kips}$$

$\therefore$ use $\phi R_n = 29.4$ kips/bolt > 13.7 kips/bolt (OK)

(No need to check the other bolts, since L_c will be larger.) Use $\frac{3}{4}$-in. diameter bolts.

(b) ASD solution: $P_a = D + L = 10$ kips

Direct shear components: $p_{cx} = \frac{10}{6} = 1.667 \text{ kips} \rightarrow$

Lower right bolt controls.

$$M = 10(10 + 3.5) = 135 \text{ in.-kips} \curvearrowleft$$

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{135(3.5)}{79.5} = 5.943 \text{ kips} \rightarrow$$

$$p_{my} = \frac{Mx}{\sum(x^2 + y^2)} = \frac{135(3)}{79.5} = 5.094 \text{ kips} \downarrow$$

$$\sum p_x = 1.667 + 5.943 = 7.610 \text{ kips} \rightarrow \qquad \sum p_y = 5.094 \text{ kips} \downarrow$$

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(7.610)^2 + (5.094)^2} = 9.158 \text{ kips}$$

Assuming that the threads are in shear (N bolts),

$$\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{\Omega} = \frac{48A_b}{2.00} = 9.158 \qquad \Rightarrow \qquad A_b = 0.3816 \text{ in.}^2$$

From $\frac{\pi d^2}{4} = 0.3816,\ d = 0.697$ in. Try $d = \frac{3}{4}$ inch.

Check bearing. $h = d + \frac{1}{16} = \frac{3}{4} + \frac{1}{16} = \frac{13}{16}$ in. For the holes nearest the edge,

$$L_c = L_e - \frac{h}{2} = 2 - \frac{13/16}{2} = 1.594 \text{ in.}$$

$$\frac{R_n}{\Omega} = \frac{1.2L_ctF_u}{\Omega} = \frac{1.2(1.594)(3/8)(58)}{2.00} = 20.8 \text{ kips}$$

$$\frac{2.4dtF_u}{\Omega} = \frac{2.4(3/4)(3/8)(58)}{2.00} = 19.6 \text{ kips} < 20.8 \text{ kips}$$

$$\therefore \text{ use } \frac{R_n}{\Omega} = 19.6 \text{ kips/bolt} > 9.16 \text{ kips/bolt} \qquad \text{(OK)}$$

(No need to check the other bolts, since L_c will be larger.) Use $\frac{3}{4}$-in. diameter bolts.

8.2-10

(a) Direct shear components: $p_{cx} = \frac{43}{6} = 7.167 \text{ kips} \leftarrow$

$$M = 43(8) = 344 \text{ in.-kips} \curvearrowright$$

$$\sum(x^2 + y^2) = 2(3)^2 + 2(9)^2 + 2(15)^2 = 630 \text{ in.}^2$$

Right-hand bolt controls.

$$p_{my} = \frac{Mx}{\sum(x^2 + y^2)} = \frac{344(15)}{630} = 8.19 \text{ kips} \downarrow$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(7.167)^2 + (8.19)^2} = 10.9 \text{ kips} \qquad \underline{p = 10.9 \text{ kips}}$$

(b) Use Table 7-7, Angle = 0°. For $e_x = 8$ in., $n = 6$, and $s = 6$ in.,

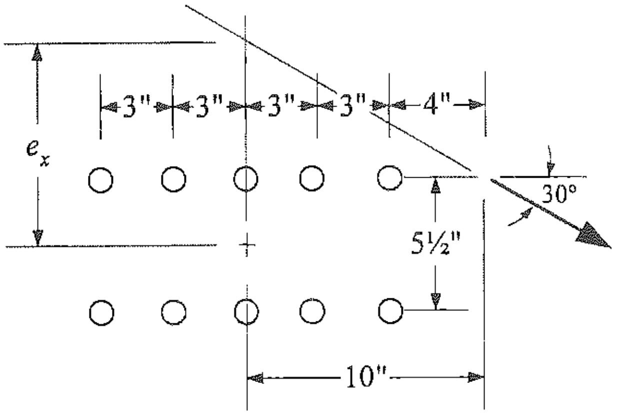
$C = 4.47$

Using the table notation, Bolt force $= r_n = \frac{R_n}{C} = \frac{43}{4.47} = 9.62$ kips

$p = 9.62$ kips (11.7% less than force from elastic analysis)

8.2-11

Use Table 7-9, Angle = 30°. $n = 5$ and $s = 3$ in.



$$e_x = 10\tan 30° + \frac{5.5}{2} = 8.524 \text{ in.}$$

$C = 4.756$

Slip-critical strength:

$r_n = \mu D_u h_{sc} T_b N_s = 0.35(1.13)(1.0)(28)(1) = 11.07$ kips/bolt

For the connection, $R_n = Cr_n = 4.756(11.07) = 52.65$ kips

(a) LRFD solution

$P_u = \phi R_n = 0.75(52.65) = 39.5$ kips $\qquad$ $\underline{P_u = 39.5 \text{ kips}}$

(b) ASD solution

$P_a = \dfrac{R_n}{\Omega} = \dfrac{52.65}{2.00} = 26.3$ kips $\qquad$ $\underline{P_a = 26.3 \text{ kips}}$

8.2-12

Use Table 7-14, Angle = 30°. $n = 3$ and $s = 3$ in.

$e_x = 4 + 4 + 2 - 3\tan 30° = 8.268$ in.

$C = 5.687$

$r_n = F_{nv}A_b = 48(0.4418) = 21.21$ kips/bolt

$R_n = Cr_n = 5.687(21.21) = 120.6$ kips

(a) $\phi R_n = 0.75(120.6) = 90.5$ kips $\qquad$ $\underline{P_u = 90.5 \text{ kips}}$

(b) $R_n/\Omega = 120.6/2.00 = 60.3$ kips $\qquad$ $\underline{P_a = 60.3 \text{ kips}}$

8.2-13

From the solution to problem 8.2-12, Use Table 7-14, Angle = 30°, $s = 3$ in.

$$e_x = 4+4+2-3\tan 30^\circ = 8.268 \text{ in.}$$

$$r_n = F_{nv}A_b = 48(0.4418) = 21.21 \text{ kips/bolt}$$

(a) $P_u = 1.2D + 1.6L = 1.2(40) + 1.6(90) = 192.0$ kips

$$C_{req} = \frac{P_u}{\phi r_n} = \frac{192}{0.75(21.21)} = 12.07$$

For $n = 6$, $C = 13.8$ Use $n = 6$ bolts per vertical row

(b) $P_a = D + L = 40 + 90 = 130$ kips

$$C_{req} = \frac{\Omega P_a}{r_n} = \frac{2.00(130)}{21.21} = 12.26$$

For $n = 6$, $C = 13.8$ Use $n = 6$ bolts per vertical row

8.3-1

Nominal bearing strength: $h = 1 + \frac{1}{16} = 1.063$ in.

For edge bolts, $L_c = L_e - \frac{h}{2} = 2 - \frac{1.063}{2} = 1.469$ in.

For other bolts, $L_c = s - h = 14 - 1.063 = 12.94$ in.

Check strength with $L_c = 1.469$ in.

$t_f = 1.12$ in. for W10 × 100 or 0.615 in. for WT5 × 27 (controls)

$$R_n = 1.2L_ctF_u = 1.2(1.469)(0.615)(65) = 70.47 \text{ kips}$$

$$2.4dtF_u = 02.4(1.0)(0.560)(65) = 87.36 \text{ kips}$$

$\therefore$ use $R_n = 70.47$ kips/bolt

Shear strength: $A_b = \pi d^2/4 = \pi(1.0)^2/4 = 0.7854 \text{ in.}^2$

Assume that threads are in the plane of shear.

$$R_n = F_{nv}A_b = 48(0.7854) = 37.70 \text{ kips/bolt}$$

Shear strength, not bearing, controls.

(a) LRFD solution

Factored load = $P_u = 1.2D + 1.6L = 1.2(18) + 1.6(50) = 101.6$ kips

Shear/bearing load per bolt = $101.6/4 = 25.4$ kips

Shear strength per bolt = $\phi R_n = 0.75(37.70) = 28.3 \text{ kips} > 25.4 \text{ kips}$ (OK)

Tension:

$$\sum M_{NA} = 2r_t(7+7) = 28.0r_t, \qquad M_u = P_u e = 101.6(3) = 304.8 \text{ in.-kips}$$

From $28r_t = 304.8$, $r_t = 10.9$ kips

$$f_v = \frac{25.4}{0.7854} = 32.34 \text{ ksi}$$

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_v \le F_{nt}$$

$$= 1.3(90) - \frac{90}{0.75(48)}(32.34) = 36.15 \text{ ksi} < 90 \text{ ksi}$$

$$\phi R_n = \phi F'_{nt} A_b = 0.75(36.15)(0.7854) = 21.3 \text{ kips/bolt} > 10.9 \text{ kips/bolt} \quad \text{(OK)}$$

Bolts are adequate

(b) ASD solution

Service load = $P_a = D + L = 18 + 50 = 68$ kips

Shear/bearing load per bolt = $68/4 = 17$ kips

Shear strength per bolt = $\dfrac{R_n}{\Omega} = \dfrac{37.70}{2.00} = 18.9 \text{ kips} > 17 \text{ kips}$ (OK)

Tension:

$$\sum M_{NA} = 2r_t(7+7) = 28.0r_t, \qquad M_a = P_a e = 68(3) = 204 \text{ in.-kips}$$

From $28r_t = 204$, $r_t = 7.29$ kips

$$f_v = \frac{17}{0.7854} = 21.65 \text{ ksi}$$

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_v \le F_{nt}$$

$$= 1.3(90) - \frac{2.00(90)}{48}(21.65) = 35.81 \text{ ksi} < 90 \text{ ksi}$$

The allowable tensile strength is

$$\frac{R_n}{\Omega} = \frac{F'_{nt}A_b}{\Omega} = \frac{35.81(0.7854)}{2.00} = 14.1 \text{ kips} > 7.29 \text{ kips} \quad \text{(OK)}$$

Bolts are adequate

8.3-2

LRFD solution

$P_u = 1.2D + 1.6L = 1.2(24) + 1.6(58) = 121.6$ kips

Shear/bearing load per bolt = $121.6/8 = 15.2$ kips

Bearing strength: $h = \dfrac{7}{8} + \dfrac{1}{16} = \dfrac{15}{16}$ in.

For edge bolts, $L_c = L_e - \frac{h}{2} = 2 - \frac{15/16}{2} = 1.531$ in.

For other bolts, $L_c = s - h = 3 - 15/16 = 2.063$ in.

Check strength with $L_c = 1.531$ in.

$t_f = 1.11$ in. for W12 × 120 or 0.575 in. for WT6 × 26.5 (controls)

$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(1.531)(0.575)(65) = 51.5$ kips

$\phi(2.4dtF_u) = 0.75(2.4)(7/8)(0.575)(65) = 58.9$ kips > 51.5 kips

$\therefore$ use $\phi R_n = 51.5$ kips/bolt > 15.2 kips/bolt (OK)

Shear strength: $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013$ in.^{2}

Assume that threads are in the plane of shear.

$\phi R_n = \phi F_{nv} A_b = 0.75(48)(0.6013) = 21.6$ kips/bolt > 15.2 kips/bolt (OK)

Tension:

$\sum M_{NA} = 2r_t(4.5 + 1.5 + 1.5 + 4.5) = 24r_t,$

$M_u = P_u e = 121.6(3) = 364.8$ in.-kips

From $24r_t = 364.8,\ r_t = 15.2$ kips

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_v \le F_{nt}, \qquad f_v = \frac{15.2}{0.6013} = 23.91 \text{ ksi}$$

$$= 1.3(90) - \frac{90}{0.75(48)}(23.91) = 57.23 \text{ ksi} < 90 \text{ ksi}$$

$\phi R_n = \phi F'_{nt} A_b = 0.75(57.23)(0.6013) = 25.8$ kips/bolt > 15.2 kips/bolt (OK)

Bolts are adequate

8.3-3

Nominal bearing strength: $h = \frac{7}{8} + \frac{1}{16} = 0.9375$ in.

$t_f = 0.605$ in. for W12 × 65 or 0.670 in. for WT15 × 49.5

For edge bolts (applies to WT only), $L_c = L_e - \frac{h}{2} = 2 - \frac{0.9375}{2} = 1.531$ in.

$R_n = 1.2L_c t F_u = 1.2(1.531)(0.670)(65) = 80.01$ kips

$2.4dtF_u = 2.4(7/8)(0.670)(65) = 91.46$ kips

$\therefore$ use $R_n = 80.01$ kips/bolt for these bolts

For other bolts (use t_w for W), $L_c = s - h = 3 - 0.9375 = 2.063$ in.

$R_n = 1.2L_c t F_u = 1.2(2.063)(0.605)(65) = 97.35$ kips

$2.4dtF_u = 2.4(7/8)(0.605)(65) = 82.58$ kips

$\therefore$ use $R_n = 82.58$ kips/bolt for these bolts

Shear strength: $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Assume that threads are in the plane of shear.

$R_n = F_{nv}A_b = 48(0.6013) = 28.86$ kips/bolt

Shear strength, not bearing, controls.

(a) LRFD solution

Factored load = $P_u = 1.2D + 1.6L = 1.2(25) + 1.6(75) = 150.0$ kips

Shear/bearing load per bolt = $150/10 = 15$ kips

Shear strength per bolt = $\phi R_n = 0.75(28.86) = 21.6 \text{ kips} > 15 \text{ kips}$ (OK)

Tension:

$\sum M_{NA} = 2r_t(3+3+6+6) = 36r_t$, $M_u = P_u e = 150(6) = 900$ in.-kips

From $36r_t = 900$, $r_t = 25$ kips

$$f_v = \frac{15}{0.6013} = 24.95 \text{ ksi}$$

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_v \le F_{nt}$$

$$= 1.3(90) - \frac{90}{0.75(48)}(24.95) = 54.63 \text{ ksi} < 90 \text{ ksi}$$

$\phi R_n = \phi F'_{nt}A_b = 0.75(54.63)(0.6013) = 24.6 \text{ kips/bolt} < 25 \text{ kips/bolt}$ (N.G.)

Bolts are not adequate

(b) ASD solution

Service load = $P_a = D + L = 25 + 75 = 100$ kips

Shear/bearing load per bolt = $100/10 = 10$ kips

Shear strength per bolt = $\dfrac{R_n}{\Omega} = \dfrac{28.86}{2.00} = 14.4 \text{ kips} > 10 \text{ kips}$ (OK)

Tension:

$\sum M_{NA} = 2r_t(3+3+6+6) = 36r_t$, $M_a = P_a e = 100(6) = 600$ in.-kips

From $36r_t = 600$, $r_t = 16.7$ kips

$$f_v = \frac{10}{0.6013} = 16.63 \text{ ksi}$$

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_v \le F_{nt}$$

$$= 1.3(90) - \frac{2.00(90)}{48}(16.63) = 54.64 \text{ ksi} < 90 \text{ ksi}$$

The allowable tensile strength is

$$\frac{R_n}{\Omega} = \frac{F'_{nt}A_b}{\Omega} = \frac{54.64(0.6013)}{2.00} = 16.4 \text{ kips} < 16.7 \text{ kips} \qquad \text{(N.G.)}$$

Bolts are not adequate

8.3-4

LRFD solution

Factored load $= P_u = 1.2D + 1.6L = 1.2(10) + 1.6(28) = 56.8$ kips

Shear/bearing load per bolt $= 56.8/6 = 9.467$ kips

Bearing strength: $t_f = 0.560$ in. for W10 × 49 or 0.620 in. for WT5 × 22.5

$$h = \frac{3}{4} + \frac{1}{16} = \frac{13}{16} \text{ in.}$$

For edge bolts (applies to WT only), $L_c = L_e - \frac{h}{2} = 1.5 - \frac{13/16}{2} = 1.094$ in.

$$\phi R_n = \phi(1.2L_c tF_u) = 0.75(1.2)(1.094)(0.620)(65) = 39.7 \text{ kips}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(3/4)(0.620)(65) = 54.4 \text{ kips} > 39.7 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 39.7 \text{ kips/bolt} > 9.467 \text{ kips/bolt} \qquad \text{(OK)}$$

For other bolts, $L_c = s - h = 3 - 13/16 = 2.188$ in.

$$\phi R_n = \phi(1.2L_c tF_u) = 0.75(1.2)(2.188)(0.560)(65) = 71.7 \text{ kips}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(3/4)(0.560)(65) = 49.1 \text{ kips} < 71.7 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 49.1 \text{ kips/bolt} > 9.467 \text{ kips/bolt} \qquad \text{(OK)}$$

Shear strength: $A_b = \pi d^2/4 = \pi(3/4)^2/4 = 0.4418 \text{ in.}^2$

Assume that threads are in the plane of shear.

$$\phi R_n = \phi F_{nv} A_b = 0.75(48)(0.4418) = 15.9 \text{ kips/bolt} > 9.467 \text{ kips (OK)}$$

Tension: Determine location of centroid of bolt areas (neutral axis), measured from top bolts.

$$\bar{y} = \frac{2(6) + 2(9)}{6} = 5.0 \text{ in.}$$

$$\sum M_{NA} = 2r_t(5 + 1 + 4) = 20r_t$$

$$M_u = P_u e = 56.8(3) = 170.4 \text{ in.-kips}$$

From $20r_t = 170.4$, $r_t = 8.52$ kips

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_v \le F_{nt}, \qquad f_v = \frac{9.467}{0.4418} = 21.43 \text{ ksi}$$

$$= 1.3(90) - \frac{90}{0.75(48)}(21.43) = 63.43 \text{ ksi} < 90 \text{ ksi}$$

$$\phi R_n = \phi F'_{nt} A_b = 0.75(63.43)(0.4418) = 21.0 \text{ kips/bolt} > 8.52 \text{ kips/bolt (OK)}$$

Bolts are adequate

8.3-5

(a) LRFD solution:

$$P_u = 1.2D + 1.6L = 1.2(0.33 \times 110) + 1.6(0.67 \times 110) = 161.5 \text{ kips}$$

Vertical component of load = $\frac{3}{5}(161.5) = 96.90$ kips

Horizontal component of load = $\frac{4}{5}(161.5) = 129.2$ kips

Shear load per bolt = $96.90/10 = 9.690$ kips

Shear strength: $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Assume that threads are in the plane of shear.

$$\phi R_n = \phi F_{nv} A_b = 0.75(48)(0.6013) = 21.6 \text{ kips/bolt} > 9.69 \text{ kips} \quad \text{(OK)}$$

Tension: Direct tensile load per bolt = $129.2/10 = 12.92$ kips

$$\sum M_{NA} = 2r_t(3+6) \times 2 = 36r_t$$

$$M_u = P_u e = 96.90(6) = 581.4 \text{ in.-kips}$$

From $36r_t = 581.4$, $r_t = 16.15$ kips

Total tensile load per bolt = $12.92 + 16.15 = 29.1$ kips

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_v \le F_{nt}, \qquad f_v = \frac{9.690}{0.6013} = 16.12 \text{ ksi}$$

$$= 1.3(90) - \frac{90}{0.75(48)}(16.15) = 76.63 \text{ ksi} < 90 \text{ ksi}$$

$$\phi R_n = \phi F'_{nt} A_b = 0.75(76.63)(0.6013) = 34.6 \text{ kips/bolt} > 29.1 \text{ kips/bolt} \quad \text{(OK)}$$

Bolts are adequate

(b) ASD solution

$$P_a = D + L = 110 \text{ kips}$$

Vertical component of load = $\frac{3}{5}(110) = 66$ kips

Horizontal component of load = $\frac{4}{5}(110) = 88$ kips

Shear load per bolt = $66/10 = 6.6$ kips

Shear strength: $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Assume that threads are in the plane of shear.

$$R_n = \frac{F_{nv}A_b}{\Omega} = \frac{48(0.6013)}{2.00} = 14.4 \text{ kips/bolt} > 6.6 \text{ kips} \qquad \text{(OK)}$$

Tension: Direct tensile load per bolt = 88/10 = 8. 8 kips

$$\sum M_{NA} = 2r_t(3+6) \times 2 = 36r_t$$

$$M_a = P_a e = 66(6) = 396 \text{ in.-kips}$$

From $36r_t = 396$, $r_t = 11$ kips

Total tensile load per bolt = 8. 8 + 11 = 19. 8 kips

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}}f_v \le F_{nt}, \qquad f_v = \frac{6.6}{0.6013} = 10.98 \text{ ksi}$$

$$= 1.3(90) - \frac{2.00(90)}{48}(10.98) = 75.83 \text{ ksi} < 90 \text{ ksi}$$

The allowable tensile strength is

$$\frac{R_n}{\Omega} = \frac{F'_{nt}A_b}{\Omega} = \frac{75.83(0.6013)}{2.00} = 22.8 \text{ kips} > 19.8 \text{ kips} \qquad \text{(OK)}$$

Bolts are adequate

8.3-6

$$P_u = 1.2D + 1.6L = 1.2(15) + 1.6(45) = 90.0 \text{ kips}$$

$$\text{Vertical component of load} = \frac{2}{\sqrt{5}}(90) = 80.50 \text{ kips}$$

$$\text{Horizontal component of load} = \frac{1}{\sqrt{5}}(90) = 40.25 \text{ kips}$$

Direct tensile load per bolt = 40. 25/6 = 6. 708 kips. Determine location of centroid of bolt areas (neutral axis), measured from top bolts.

$$\bar{y} = \frac{2(3) + 2(9)}{6} = 4.0 \text{ in.}$$

$$\sum M_{NA} = 2r_{ut}(5 + 1 + 4) = 20\, r_{ut}$$

$$M_u = 80.5(6) + 40.25(3 + 4) = 764.8 \text{ in.-kips}$$

From $20r_t = 764.8$, $r_t = 38.24$ kips

Total tensile load per bolt = 6. 708 + 38. 24 = 45. 0 kips

Shear/bearing load per bolt = 80. 5/6 = 13. 42 kips

$$A_b = \pi d^2/4 = \pi(1.0)^2/4 = 0.7854 \text{ in.}^2$$

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}}f_v \le F_{nt}, \qquad f_v = \frac{13.42}{0.7854} = 17.09 \text{ ksi}$$

$$= 1.3(90) - \frac{2.00(90)}{48}(17.09) = 52.91 \text{ ksi} < 90 \text{ ksi ksi} < 90 \text{ ksi ksi}$$

$$\phi R_n = \phi F'_{nt} A_b = 0.75(52.91)(0.7854) = 31.2 \text{ kips/bolt } < 45.0 \text{ kips/bolt} \qquad \text{(N.G.)}$$

Bolts are not adequate

Note that, for this location of the neutral axis, $\Sigma F \neq 0$. Alternative solutions that do not violate equilibrium include the following:

1. Use the same method for finding r_t but place the neutral axis at the second line of bolts. This makes $\Sigma F = 0$.

2. Use the method of Case I in Part 7 of the *Manual.*

8.3-7

Nominal bearing strength: $h = \frac{3}{4} + \frac{1}{16} = \frac{13}{16}$ in.

For edge bolts, $L_c = L_e - \frac{h}{2} = 1.5 - \frac{13/16}{2} = 1.094$ in.

For other bolts, $L_c = s - h = 3 - 13/16 = 2.188$ in.

Check strength with $L_c = 1.094$ in. (the angle controls)

$$R_n = 1.2 L_c t F_u = 1.2(1.094)(5/16)(58) = 23.79 \text{ kips}$$

$$2.4 d t F_u = 2.4(3/4)(5/16)(58) = 32.63 \text{ kips}$$

$$\therefore \text{ use } R_n = 23.79 \text{ kips/bolt}$$

Shear strength: $A_b = \pi d^2/4 = \pi(3/4)^2/4 = 0.4418 \text{ in.}^2$

Assume that threads are in the plane of shear.

$$R_n = F_{nv} A_b = 48(0.4418) = 21.21 \text{ kips/bolt}$$

Slip-critical strength:

$$R_n = \mu D_u h_{sc} T_b N_s = 0.35(1.13)(1.0)(28)(1) = 11.07 \text{ kips/bolt}$$

(a) LRFD solution

Shear/bearing load per bolt $= R_u/10 = 0.1R_u$

Bearing: $\phi R_n = 0.75(23.79) = 17.84$ kips

Shear: $\phi R_n = 0.75(21.21) = 15.91$ kips

Slip: $\phi R_n = 1.00(11.07) = 11.07$ kips

Slip is more critical than bearing or shear, and in this type of connection, the tensile force does not reduce the slip-critical strength. Let

$$0.1R_u = 11.07, \qquad R_u = 111 \text{ kips}$$

Tension:

$$\sum M_{NA} = 2r_t(3+6) \times 2 = 36r_t$$

$$M_u = P_u e = 2.25R_u$$

From $36r_t = 2.25R_u$, $\quad r_t = 0.0625R_u$

Interaction of tension and shear is not a consideration with slip-critical bolts. The design tensile strength per bolt is

$$\phi R_n = \phi F_{nt} A_b = 0.75(90)(0.4418) = 29.82 \text{ kips}$$

Let $\quad 0.0625R_u = 29.82, \quad R_u = 477 \text{ kips}$

The reaction capacity is therefore based on the slip-critical strength. $\underline{R_u = 111 \text{ kips}}$

(b) ASD solution

Shear/bearing load per bolt $= R_a/10 = 0.1R_a$

Bearing: $\dfrac{R_n}{\Omega} = \dfrac{23.79}{2.00} = 11.90 \text{ kips}$

Shear: $\dfrac{R_n}{\Omega} = \dfrac{21.21}{2.00} = 10.61 \text{ kips}$

Slip: $\dfrac{R_n}{\Omega} = \dfrac{11.07}{1.50} = 7.380 \text{ kips}$

Slip is more critical than bearing or shear, and in this type of connection, the tensile force does not reduce the slip-critical strength. Let

$$0.1R_a = 7.380, \quad R_a = 73.8 \text{ kips}$$

Tension:

$$\sum M_{NA} = 2r_t(3+6) \times 2 = 36r_t$$

$$M_a = P_a e = 2.25R_a$$

From $36r_t = 2.25R_a$, $\quad r_t = 0.0625R_a$

Interaction of tension and shear is not a consideration with slip-critical bolts. The allowable tensile strength per bolt is

$$\frac{R_n}{\Omega} = \frac{F_{nt}A_b}{\Omega} = \frac{90(0.4418)}{2.00} = 19.88 \text{ kips}$$

Let $\quad 0.0625R_a = 19.88, \quad R_a = 318 \text{ kips}$

The reaction capacity is therefore based on the slip-critical strength. $\underline{R_a = 73.8 \text{ kips}}$

8.3-8

LRFD solution

$$P_{ux} = 1.2D + 1.6L = 1.2(0.30 \times 47) + 1.6(0.70 \times 47) = 69.56 \text{ kips}$$

$$P_{uy} = 1.2D + 1.6L = 1.2(0.30 \times 60) + 1.6(0.70 \times 60) = 88.8 \text{ kips}$$

Assume that tension controls. Select bolt size based on tension, then check the other limit states.

$$\text{Direct tension} = \frac{69.56}{10} = 6.956 \text{ kips/bolt}$$

$$\sum M_{NA} = 2r_t(7 + 3.5) \times 2 = 42r_t, \qquad M_u = 69.56(9) + 88.8(8) = 1336 \text{ in.-kips}$$

From $42r_t = 1336$, $r_t = 31.81$ kips

Total tensile load per bolt $= 6.956 + 31.81 = 38.77$ kips. Let

$$\phi F_{nt} = \frac{38.77}{A_b}$$

$$A_b = \frac{38.77}{\phi F_{nt}} = \frac{38.77}{0.75(90)} = 0.5744 \text{ in.}^2$$

$$\text{Required diameter} = d_b = \sqrt{\frac{4A_b}{\pi}} = \sqrt{\frac{4(0.5744)}{\pi}} = 0.8552 \text{ in.}$$

Try $\frac{7}{8}$-in. diameter bolts, with $A_b = \pi(7/8)^2/4 = 0.6013$ in.^{2}

Slip-critical strength will control over shear. In this type of connection, the tensile force does not normally reduce the slip-critical strength, but since there is a direct tension component, apply the reduction factor.

$$R_n = \mu D_u h_{sc} T_b N_s = 0.35(1.13)(1.0)(39)(1) = 15.42 \text{ kips/bolt}$$

$$\phi R_n = 1.0(15.42) = 15.42 \text{ kips/bolt}$$

$$k_s = 1 - \frac{T_u}{D_u T_b N_b} = 1 - \frac{69.56}{1.13(39)(10)} = 0.8422$$

$$k_s(15.42) = 0.8422(15.42) = 13.0 \text{ kips/bolt} > 8.88 \text{ kips/bolt (OK)}$$

Check bearing (flange of WT controls). $h = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$ in.

For the holes nearest the edge, $L_c = L_e - \frac{h}{2} = 2 - \frac{15/16}{2} = 1.531$ in.

$$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(1.531)(0.590)(65) = 52.8 \text{ kips} > 8.88 \text{ kips (OK)}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(7/8)(0.590)(65) = 60.4 \text{ kips/bolt} > 50.7 \text{ kips}$$

For the other holes, $L_c = s - h = 3.5 - 15/16 = 2.56$ in. Since this is larger than L_c for the edge bolts, no further check is necessary.

Use $\frac{7}{8}$-inch diameter bolts

8.3-9

(a) Factored load: Neglect the beam weight initially, and account for it later.

$$w_u = 1.2w_D + 1.6w_L = 1.6(4) = 6.4 \text{ kips/ft}$$

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(6.4)(30)^2 = 720 \text{ ft-kips}$$

From the Z_x table, Try a W24 × 76:

$$\phi_b M_n = \phi_b M_p = 750 \text{ ft-kips} > 720 \text{ ft-kips (OK)}$$

Check beam weight: $w_u = 1.2w_D + 1.6w_L = 1.2(0.076) + 1.6(4) = 6.491$ kips/ft

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(6.491)(30)^2 = 730 \text{ ft-kips} < 750 \text{ ft-kips (OK)}$$

Shear: $V_u = \dfrac{6.491(30)}{2} = 97.4$ kips

From the Z_x table, $\phi_v V_n = 316 \text{ kips} > 97.4 \text{ kips}$ (OK)

Use a W24 × 76

(b) Use A325-N bearing type bolts.

For the beam-to-angle bolts, design for shear, then check bearing. Try $\frac{3}{4}$-in. diameter bolts.

$$A_b = \pi d^2/4 = \pi(3/4)^2/4 = 0.4418 \text{ in.}^2$$

$$\phi R_n = \phi F_{nv} A_b = 0.75(48)(0.4418) \times 2 \text{ shear planes} = 31.81 \text{ kips/bolt}$$

Let $31.81N_b = V_u$: $\quad 31.81N_b = 97.4 \quad \Rightarrow \quad N_b = 3.06$

Try 4 bolts in beam-to-angle connection and 8 bolts in angle-to-column connection.

Use a minimum spacing of $3d = 3(3/4) = 2.25$ in.

Minimum edge distance from AISC Table J3.4 = $1\frac{1}{4}$ in.

Try $s = 3$ in. and $L_e = 1\frac{1}{2}$ in. Total length of angles = $3(3) + 2(1.5) = 12$ in.

Bearing: Check angles first: $h = d + \dfrac{1}{16} = \dfrac{3}{4} + \dfrac{1}{16} = 0.8125$ in.

For edge bolt, $L_c = L_e - \dfrac{h}{2} = 1.5 - \dfrac{0.8125}{2} = 1.094$ in.

$$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(1.094)t(58) = 57.11t$$

$$\phi(2.4dtF_u) = 0.75(2.4)(3/4)t(58) = 78.3t > 57.11t$$

$\therefore$ use $\phi R_n = 57.11t$ kips/bolt

For other bolts, $L_c = s - h = 3 - 0.8125 = 2.188$ in.

$$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(2.188)t(58) = 114.2t$$

$\phi(2.4dtF_u) = 78.3t < 114.2t$ $\therefore$ use $\phi R_n = 78.3t$ kips/bolt

Total bearing strength (per angle leg) = $57.11t + 3(78.3t) = 292.0t$

For two angles, $292.0t \times 2 = 584.0t$

Let $584.0t = V_u = 97.4 \quad \Rightarrow \quad t = 0.167$ in.

Check beam web. $t_w = 0.440$ in. There are no edge bolts. For the other bolts,

$$\phi R_n = \phi(1.2L_ctF_u) = 0.75(1.2)(2.188)(0.440)(65) = 56.32 \text{ kips/bolt}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(3/4)(0.440)(65) = 38.61 \text{ kips/bolt} < 56.32 \text{ kips/bolt}$$

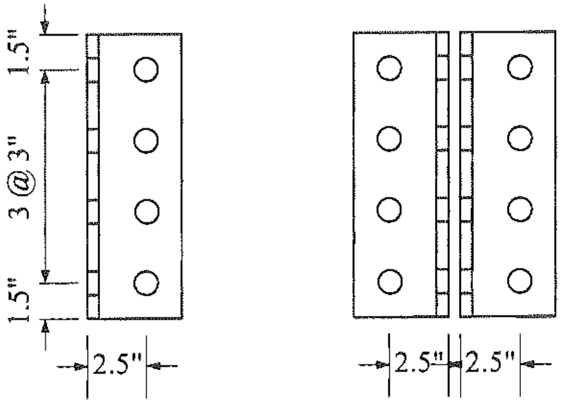
$$\therefore \text{ use } \phi R_n = 38.61 \text{ kips/bolt}$$

Total bearing strength = $4(38.61) = 154.4$ kips > 97.4 kips (OK)

Bearing on column flange OK (column flange is thicker than beam flange, and there are twice as many bolts).

Try 2L4 × 4 × $\frac{1}{4}$ $\left(\frac{1}{4} \text{ in.} > 0.167 \text{ in. required}\right)$

Assume a $\frac{1}{2}$-in. setback and the usual gage distance.



Beam connection Column connection

Shear yield strength of angles (AISC J4.2):

$$\phi R_n = \phi(0.60A_gF_y) \times 2 = 1.0(0.60)(12 \times \tfrac{1}{4})(36) \times 2$$

$$= 130 \text{ kips} > 97.4 \text{ kips} \quad \text{(OK)}$$

Shear rupture strength of angle (AISC J4.2):

Use hole diameter $= \frac{3}{4} + \frac{1}{8} = 0.875$ in.

$$A_{nv} = 2 \times (12 - 4 \times 0.875)\left(\frac{1}{4}\right) = 4.25 \text{ in.}^2$$

$$\phi R_n = \phi(0.60A_{nv}F_u) = 0.75(0.60)(4.25)(58) = 111 \text{ kips} > 97.4 \text{ kips (OK)}$$

Block shear strength of angles:

Shear areas: $A_{gv} = \frac{1}{4}(10.5) \times 2 = 5.25 \text{ in.}^2$

$$A_{nv} = \frac{1}{4}[10.5 - 3.5(0.875)] \times 2 = 3.719 \text{ in.}^2$$

Tension area: $A_{nt} = \frac{1}{4}[1.5 - 0.5(0.875)] \times 2 = 0.5313 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(3.719) + 1.0(58)(0.5313) = 160.2 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(5.25) + 1.0(58)(0.5313) = 144.2 \text{ kips}$$

$$\phi R_n = 0.75(144.2) = 108 \text{ kips} > 97.4 \text{ kips} \qquad \text{(OK)}$$

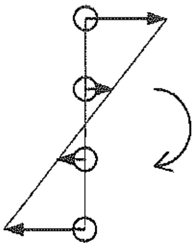
Use 2L4 × 4 × $\frac{1}{4}$ as shown, with

four $\frac{3}{4}$-in. diameter A325 bearing-type bolts in the beam web, and

eight $\frac{3}{4}$-in. diameter A325 bearing-type bolts in the column flange

(c) Check the beam-to-angle connection (shear only), accounting for eccentricity. The direct shear component is

$$p_{cy} = \frac{V_u}{4} = \frac{97.4}{4} = 24.35 \text{ kips}$$



Eccentric shear component:

$$M = V_ue = 97.4(2.5) = 243.5 \text{ in.-kips}$$

$$\sum(x^2 + y^2) = 2(4.5)^2 + 2(1.5)^2 = 45 \text{ in.}^2$$

Upper right bolt is critical:

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{243.5(4.5)}{45} = 24.35 \text{ kips}$$

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(24.35)^2 + (24.35)^2} = 34.4 \text{ kips}$$

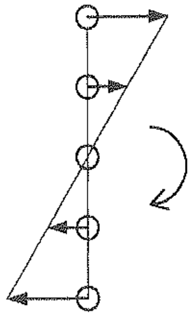
$$\phi R_n = \phi F_{nv}A_b = 0.75(48)(0.4418) \times 2 \text{ shear planes}$$

$$= 31.8 \text{ kips/bolt} < 34.4 \text{ kips/bolt} \qquad \text{(N.G.)}$$

Try 5 bolts in the beam-to-angle connection and 10 bolts in the angle-to-column connection.

Check for eccentricity:

Direct shear component: $p_{cy} = \frac{V_u}{5} = \frac{97.4}{5} = 19.48 \text{ kips}$



Eccentric shear component:

$$M = V_u e = 97.4(2.5) = 243.5 \text{ in.-kips}$$

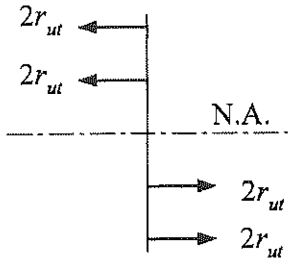
$$\sum(x^2 + y^2) = 2(6)^2 + 2(3)^2 = 90 \text{ in.}^2$$

Upper right bolt is critical:

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{243.5(6)}{90} = 16.23 \text{ kips}$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(16.23)^2 + (19.48)^2} = 25.4 \text{ kips} < 31.8 \text{ kips (OK)}$$

Check the angle-to-column connection, accounting for eccentricity (tension):



$$\sum M_{NA} = 2r_t(6 + 3 + 3 + 6) = 36r_t$$

$$M_u = 97.4(2.5) = 243.5 \text{ in.-kips}$$

From $36r_t = 243.5$, $r_t = 6.76$ kips

$$f_v = \frac{97.4/10}{0.4418} = 22.05 \text{ ksi}$$

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_v \leq F_{nt}$$

$$= 1.3(90) - \frac{90}{0.75(48)}(22.05) = 61.88 \text{ ksi} < 90 \text{ ksi}$$

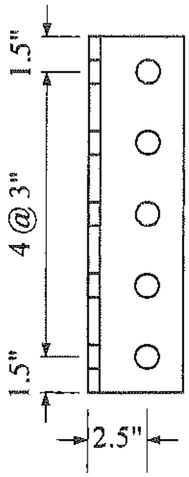
$$\phi R_n = \phi F'_{nt} A_b = 0.75(61.88)(0.4418) = 20.5 \text{ kips/bolt} > 6.76 \text{ kips/bolt} \qquad \text{(OK)}$$

Use 2L4 × 4 × $\frac{1}{4}$ as shown, with

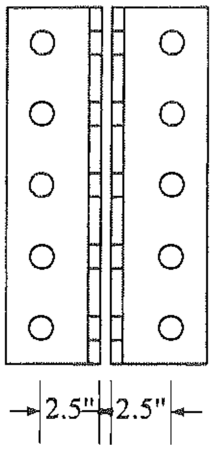
five $\frac{3}{4}$ in. diameter A325 bearing-type bolts in the beam web, and

ten $\frac{3}{4}$ in. diameter A325 bearing-type bolts in the column flange

(d)



Beam connection



Column connection

8.3-10

(a) Neglect the beam weight initially, and account for it later.

$w_a = 4$ kips/ft

$$M_a = \frac{1}{8}w_aL^2 = \frac{1}{8}(4)(30)^2 = 450 \text{ ft-kips}$$

From the Z_x table, Try a W24 × 76:

$$\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 499 \text{ ft-kips} > 450 \text{ ft-kips (OK)}$$

Check beam weight: $w_a = w_D + w_L = 0.076 + 4 = 4.076$ kips/ft

$$M_a = \frac{1}{8}w_aL^2 = \frac{1}{8}(4.076)(30)^2 = 459 \text{ ft-kips} < 499 \text{ ft-kips (OK)}$$

Shear: $V_a = \dfrac{4.076(30)}{2} = 61.14$ kips

From the Z_x table, $\dfrac{V_n}{\Omega_v} = 210$ kips > 61.14 kips (OK)

Use a W24 × 76

(b) Use A325-N bearing type bolts.

For the beam-to-angle bolts, design for shear, then check bearing. Try $\frac{3}{4}$-in. diameter bolts.

$$A_b = \pi d^2/4 = \pi(3/4)^2/4 = 0.4418 \text{ in.}^2$$

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}F_{nv}A_b = \frac{1}{2.00}(48)(0.4418) \times 2 \text{ shear planes} = 21.21 \text{ kips/bolt}$$

Let $21.21N_b = V_a$: $\quad 21.31N_b = 61.14 \quad \Rightarrow \quad N_b = 2.87$

Try 4 bolts in beam-to-angle connection and 8 bolts in angle-to-column connection.

Use a minimum spacing of $3d = 3(3/4) = 2.25$ in.

Minimum edge distance from AISC Table J3.4 = $1\frac{1}{4}$ in.

Try $s = 3$ in. and $L_e = 1\frac{1}{2}$ in. Total length of angles $= 3(3) + 2(1.5) = 12$ in.

Bearing: Check angles first: $h = d + \frac{1}{16} = \frac{3}{4} + \frac{1}{16} = 0.8125$ in.

For edge bolt, $L_c = L_e - \frac{h}{2} = 1.5 - \frac{0.8125}{2} = 1.094$ in.

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2L_ctF_u) = \frac{1}{2.00}(1.2)(1.094)t(58) = 38.07t$$

$$\frac{1}{\Omega}(2.4dtF_u) = \frac{1}{2.00}(2.4)(3/4)t(58) = 52.2t > 38.07t$$

$$\therefore \text{ use } \frac{R_n}{\Omega} = 38.07t \text{ kips/bolt}$$

For other bolts, $L_c = s - h = 3 - 0.8125 = 2.188$ in.

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2L_ctF_u) = \frac{1}{2.00}(1.2)(2.188)t(58) = 76.14t$$

$$\frac{1}{\Omega}(2.4dtF_u) = 52.2t < 76.14t \ \therefore \text{ use } \frac{R_n}{\Omega} = 52.2t \text{ kips/bolt}$$

Total bearing strength (per angle leg) $= 38.07t + 3(52.2t) = 194.7t$

For two angles, $194.2t \times 2 = 388.4t$

Let $388.4t = V_a = 61.14 \quad \Rightarrow \quad t = 0.157$ in.

Check beam web. $t_w = 0.440$ in. There are no edge bolts. For the other bolts,

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(1.2L_ctF_u) = \frac{1}{2.00}(1.2)(2.188)(0.440)(65) = 37.55 \text{ kips/bolt}$$

$$\frac{1}{\Omega}(2.4dtF_u) = \frac{1}{2.00}(2.4)(3/4)(0.440)(65) = 25.74 \text{ kips/bolt} < 37.55 \text{ kips/bolt}$$

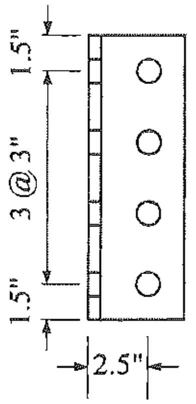
$$\therefore \text{ use } \frac{R_n}{\Omega} = 25.74 \text{ kips/bolt}$$

Total bearing strength $= 4(25.74) = 103$ kips > 61.14 kips (OK)

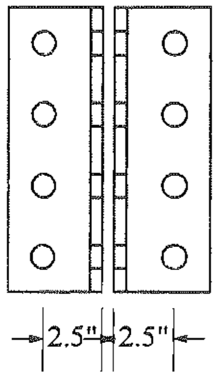
Bearing on column flange OK (column flange is thicker than beam flange, and there are twice as many bolts).

Try 2L4 × 4 × $\frac{1}{4}$ $\left(\frac{1}{4}\text{ in.} > 0.157 \text{ in. required}\right)$

Assume a $\frac{1}{2}$-in. setback and the usual gage distance.



Beam connection



Column connection

Shear yield strength of angles (AISC J4.2):

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(0.60A_gF_y) \times 2 = \frac{1}{1.5}(0.60)(12 \times \tfrac{1}{4})(36) \times 2$$

$$= 86.4 \text{ kips} > 61.14 \text{ kips} \qquad \text{(OK)}$$

Shear rupture strength of angle (AISC J4.2):

Use hole diameter $= \frac{3}{4} + \frac{1}{8} = 0.875$ in.

$$A_{nv} = 2 \times (12 - 4 \times 0.875)\left(\frac{1}{4}\right) = 4.25 \text{ in.}^2$$

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(0.60A_{nv}F_u) = \frac{1}{2.00}(0.60)(4.25)(58) = 111 \text{ kips} > 61.14 \text{ kips (OK)}$$

Block shear strength of angles:

Shear areas: $A_{gv} = \frac{1}{4}(10.5) \times 2 = 5.25 \text{ in.}^2$

$$A_{nv} = \frac{1}{4}[10.5 - 3.5(0.875)] \times 2 = 3.719 \text{ in.}^2$$

Tension area: $A_{nt} = \frac{1}{4}[1.5 - 0.5(0.875)] \times 2 = 0.5313 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(3.719) + 1.0(58)(0.5313) = 160.2 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(5.25) + 1.0(58)(0.5313) = 144.2 \text{ kips}$$

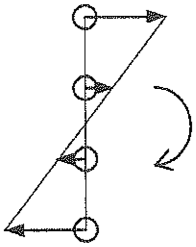
$$\frac{R_n}{\Omega} = \frac{1}{2.00}(144.2) = 72.1 \text{ kips} > 61.14 \text{ kips} \qquad \text{(OK)}$$

Use 2L4 × 4 × $\frac{1}{4}$ as shown, with four $\frac{3}{4}$-in. diameter A325 bearing-type bolts in the beam web, and

eight $\frac{3}{4}$-in. diameter A325 bearing-type bolts in the column flange

(c) Check the beam-to-angle connection (shear only), accounting for eccentricity. The direct shear component is

$$p_{cy} = \frac{V_a}{4} = \frac{61.14}{4} = 15.29 \text{ kips}$$



Eccentric shear component:

$$M = V_a e = 61.14(2.5) = 152.9 \text{ in.-kips}$$

$$\sum(x^2 + y^2) = 2(4.5)^2 + 2(1.5)^2 = 45 \text{ in.}^2$$

Upper right bolt is critical:

$$p_{mx} = \frac{My}{\sum(x^2+y^2)} = \frac{152.9(4.5)}{45} = 15.29 \text{ kips}$$

$$p = \sqrt{(\sum p_x)^2 + (\sum p_y)^2} = \sqrt{(15.29)^2 + (15.29)^2} = 21.6 \text{ kips}$$

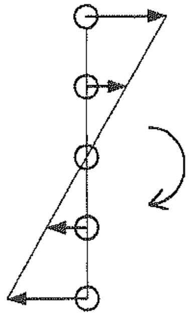
$$\frac{R_n}{\Omega} = \frac{1}{\Omega} F_{nv} A_b = \frac{1}{2.00}(48)(0.4418) \times 2 \text{ shear planes}$$

$$= 21.2 \text{ kips/bolt} < 21.6 \text{ kips/bolt} \qquad \text{(N.G.)}$$

Try 5 bolts in the beam-to-angle connection and 10 bolts in the angle-to-column connection.

Check for eccentricity:

Direct shear component: $p_{cy} = \frac{V_a}{5} = \frac{61.14}{5} = 12.23$ kips



Eccentric shear component:

$$M = V_a e = 61.14(2.5) = 152.9 \text{ in.-kips}$$

$$\sum(x^2 + y^2) = 2(6)^2 + 2(3)^2 = 90 \text{ in.}^2$$

Upper right bolt is critical:

$$p_{mx} = \frac{My}{\sum(x^2 + y^2)} = \frac{152.9.5(6)}{90} = 5.097 \text{ kips}$$

$$p = \sqrt{\left(\sum p_x\right)^2 + \left(\sum p_y\right)^2} = \sqrt{(5.097)^2 + (12.23)^2} = 13.25 \text{ kips} < 21.2 \text{ kips (OK)}$$

Check the angle-to-column connection, accounting for eccentricity (tension):

$$\sum M_{NA} = 2r_t(6 + 3 + 3 + 6) = 36r_t$$

$$M_a = 61.14(2.5) = 152.9 \text{ in.-kips}$$

From $36r_t = 152.9$, $r_t = 4.25$ kips

$$f_v = \frac{61.14/10}{0.4418} = 13.84 \text{ ksi}$$

$$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_v \leq F_{nt}$$

$$= 1.3(90) - \frac{2.00(90)}{48}(13.84) = 65.1 \text{ ksi} < 90 \text{ ksi}$$

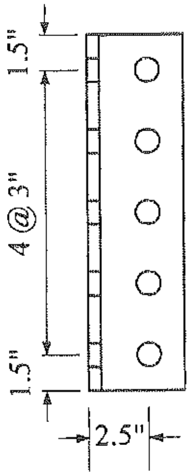
$$\frac{R_n}{\Omega} = \frac{1}{\Omega} F'_{nt} A_b = \frac{1}{2.00}(65.1)(0.4418) = 14.4 \text{ kips/bolt} > 4.25 \text{ kips/bolt} \qquad \text{(OK)}$$

Use 2L4 × 4 × $\frac{1}{4}$ as shown, with

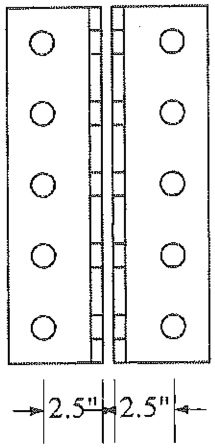
five $\frac{3}{4}$ in. diameter A325 bearing-type bolts in the beam web, and

ten $\frac{3}{4}$ in. diameter A325 bearing-type bolts in the column flange

(d)



Beam connection



Column connection

8.4-1

Direct shearing stress: $f_{1y} = \dfrac{9}{7+8} = 0.6$ kips/in.↓

Shearing stress caused by moment: Locate centroid with respect to upper left corner.

$$\bar{x} = \frac{7(3.5)}{15} = 1.633 \text{ in.}, \qquad \bar{y} = \frac{8(4)}{15} = 2.133 \text{ in.}$$

$$M = Pe = 9(15 - 1.633) = 120.3 \text{ in.-kips} \curvearrowright$$

$$I_x = \frac{(8)^3}{12} + 8(4 - 2.133)^2 + 7(2.133)^2 = 102.4 \text{ in.}^4$$

$$I_y = 8(1.633)^2 + \frac{(7)^3}{12} + 7(3.5 - 1.633)^2 = 74.32 \text{ in.}^4$$

$$J = I_x + I_y = 102.4 + 74.32 = 176.7 \text{ in.}^4$$

Lower left corner:

$$f_{2x} = \frac{My}{J} = \frac{120.3(2.133)}{176.7} = 1.452 \text{ kips/in.} \rightarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{120.3(7 - 1.633)}{176.7} = 3.654 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{\left(\sum f_x\right)^2 + \left(\sum f_y\right)^2} = \sqrt{(1.452)^2 + (0.6 + 3.654)^2} = 4.50 \text{ kips/in.}$$

$$\underline{f_v = 4.50 \text{ kips/in.}}$$

8.4-2

Direct shearing stress: $f_{1y} = \dfrac{15}{5+3+5} = 1.154$ kips/in.↓

Shearing stress caused by moment: Locate centroid with respect to lower left corner.

$$\bar{x} = \frac{5(2.5)\times 2}{13} = 1.923 \text{ in.,} \qquad \bar{y} = \frac{5(12)+3(10.5)}{13} = 7.038 \text{ in.}$$

$$M = Pe = 15(12-1.923) = 151.2 \text{ in.-kips} \curvearrowright$$

$$I_x = \frac{(3)^3}{12} + 3(10.5-7.038)^2 + 5(7.038)^2 + 5(12-7.038)^2 = 409.0 \text{ in.}^4$$

$$I_y = 3(1.923)^2 + 2\left[\frac{(5)^3}{12} + 5(2.5-1.923)^2\right] = 35.26 \text{ in.}^4$$

$$J = I_x + I_y = 409.0 + 35.26 = 444.3 \text{ in.}^4$$

Lower right corner:

$$f_{2x} = \frac{My}{J} = \frac{151.2(7.038)}{444.3} = 2.395 \text{ kips/in.} \leftarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{151.2(5-1.923)}{444.3} = 1.047 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{\left(\sum f_x\right)^2 + \left(\sum f_y\right)^2} = \sqrt{(2.395)^2 + (1.154+1.047)^2} = 3.25 \text{ kips/in.}$$

$$\underline{f_v = 3.25 \text{ kips/in.}}$$

8.4-3

$$P_x = 75\cos 75^\circ = 19.41 \text{ kips} \leftarrow \qquad P_y = 75\sin 75^\circ = 72.44 \text{ kips} \downarrow$$

$$f_{1x} = \frac{19.41}{18} = 1.078 \text{ kips/in.} \leftarrow \qquad f_{1y} = \frac{72.44}{18} = 4.024 \text{ kips/in.} \downarrow$$

Shearing stress caused by moment: Locate centroid with respect to lower left corner.

$$\bar{x} = \frac{9(4.5)+3(1.5)+3(7.5)+3(9)}{18} = 5.25 \text{ in.}$$

$$\bar{y} = \frac{3(12)+3(12)+3(10.5)}{18} = 5.75 \text{ in.}$$

$$M = 19.41(12-5.75) + 72.44(6+5.25) = 936.3 \text{ in.-kips} \curvearrowleft$$

$$I_x = 9(5.75)^2 + 3(12-5.75)^2 \times 2 + \frac{(3)^3}{12} + 3(12-1.5-5.75)^2 = 601.9 \text{ in.}^4$$

$$I_y = \frac{(9)^3}{12} + 9(0.75)^2 + \frac{(3)^3}{12} + 3(3.75)^2 + \frac{(3)^3}{12} + 3(2.25)^2 + 3(3.75)^2$$

$$= 169.9 \text{ in.}^4$$

$$J = I_x + I_y = 601.9 + 169.9 = 771.8 \text{ in.}^4$$

Stress at upper left:

$$f_{2x} = \frac{My}{J} = \frac{936.3(12-5.75)}{771.8} = 7.582 \text{ kips/in.} \leftarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{936.3(5.25)}{771.8} = 6.369 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(1.078+7.582)^2 + (4.024+6.369)^2}$$

$= 13.5$ kips/in. $\underline{f_v = 13.5 \text{ kips/in.}}$

8.4-4

LRFD solution:

$$P_u = 1.2D + 1.6L = 1.2(2.5) + 1.6(7.5) = 15.0 \text{ kips}$$

Load on one angle $= \frac{15}{2} = 7.5$ kips, $f_y = \frac{7.5}{3(5)} = 0.5$ kips/in. $\downarrow$

Shearing stress caused by moment: Locate centroid with respect to right side of weld.

$$\bar{x} = \frac{5(2.5)\times 2}{15} = 1.667 \text{ in.}$$

$$M = 7.5(17-1.667) = 115.0 \text{ in.-kips} \curvearrowleft$$

$$I_x = \frac{(5)^3}{12} + 2(5)(2.5)^2 = 72.92 \text{ in.}^4$$

$$I_y = 5(1.667)^2 + 2\left[\frac{(1.667)^3}{3} + \frac{(3.333)^3}{3}\right] = 41.67 \text{ in.}^4$$

$$J = I_x + I_y = 72.92 + 41.67 = 114.6 \text{ in.}^4$$

Stress at upper left:

$$f_{2x} = \frac{My}{J} = \frac{115(2.5)}{114.6} = 2.509 \text{ kips/in.} \leftarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{115(5-1.667)}{114.6} = 3.345 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(2.509)^2 + (0.5+3.345)^2} = 4.59 \text{ kips/in.}$$

Weld strength is

$$\phi R_n = \phi[0.707w(0.6)F_{EXX}] = 0.75(0.707)(3/16)(0.6)(70) = 4.18 \text{ kips/in.}$$

(Alternate: $\phi R_n = 1.392 \times 3$ sixteenths $= 4.18$ kips/in.)

Since 4.59 kips/in. > 4.18 kips/in., weld is not adequate.

8.4-5

Locate centroid with respect to left side.

$$\bar{x} = \frac{6(4+3)(2)}{18} = 4.667 \text{ in.}$$

$$I_x = \frac{(6)^3}{12} + 6(6)^2 \times 2 = 450.0 \text{ in.}^4$$

$$I_y = 6(4.667)^2 + 2\left[\frac{(6)^3}{12} + 6(7-4.667)^2\right] = 232.0 \text{ in.}^4$$

$$J = I_x + I_y = 450 + 232 = 682 \text{ in.}^4$$

(a) LRFD solution

$$P_u = 1.2D + 1.6L = 1.2(10) + 1.6(10) = 28.0 \text{ kips}$$

$$P_{ux} = 28\sin 30^\circ = 14.00 \text{ kips} \rightarrow \qquad P_{uy} = 28\cos 30^\circ = 24.25 \text{ kips} \downarrow$$

$$f_{1x} = \frac{14}{18} = 0.7778 \text{ kips/in.} \rightarrow \qquad f_{1y} = \frac{24.25}{18} = 1.347 \text{ kips/in.} \downarrow$$

Shearing stress caused by moment:

$$M = 14(6) + 24.25(10 + 12 - 4.667) = 504.3 \text{ in.-kips} \curvearrowright$$

Stress at upper right:

$$f_{2x} = \frac{My}{J} = \frac{504.3(6)}{682} = 4.437 \text{ kips/in.} \rightarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{504.3(10-4.667)}{682} = 3.943 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(0.7778 + 4.437)^2 + (1.347 + 3.943)^2}$$

$$= 7.428 \text{ kips/in.}$$

Required weld size $= \dfrac{7.428}{1.392} = 5.34$ sixteenths.

For $t = 1/2$ in., min. $w = 3/16$ in. and max $w = 1/2 - 1/16 = 7/16$ in.

Use a $\frac{3}{8}$-in. fillet weld.

(b) ASD solution

$$P_a = D + L = 10 + 10 = 20 \text{ kips}$$

$$P_{ax} = 20\sin 30^\circ = 10 \text{ kips} \rightarrow \qquad P_{ay} = 20\cos 30^\circ = 17.32 \text{ kips} \downarrow$$

$$f_{1x} = \frac{10}{18} = 0.5556 \text{ kips/in.} \rightarrow \qquad f_{1y} = \frac{17.32}{18} = 0.9622 \text{ kips/in.} \downarrow$$

Shearing stress caused by moment:

$$M = 10(6) + 17.32(10 + 12 - 4.667) = 360.2 \text{ in.-kips} \curvearrowright$$

Stress at upper right:

$$f_{2x} = \frac{My}{J} = \frac{360.2(6)}{682} = 3.169 \text{ kips/in.} \rightarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{360.2(10-4.667)}{682} = 2.817 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(0.5556+3.169)^2 + (0.9622+2.817)^2} = 5.306 \text{ kips/in.}$$

Required weld size $= \frac{5.306}{0.9279} = 5.72$ sixteenths.

For $t = 1/2$ in., min. $w = 3/16$ in. and max $w = 1/2 - 1/16 = 7/16$ in.

Use a $\frac{3}{8}$-in. fillet weld.

8.4-6

LRFD solution

$$D + L = D + 2D = 20 \qquad \Rightarrow \qquad D = 6.667 \text{ kips}, \ L = 13.33 \text{ kips}$$

$$P_u = 1.2D + 1.6L = 1.2(6.667) + 1.6(13.33) = 29.33 \text{ kips}$$

$$f_{1y} = \frac{29.33}{5} = 5.866 \text{ kips/in.} \downarrow$$

Shearing stress caused by moment: Locate centroid with respect to lower right.

$$\bar{x} = \frac{3(1.5) + 2(1)}{3+2} = 1.3 \text{ in.}, \qquad \bar{y} = \frac{3(6)}{3+2} = 3.6 \text{ in.}$$

$$M = 29.33(2 + 1.3) = 96.79 \text{ in.-kips} \curvearrowright$$

$$I_x = 3(6-3.6)^2 + 2(3.6)^2 = 43.2 \text{ in.}^4$$

$$I_y = \frac{(3)^3}{12} + 3(1.5-1.3)^2 + \frac{(2)^3}{12} + 2(0.3)^2 = 3.217 \text{ in.}^4$$

$$J = I_x + I_y = 43.2 + 3.217 = 46.42 \text{ in.}^4$$

Stress at upper right:

$$f_{2x} = \frac{My}{J} = \frac{96.79(3.6)}{46.42} = 7.506 \text{ kips/in.} \rightarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{96.79(1.3)}{46.42} = 2.711 \text{ kips/in.} \downarrow$$

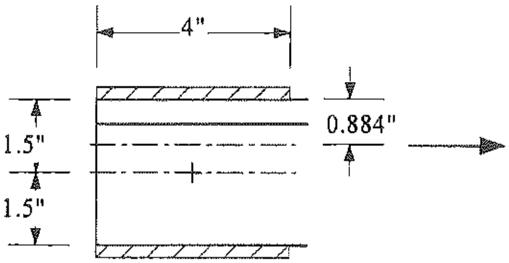
$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(7.506)^2 + (5.866+2.711)^2} = 11.4 \text{ kips/in.}$$

Weld strength: $\phi R_n = 1.392 \times 8$ sixteenths $= 11.1$ kips/in. < 11.4 kips/in. (N.G.)

Weld is not adequate.

8.4-7

If eccentricity is not considered, $f_{1x} = \dfrac{112}{2(4)} = 14.0$ kips/in. →



Load when eccentricity is considered:

$$I_x = 2(4)(1.5)^2 = 18.0 \text{ in.}^4, \qquad I_y = 2\left[\frac{(4)^3}{12}\right] = 10.67 \text{ in.}^4$$

$$J = I_x + I_y = 18.0 + 10.67 = 28.67 \text{ in.}^4$$

$$e = 1.5 - 0.884 = 0.616 \text{ in.}$$

$$M = Pe = 112(0.616) = 68.99 \text{ in.-kips}$$

Stress at upper right:

$$f_{2x} = \frac{My}{J} = \frac{68.99(1.5)}{28.67} = 3.610 \text{ kips/in.}\rightarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{68.99(2)}{28.67} = 4.813 \text{ kips/in.}\downarrow$$

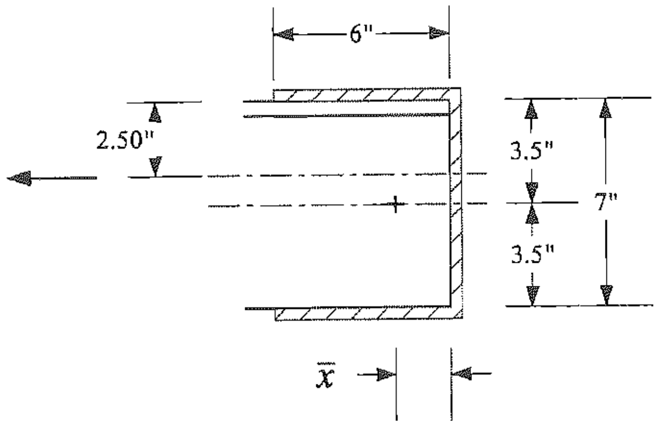
$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(14.0 + 3.610)^2 + (4.813)^2} = 18.26 \text{ kips/in.}$$

Load caused by eccentricity = 18.26 − 14.0 = 4.26 kips/in.

$\underline{f_v = 4.26 \text{ kips/in.}}$

8.4-8

If eccentricity is not considered, $f_{1x} = \dfrac{150}{2(6)+7} = 7.895$ kips/in. ←



Load when eccentricity is considered: Locate centroid with respect to right side.

$$\bar{x} = \frac{6(3)\times 2}{19} = 1.895 \text{ in.,} \qquad \bar{y} = 3.5 \text{ in.}$$

$$I_x = 2(6)(3.5)^2 + \frac{(7)^3}{12} = 175.6 \text{ in.}^4$$

$$I_y = 2\left[\frac{(6)^3}{12} + 6(3-1.895)^2\right] + 7(1.895)^2 = 75.79 \text{ in.}^4$$

$$J = I_x + I_y = 175.6 + 75.79 = 251.4 \text{ in.}^4$$

$$M = 150(3.5 - 2.50) = 150 \text{ in.-kips}$$

Stress at upper left:

$$f_{2x} = \frac{My}{J} = \frac{150(3.5)}{251.4} = 2.088 \text{ kips/in.} \leftarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{150(6-1.895)}{251.4} = 2.449 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{(\textstyle\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(7.895+2.088)^2 + (2.449)^2} = 10.28 \text{ kips/in.}$$

Load caused by eccentricity = 10.28 − 7.895 = 2.39 kips/in.

$\underline{f_v = 2.39 \text{ kips/in.}}$

8.4-9

LRFD Solution

Compute the tension member available strength. For the gross section:

$$\phi_t P_n = 0.90 F_y A_g = 0.90(36)(4.38) = 141.9 \text{ kips}$$

Net section: Assume that $U = 0.80$. Check once the connection is designed.

$$A_e = A_g U = 4.38(0.80) = 3.504 \text{ in.}^2$$

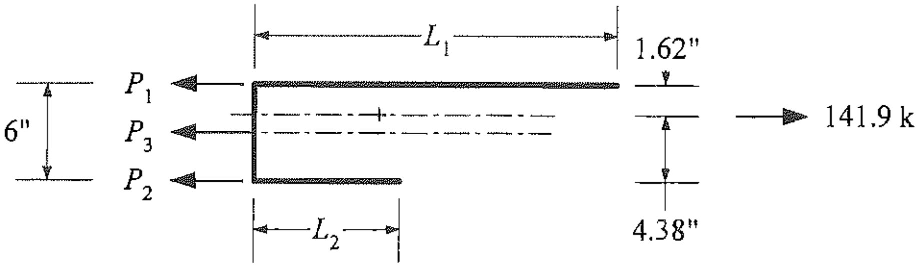
$$\phi_t P_n = 0.75 F_u A_e = 0.75(58)(3.504) = 152.4 \text{ kips. Gross section controls.}$$

Determine weld size. Based a thickness of 3/8 inch, min. $w = 3/16$ in.

$$\max w = 3/8 - 1/16 = 5/16 \text{ in.}$$

To minimize the length of the connection, try 5/16 in. fillet welds. Investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\phi R_n = 1.392D = 1.392(5) = 6.96 \text{ kips/in.}$$



$$P_3 = 6(6.96) = 41.76 \text{ kips}$$

$$\sum M_{L2} = 141.9(4.38) - 41.76\left(\frac{6}{2}\right) - P_1(6) = 0 \quad \Rightarrow \quad P_1 = 82.71 \text{ kips}$$

$$\sum F = 141.9 - 41.76 - 82.71 - P_2 = 0 \quad \Rightarrow \quad P_2 = 17.43 \text{ kips}$$

$$L_1 = \frac{P_1}{6.96} = \frac{82.71}{6.96} = 11.9 \text{ in. Use 12 in.}$$

$$L_2 = \frac{P_2}{6.96} = \frac{17.43}{6.96} = 2.50 \text{ in. Use } 2\tfrac{1}{2} \text{ in.}$$

For the second option, the strength of the longitudinal welds is $0.85(6.96) = 5.916$ kips/in. and the strength of the transverse weld is $1.5(6.96) = 10.44$ kips/in.

$$P_3 = 6(10.44) = 62.64 \text{ kips}$$

$$\sum M_{L2} = 141.9(4.38) - 62.64\left(\frac{6}{2}\right) - P_1(6) = 0 \quad \Rightarrow \quad P_1 = 72.27 \text{ kips}$$

$$\sum F = 141.9 - 62.64 - 72.27 - P_2 = 0 \quad \Rightarrow \quad P_2 = 6.99 \text{ kips}$$

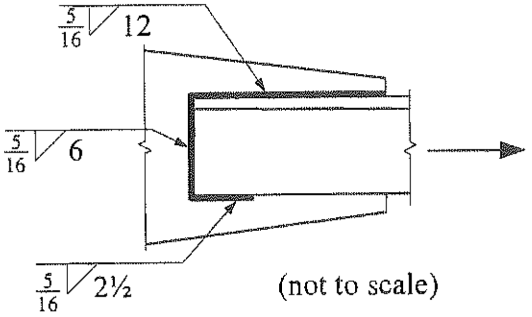
$$L_1 = \frac{P_1}{5.916} = \frac{72.27}{5.916} = 12.22 \text{ in. Use } 12\tfrac{1}{2} \text{ in.}$$

$$L_2 = \frac{P_2}{5.916} = \frac{6.99}{5.916} = 1.182 \text{ in. Use } 1\tfrac{1}{2} \text{ in.}$$

(Min. length $= 4w = 4(5/16) = 1.25$ in.)

The total length of weld is nearly the same for both options. Try the first option and check the value of U.

$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.62}{12} = 0.865 \quad > 0.80 \qquad \text{(OK)}$ Use the weld shown.



8.4-10

Use Table 8-10, Angle = 0°

$k\ell = 7 \text{ in.}, \qquad k = \frac{k\ell}{\ell} = \frac{7}{8} = 0.875$

$x = 0.2043$ (by interpolation), $\qquad x\ell = 0.2043(8) = 1.634$ in.

$e_x = 7 + 8 - 1.634 = 13.37$ in.

$a\ell = e_x, \qquad a = \frac{e_x}{\ell} = \frac{13.37}{8} = 1.671$

$C = 1.006$ by interpolation.

If the 9-kip load is a factored load,

$$D_{\text{min}} = \frac{P_u}{\phi C C_1 \ell} = \frac{9}{0.75(1.006)(1.0)(8)} = 1.491$$

$\phi R_n = 1.392D = 1.392(1.494) = 2.08$ kips/in.

If the 9-kip load is a service load,

$$D_{\text{min}} = \frac{\Omega P_a}{C C_1 \ell} = \frac{2.00(9)}{1.006(1.0)(8)} = 2.237$$

$\frac{R_n}{\Omega} = 0.9279D = 0.9279(2.237) = 2.08$ kips/in. $p = 2.08$ kips/in.

8.4-11

Consider the weld on one angle. Use Table 8-8, Angle = 0°

$k\ell = 5 \text{ in.}, \qquad k = \frac{k\ell}{\ell} = \frac{5}{5} = 1.0$

$x = 0.333, \qquad x\ell = 0.333(5) = 1.665$ in.

$e_x = 12 + 5 - 1.665 = 15.34$ in.

$$a\ell = e_x, \qquad a = \frac{e_x}{\ell} = \frac{15.34}{5} = 3.068$$

Maximum value of a in the table is 3.00. By extrapolation,

$$C = 1.12 + \left(\frac{1.12 - 1.19}{3.00 - 2.80}\right)(3.068 - 3.00) = 1.096$$

(a) LRFD solution

$$P_u = 1.2D + 1.6L = 1.2(0.25 \times 10) + 1.6(0.75 \times 10) = 15.0 \text{ kips}$$

For one angle, $P_u = \frac{15}{2} = 7.5$ kips

$$D_{\min} = \frac{P_u}{\phi C C_1 \ell} = \frac{7.5}{0.75(1.096)(1)(5)}$$

$$= 1.83 \text{ sixteenths} < 3 \text{ sixteenths furnished} \qquad \text{(OK)}$$

Weld is adequate.

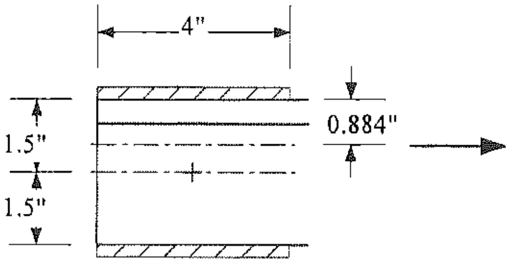
(b) ASD solution

$P_a = 10$ kips; For one angle, $P_a = 10/2 = 5$ kips

$$D_{\min} = \frac{\Omega P_a}{C C_1 \ell} = \frac{2.00(5)}{1.096(1)(5)} = 1.83 \text{ sixteenths} < 3 \text{ sixteenths furnished} \qquad \text{(OK)}$$

Weld is adequate.

8.4-12



Use Table 8-4, Angle = 0°

$$k\ell = 3 \text{ in.}, \qquad k = \frac{k\ell}{\ell} = \frac{3}{4} = 0.75$$

$$e_x = 1.5 - 0.884 = 0.616 \text{ in.}$$

$$a\ell = e_x, \qquad a = \frac{0.616}{\ell} = \frac{0.616}{4} = 0.154$$

$C = 3.540$ by interpolation. If the 112-kip load is treated as a factored load,

$$D_{\min} = \frac{P_u}{\phi C C_1 \ell} = \frac{112}{0.75(3.540)(1.0)(4)} = 10.55$$

$$\phi R_n = 1.392D = 1.392(10.55) = 14.69 \text{ kips/in.}$$

When eccentricity is not accounted for, $e_x = 0$, $a = 0$, and $C = 3.71$

$$D_{\min} = \frac{P_u}{\phi C C_1 \ell} = \frac{112}{0.75(3.71)(1.0)(4)} = 10.06$$

$$\phi R_n = 1.392D = 1.392(10.06) = 14.00 \text{ kips/in.}$$

Difference = $14.69 - 14.00 = 0.69$ kips/in. (one significant figure is lost in the subtraction)

0.69 kips/inch

8.4-13

(a) $P_u = 1.2D + 1.6L = 1.2(18) + 1.6(43) = 90.4$ kips

$$P_{ux} = 90.4 \sin 15° = 23.40 \text{ kips} \leftarrow \qquad P_{uy} = 90.4 \cos 15° = 87.32 \text{ kips} \downarrow$$

$$f_{1x} = \frac{23.40}{2(12+4)} = 0.7313 \text{ kips/in.} \leftarrow \qquad f_{1y} = \frac{87.32}{32} = 2.729 \text{ kips/in.} \downarrow$$

Shearing stress caused by moment:

$$M = 87.32(2 + 2.5) - 23.40(6) = 252.5 \text{ in.-kips} \curvearrowright$$

$$I_x = 2 \times \frac{(12)^3}{12} + 2(4)(6)^2 = 576.0 \text{ in.}^4, \qquad I_y = 12(2)^2(2) + 2\frac{(4)^3}{12} = 106.7 \text{ in.}^4$$

$$J = I_x + I_y = 576.0 + 106.7 = 682.7 \text{ in.}^4$$

Stress at lower right:

$$f_{2x} = \frac{My}{J} = \frac{252.5(6)}{682.7} = 2.219 \text{ kips/in.} \leftarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{252.5(2)}{682.7} = 0.7397 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(0.7313 + 2.219)^2 + (2.729 + 0.7397)^2}$$

$$= 4.554 \text{ kips/in.}$$

Required weld size = $\dfrac{4.554}{1.392} = 3.27$ sixteenths. Use a ¼-in. fillet weld.

(b) Use Table 8-6, Angle = 15°

$$k\ell = 4 \text{ in.}, \qquad k = \frac{k\ell}{\ell} = \frac{4}{12} = 0.3333$$

$$6 \tan 15° = 1.608 \text{ in.}, \qquad e_x = \frac{4}{2} + 2.5 - 1.608 = 2.892 \text{ in.}$$

$$a\ell = e_x, \qquad a = \frac{e_x}{\ell} = \frac{2.892}{12} = 0.241$$

$C = 4.79$ by interpolation

$$D_{\min} = \frac{P_u}{\phi C C_1 \ell} = \frac{90.4}{0.75(4.79)(1)(12)} = 2.10 \text{ sixteenths}$$

Use a $\frac{3}{16}$-in. fillet weld.

8.4-14

(a) $P_a = D + L = 18 + 43 = 61$ kips

$P_{ax} = 61 \sin 15° = 15.79 \text{ kips} \leftarrow \qquad P_{ay} = 61 \cos 15° = 58.92 \text{ kips} \downarrow$

$f_{1x} = \dfrac{15.79}{2(12+4)} = 0.4934 \text{ kips/in.} \leftarrow \qquad f_{1y} = \dfrac{58.92}{32} = 1.841 \text{ kips/in.} \downarrow$

Shearing stress caused by moment:

$M = 58.92(2 + 2.5) - 15.79(6) = 170.4 \text{ in.-kips} \curvearrowright$

$I_x = (2)\dfrac{(12)^3}{12} + 2(4)(6)^2 = 288.0 \text{ in.}^4, \qquad I_y = 12(2)^2(2) + 2\dfrac{(4)^3}{12} = 106.7 \text{ in.}^4$

$J = I_x + I_y = 288.0 + 106.7 = 394.7 \text{ in.}^4$

Stress at lower right:

$f_{2x} = \dfrac{My}{J} = \dfrac{170.4(6)}{394.7} = 2.59 \text{ kips/in.} \leftarrow$

$f_{2y} = \dfrac{Mx}{J} = \dfrac{170.4(2)}{394.7} = 0.8634 \text{ kips/in.} \downarrow$

$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(0.4934 + 2.59)^2 + (1.841 + 0.8634)^2}$

$= 4.101$ kips/in.

Required weld size $= \dfrac{4.101}{0.9279} = 4.42$ sixteenths. Use a $\frac{5}{16}$-in. fillet weld.

(b) Use Table 8-6, Angle = 15°

$k\ell = 4 \text{ in.}, \qquad k = \dfrac{k\ell}{\ell} = \dfrac{4}{12} = 0.3333$

$6 \tan 15° = 1.608 \text{ in.}, \qquad e_x = \dfrac{4}{2} + 2.5 - 1.608 = 2.892 \text{ in.}$

$a\ell = e_x, \qquad a = \dfrac{e_x}{\ell} = \dfrac{2.892}{12} = 0.241$

$C = 4.79$ by interpolation

$D_{\min} = \dfrac{\Omega P_a}{C C_1 \ell} = \dfrac{2.00(61)}{4.79(1)(12)} = 2.12$ sixteenths Use a $\frac{3}{16}$-in. fillet weld.

8.4-15

(a) $P_u = 1.2D + 1.6L = 1.2(31) + 1.6(31) = 86.8$ kips

For $t = 5/16$ in., min. $w = 3/16$ in. and max $w = 5/16 - 1/16 = 1/4$ in.

Use E70 fillet welds. Try $w = 3/16$ in., one transverse weld and two longitudinal welds. Investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\phi R_n = 1.392D = 1.392(3) = 4.176 \text{ kips/in.}$$

Base metal shear strength of the angle:

$$\text{Yielding: } \phi R_n = \phi(0.6F_y t) = 1.00(0.6)(36)(5/16) = 6.75 \text{ kips/in.}$$

$$\text{Rupture: } \phi R_n = \phi(0.6F_u t) = 0.75(0.6)(58)(5/16) = 8.156 \text{ kips/in.}$$

The weld shear strength controls.

$$\text{Total length required} = \frac{86.8}{4.176} = 20.79 \text{ in.}$$

$$\text{Length of longitudinal welds} = \frac{20.79 - 6}{2} = 7.40 \text{ in.; use } 7\tfrac{1}{2} \text{ in.}$$

For the second option, the strength of the longitudinal welds is $0.85(4.176) = 3.550$ kips/in.

and the strength of the transverse weld is $1.5(4.176) = 6.264$ kips/in.

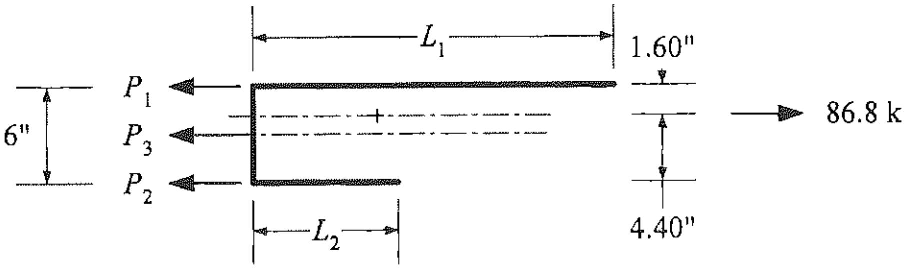
$$\text{Length of longitudinal welds} = \frac{86.8 - 6.264(6)}{3.550(2)} = 6.93 \text{ in.; use 7 in.}$$

The second options requires shorter longitudinal welds:

Use $\frac{3}{16}$-in. E70 fillet welds, across the end and 7 inches on each side

(b) Investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\phi R_n = 4.176 \text{ kips/in.}$$



$$P_3 = 6(4.176) = 25.06 \text{ kips}$$

$$\sum M_{L2} = 86.8(4.40) - 25.06\left(\frac{6}{2}\right) - P_1(6) = 0 \quad \Rightarrow \quad P_1 = 51.12 \text{ kips}$$

$$\sum F = 86.8 - 25.06 - 51.12 - P_2 = 0 \quad \Rightarrow \quad P_2 = 10.62 \text{ kips}$$

$$L_1 = \frac{P_1}{4.176} = \frac{51.12}{4.176} = 12.2 \text{ in. Use } 12\tfrac{1}{2} \text{ in.}$$

$$L_2 = \frac{P_2}{4.176} = \frac{10.62}{4.176} = 2.54 \text{ in. Use 3 in.}$$

For the second option, the strength of the longitudinal welds is $0.85(4.176) = 3.550$ kips/in.

and the strength of the transverse weld is $1.5(4.176) = 6.264$ kips/in.

$$P_3 = 6(6.264) = 37.58 \text{ kips}$$

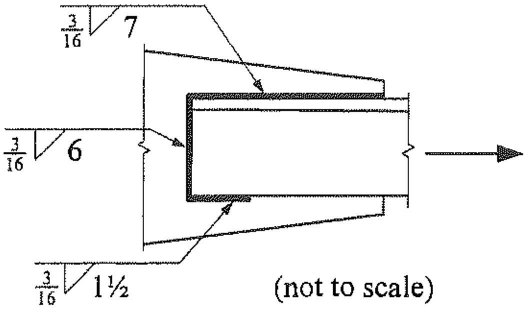
$$\sum M_{L2} = 86.8(4.40) - 37.58\left(\frac{6}{2}\right) - P_1(6) = 0 \quad \Rightarrow \quad P_1 = 44.86 \text{ kips}$$

$$\sum F = 86.8 - 37.58 - 44.86 - P_2 = 0 \quad \Rightarrow \quad P_2 = 4.36 \text{ kips}$$

$$L_1 = \frac{P_1}{3.550} = \frac{44.86}{3.550} = 12.6 \text{ in. Use 7 in.}$$

$$L_2 = \frac{P_2}{3.550} = \frac{4.36}{3.550} = 1.23 \text{ in. Use } 1\tfrac{1}{2} \text{ in.}$$

Use the weld shown.



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8.4-16

(a) $P_a = D + L = 31 + 31 = 62$ kips

For $t = 5/16$ in., min. $w = 3/16$ in. and max $w = 5/16 - 1/16 = 1/4$ in.

Use E70 fillet welds. Try $w = 3/16$ in., one transverse weld and two longitudinal welds. Investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$R_n/\Omega = 0.9279D = 0.9279(3) = 2.784 \text{ kips/in.}$$

Base metal shear strength of the angle:

Yielding: $R_n/\Omega = 0.4F_y t = 0.4(36)(5/16) = 4.5$ kips/in.

Rupture: $R_n/\Omega = 0.3F_u t = 0.3(58)(5/16) = 5.438$ kips/in.

The weld shear strength controls.

$$\text{Total length required} = \frac{62}{2.784} = 22.27 \text{ in.}$$

$$\text{Length of longitudinal welds} = \frac{22.27 - 6}{2} = 8.14 \text{ in.; use } 8\tfrac{1}{2} \text{ in.}$$

For the second option, the strength of the longitudinal welds is

$$0.85(2.784) = 2.366 \text{ kips/in.}$$

and the strength of the transverse weld is $1.5(2.784) = 4.176$ kips/in.

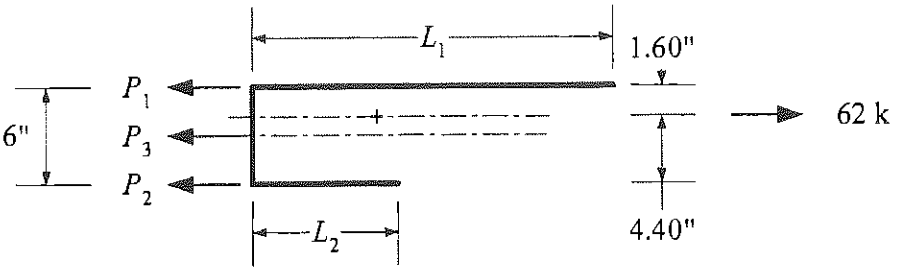
Length of longitudinal welds $= \dfrac{62-4.176(6)}{2.366(2)} = 7.81$ in.; use 8 in.

The second options requires shorter longitudinal welds:

Use $\frac{3}{16}$-in. E70 fillet welds, across the end and 8 inches on each side

(b) Investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$R_n/\Omega = 2.784$ kips/in.



$P_3 = 6(2.784) = 16.7$ kips

$\sum M_{L2} = 62(4.40) - 16.7\left(\frac{6}{2}\right) - P_1(6) = 0 \quad \Rightarrow \quad P_1 = 37.12$ kips

$\sum F = 62 - 16.7 - 37.12 - P_2 = 0 \quad \Rightarrow \quad P_2 = 8.18$ kips

$L_1 = \dfrac{P_1}{2.784} = \dfrac{37.12}{2.784} = 13.3$ in. Use 13½ in.

$L_2 = \dfrac{P_2}{2.784} = \dfrac{8.18}{2.784} = 2.94$ in. Use 3 in.

For the second option, the strength of the longitudinal welds is

$0.85(2.784) = 2.366$ kips/in.

and the strength of the transverse weld is $1.5(2.784) = 4.176$ kips/in.

$P_3 = 6(4.176) = 25.06$ kips

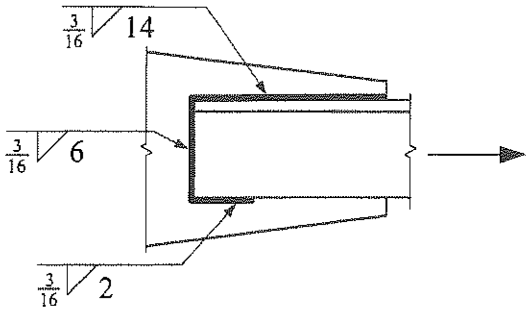
$\sum M_{L2} = 62(4.40) - 25.06\left(\frac{6}{2}\right) - P_1(6) = 0 \quad \Rightarrow \quad P_1 = 32.94$ kips

$\sum F = 62 - 25.06 - 32.94 - P_2 = 0 \quad \Rightarrow \quad P_2 = 4.00$ kips

$L_1 = \dfrac{P_1}{2.366} = \dfrac{32.94}{2.366} = 13.9$ in. Use 14 in.

$L_2 = \dfrac{P_2}{2.366} = \dfrac{4.00}{2.366} = 1.69$ in. Use 2 in.

Use the weld shown.



8.4-17

(a) $P_u = 1.2D + 1.6L = 1.2(40) + 1.6(80) = 176.0$ kips

For $t = 3/8$ in., min. $w = 3/16$ in. and max $w = 5/8 - 1/16 = 9/16$ in.

Use E70 fillet welds. Try $w = 3/16$ in., one transverse weld and two longitudinal welds. Investigate the two options specified in AISC J2.4(c). First, assuming the same strength for both the longitudinal and transverse welds,

$$\phi R_n = 1.392D = 1.392(3) = 4.176 \text{ kips/in.}$$

Base metal shear strength of the angle:

Yielding: $\phi R_n = \phi(0.6F_y t) = 1.00(0.6)(36)(3/8) = 8.1$ kips/in.

Rupture: $\phi R_n = \phi(0.6F_u t) = 0.75(0.6)(58)(3/8) = 9.788$ kips/in.

The weld shear strength controls. Total length required $= \dfrac{176}{4.176} = 42.15$ in.

Length of longitudinal welds $= \dfrac{42.15 - 5}{2} = 18.6$ in.; use 19 in.

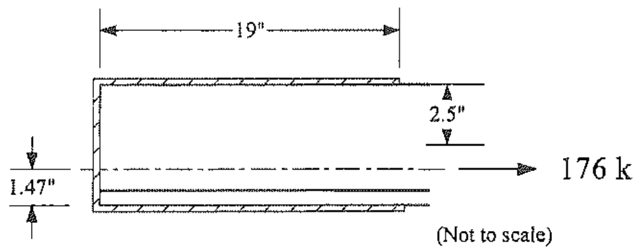
For the second option, the strength of the longitudinal welds is $0.85(4.176) = 3.550$ kips/in.

and the strength of the transverse weld is $1.5(4.176) = 6.264$ kips/in.

Length of longitudinal welds $= \dfrac{176 - 6.264(5)}{3.550(2)} = 20.4$ in.; use 20½ in.

Use 3/16-in. E70 fillet welds, across the end and 19 inches on each side

(b)



(Not to scale)

$$f_{1x} = \frac{176}{2(19)+5} = 4.093 \text{ kips/in.} \rightarrow$$

Locate centroid with respect to left side.

$$\bar{x} = \frac{19(9.5)\times 2}{2(19)+5} = 8.395 \text{ in.}$$

$$I_x = 19(2.5)^2(2) + \frac{(5)^3}{12} = 247.9 \text{ in.}^4$$

$$I_y = 2\left(\frac{(19)^3}{12} + 19(9.5-8.395)^2\right) + 5(8.395)^2 = 1542 \text{ in.}^4$$

$$J = I_x + I_y = 247.9 + 1542 = 1790 \text{ in.}^4$$

$$M = 176(2.5-1.47) = 181.3 \text{ in.-kips}$$

Stress at upper right:

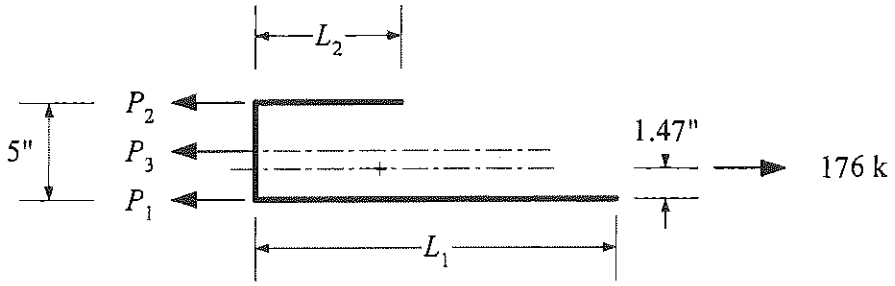
$$f_{2x} = \frac{My}{J} = \frac{181.3(2.5)}{1790} = 0.2532 \text{ kips/in.} \rightarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{181.3(19-8.395)}{1790} = 1.074 \text{ kips/in.} \downarrow$$

$$f_v = \sqrt{\left(\sum f_x\right)^2 + \left(\sum f_y\right)^2} = \sqrt{(4.093+0.2532)^2 + (1.074)^2} = 4.477 \text{ kips/in.}$$

For a 3/16-inch weld (from part a), $\phi R_n = 4.176$ kips/in. < 4.477 kips/in. (N.G.)

Use $w = 3/16$ inch and balance the welds.



Option 1: $P_3 = 5(4.176) = 20.88$ kips

$$\sum M_{L1} = 176(1.47) - 20.88\left(\frac{5}{2}\right) - P_2(5) = 0 \quad \Rightarrow \quad P_2 = 41.3 \text{ kips}$$

$$\sum F = 176 - 20.88 - 41.3 - P_1 = 0 \quad \Rightarrow \quad P_1 = 113.8 \text{ kips}$$

$$L_1 = \frac{P_1}{4.176} = \frac{113.8}{4.176} = 27.3 \text{ in. Use } 27\tfrac{1}{2} \text{ in.}$$

$$L_2 = \frac{P_2}{4.176} = \frac{41.3}{4.176} = 9.89 \text{ in. Use 10 in.}$$

Option 2: The strength of the longitudinal welds is $0.85(4.176) = 3.550$ kips/in. and the strength of the transverse weld is $1.5(4.176) = 6.264$ kips/in.

$$P_3 = 5(6.264) = 31.32 \text{ kips}$$

$$\sum M_{L1} = 176(1.47) - 31.32\left(\frac{5}{2}\right) - P_2(5) = 0 \quad \Rightarrow \quad P_2 = 36.08 \text{ kips}$$

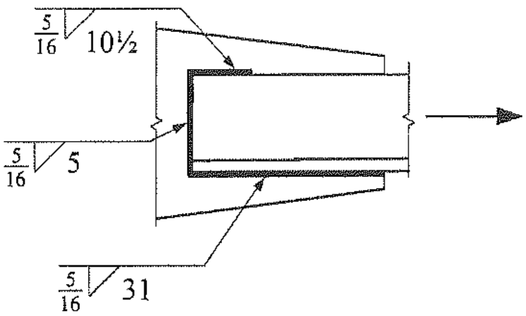
$$\sum F = 176 - 31.32 - 36.08 - P_1 = 0 \quad \Rightarrow \quad P_1 = 108.6 \text{ kips}$$

$$L_1 = \frac{P_1}{3.550} = \frac{108.6}{3.550} = 30.6 \text{ in. Use 31 in.}$$

$$L_2 = \frac{P_2}{3.550} = \frac{36.08}{3.550} = 10.2 \text{ in. Use } 10\tfrac{1}{2} \text{ in.}$$

Option 1 results in a shorter connection. Use the connection shown

(not to scale)



Note that, if a larger weld is used, the connection will be shorter.

Although the problem statement requires that the minimum weld size be used, as was done above, an alternate solution with a larger weld will be presented.

Alternate Solution

(a) Try a 5/16-in. fillet weld. First, assuming the same strength for both the longitudinal and transverse welds,

$$\phi R_n = 1.392D = 1.392(5) = 6.96 \text{ kips/in. (controls over base metal shear strength)}$$

$$\text{Total length required} = \frac{176}{6.96} = 25.29 \text{ in.}$$

$$\text{Length of longitudinal welds} = \frac{25.29 - 5}{2} = 10.2 \text{ in.; use } 10\tfrac{1}{2} \text{ in.}$$

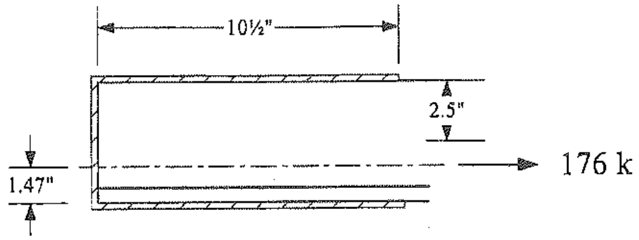
For the second option, the strength of the longitudinal welds is $0.85(6.96) = 5.916$ kips/in.

and the strength of the transverse weld is $1.5(6.96) = 10.44$ kips/in.

$$\text{Length of longitudinal welds} = \frac{176 - 10.44(5)}{5.916(2)} = 10.5 \text{ in.; use } 10\tfrac{1}{2} \text{ in.}$$

Use 5/16-in. E70 fillet welds, across the end and 10½ inches on each side

(b)



$$f_{1x} = \frac{176}{2(10.5)+5} = 6.769 \text{ kips/in.} \rightarrow$$

Locate centroid with respect to left side.

$$\bar{x} = \frac{10.5(5.25)\times 2}{2(10.5)+5} = 4.24 \text{ in.}$$

$$I_x = 10.5(2.5)^2(2) + \frac{(5)^3}{12} = 141.7 \text{ in.}^4$$

$$I_y = 2\left(\frac{(10.5)^3}{12} + 10.5(5.25-4.24)^2\right) + 5(4.24)^2 = 304.2 \text{ in.}^4$$

$$J = I_x + I_y = 141.7 + 304.2 = 445.9 \text{ in.}^4$$

$$M = 176(2.5-1.47) = 181.3 \text{ in.-kips}$$

Stress at upper right:

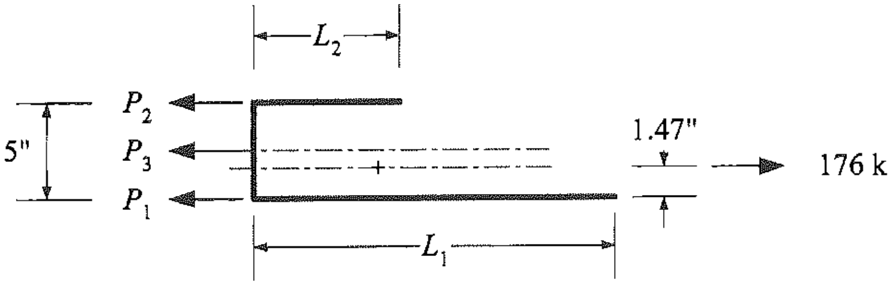
$$f_{2x} = \frac{My}{J} = \frac{181.3(2.5)}{445.9} = 1.016 \text{ kips/in.}\rightarrow$$

$$f_{2y} = \frac{Mx}{J} = \frac{181.3(10.5-4.24)}{445.9} = 2.545 \text{ kips/in.}\downarrow$$

$$f_v = \sqrt{(\sum f_x)^2 + (\sum f_y)^2} = \sqrt{(6.769+1.016)^2 + (2.545)^2} = 8.19 \text{ kips/in.}$$

For a 5/16-inch weld (from part a), $\phi R_n = 6.96$ kips/in. < 8.19 kips/in. (N.G.)

Use $w = 5/16$ inch and balance the welds.



Option 1: $P_3 = 5(6.96) = 34.8$ kips

$$\sum M_{L1} = 176(1.47) - 34.8\left(\frac{5}{2}\right) - P_2(5) = 0 \quad \Rightarrow \quad P_2 = 34.34 \text{ kips}$$

$$\sum F = 176 - 34.8 - 34.34 - P_1 = 0 \quad \Rightarrow \quad P_1 = 106.9 \text{ kips}$$

$$L_1 = \frac{P_1}{6.96} = \frac{106.9}{6.96} = 15.4 \text{ in. Use } 15\tfrac{1}{2} \text{ in.}$$

$$L_2 = \frac{P_2}{6.96} = \frac{34.34}{6.96} = 4.93 \text{ in. Use 5 in.}$$

Option 2: The strength of the longitudinal welds is $0.85(6.96) = 5.916$ kips/in. and the strength of the transverse weld is $1.5(6.96) = 10.44$ kips/in.

$$P_3 = 5(10.44) = 52.2 \text{ kips}$$

$$\sum M_{L1} = 176(1.47) - 52.2\left(\frac{5}{2}\right) - P_2(5) = 0 \quad \Rightarrow \quad P_2 = 25.64 \text{ kips}$$

$$\sum F = 176 - 52.2 - 25.64 - P_1 = 0 \quad \Rightarrow \quad P_1 = 98.16 \text{ kips}$$

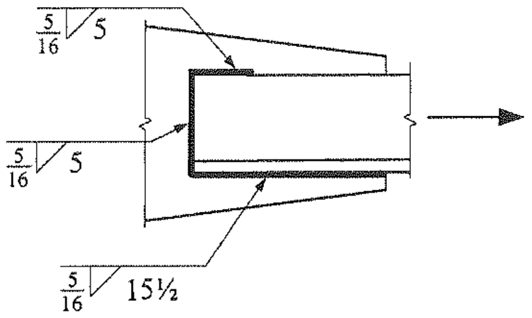
$$L_1 = \frac{P_1}{5.916} = \frac{98.16}{5.916} = 16.6 \text{ in. Use 17 in.}$$

$$L_2 = \frac{P_2}{5.916} = \frac{25.64}{5.916} = 4.33 \text{ in. Use } 4\tfrac{1}{2} \text{ in.}$$

Option 1 results in a shorter connection. Use the connection shown

(Note that, if a larger weld is used, the connection will be shorter.)

(not to scale)



8.4-19

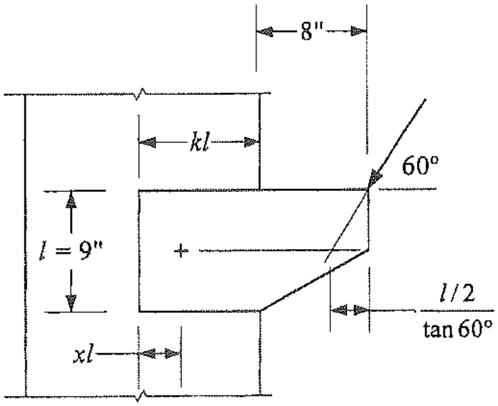
(a) $P_u = 1.2D + 1.6L = 1.2(6) + 1.6(30) = 55.2$ kips

For $t = 3/8$ in., min. $w = 3/16$ in. and max $w = 3/8 - 1/16 = 5/16$ in.

Use E70 electrodes, ultimate strength analysis, and the tables in Part 8 of the Manual.

For $D = 3$ ($w = 3/16$ in.), $C_{\min} = \dfrac{P_u}{\phi C_1 D\ell} = \dfrac{55.2}{0.75(1.0)(3)(9)} = 2.73$

Try a C-shaped weld. Use Table 8-8, Angle = 30°.



$$e_x = 8 - \frac{9/2}{\tan 60^\circ} + k\ell - xl = 5.402 + k\ell - xl$$

$$= 5.402 + 9k - 9x$$

$$a = \frac{e_x}{\ell} = \frac{5.402 + 9k - 9x}{9} = 0.6002 + k - x$$

Try $k = 1.0$.

$x = 0.333, \qquad a = 0.6002 + k - x = 0.6002 + 1.0 - 0.333 = 1.267$

$C = 2.77 > 2.73$ (OK)

(This will be the shortest total length that will work for the minimum weld size.)

Volume of weld metal $= \frac{1}{2}\left(\frac{3}{16}\right)^2(9 + 9 + 9) = 0.4746$ in.^{3}

Try an L-shaped weld. Use table 8-10, Angle = 30°.

$$e_x = 8 - \frac{y\ell}{\tan 60^\circ} + k\ell - xl = 8.0 - 0.5774y\ell + k\ell - xl$$

$$= 8.0 - 0.5774y(9) + k(9) - x(9) = 8.0 - 5.197y + 9.0k - 9.0x$$

$$a = \frac{e_x}{\ell} = \frac{8.0 - 5.197y + 9.0k - 9.0x}{9} = 0.8889 - 0.5774y + k - x$$

Try $k = 1.0$.

$x = 0.250, \qquad y = 0.250$

$a = 0.8889 - 0.5774(0.250) + 1.0 - 0.250 = 1.495$

$C = 1.38 < 2.73$ (N.G. for $D = 3$)

For $D = 5$ (maximum weld size),

$$C_{\min} = \frac{P_u}{\phi C_1 D\ell} = \frac{55.2}{0.75(1.0)(5)(9)} = 1.64 > 1.38 \qquad \text{(N.G.)}$$

This configuration will not work.

Try two parallel welds. Use table 8-5, Angle = 30°.

$$k\ell = 9 \text{ in.}, \qquad \ell = \frac{9}{k}$$

$$e_x = 8 - \frac{9/2}{\tan 60°} + \frac{\ell}{2} = 8 - \frac{9/2}{\tan 60°} + \frac{9/k}{2} = 5.402 + \frac{4.5}{k}$$

$$a = \frac{e_x}{\ell} = \frac{5.402 + 4.5/k}{\ell} = \frac{5.402 + 4.5/k}{9} = 0.6002 + \frac{0.5}{k}$$

Try $D = 3$ ($C_{\min} = 2.73$) and $k = 1.0$:

$$a = 0.6002 + \frac{0.5}{1.0} = 1.1$$

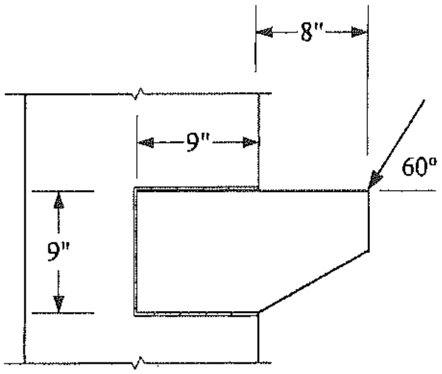
$$C = 2.05 < 2.73 \qquad \text{(N.G.)}$$

Try $D = 4$, $C_{\min} = \dfrac{P_u}{\phi C_1 D\ell} = \dfrac{55.2}{0.75(1.0)(4)(9)} = 2.04 < 2.05$ (OK)

Volume of weld metal $= \frac{1}{2}\left(\frac{1}{4}\right)^2(9+9) = 0.5625 \text{ in.}^3$

This is larger than that required for the C-shaped weld with $w = \frac{3}{16}$ inch.

Use a $\frac{3}{16}$-in. fillet weld as shown.



(b) This weld uses the smallest volume of weld that will work for the available length.

8.5-1

Direct shear: $f_v = \dfrac{15}{2(2+10)} = 0.625$ kips/in.

Tension: Locate neutral axis with respect to top.

$$\bar{y} = \frac{2(10)(5)}{24} = 4.167 \text{ in.}, \qquad M = 15(4) = 60.0 \text{ in.-kips}$$

$$I = 2\left[\frac{(10)^3}{12} + 10(5 - 4.167)^2 + 2(4.167)^2\right] = 250.0 \text{ in.}^4$$

$$f_t = \frac{Mc}{I} = \frac{60(4.167)}{250} = 1.0 \text{ kips/in.}$$

$$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{(0.625)^2 + (1.0)^2} = 1.18 \text{ kips/in.} \qquad \underline{f_r = 1.18 \text{ kips/in.}}$$

8.5-2

Direct shear: $f_v = \dfrac{15}{8 + 6(2)} = 0.75$ kips/in.

Tension: Locate neutral axis with respect to bottom.

$$\bar{y} = \frac{8(6) + 2(5)(2) + 2(1)(2)}{20} = 3.6 \text{ in.,} \qquad M = 15(3) = 45.0 \text{ in.-kips}$$

$$I = 8(6-3.6)^2 + 2\left[\frac{(2)^3}{12} + 2(5-3.6)^2\right] + 2\left[\frac{(2)^3}{12} + 2(2.6)^2 + 2(3.6)^2\right]$$

$$= 135.5 \text{ in.}^4$$

$$f_t = \frac{Mc}{I} = \frac{45(6-3.6)}{135.5} = 0.797 \text{ kips/in.}$$

$$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{(0.75)^2 + (0.797)^2} = 1.09 \text{ kips/in.} \qquad \underline{f_r = 1.09 \text{ kips/in.}}$$

8.5-3

Maximum weld size is $w = 5/8 - 1/16 = 9/16$ in

(a) LRFD solution

Weld strength: $\phi R_n = 1.392D = 1.392(9) = 12.53$ kips/in.

Base metal shear strength of the W14 × 48:

Yielding: $\phi R_n = \phi(0.6F_y t) = 1.00(0.6)(50)(0.595) = 17.85$ kips/in.

Rupture: $\phi R_n = \phi(0.6F_u t) = 0.75(0.6)(65)(0.595) = 17.40$ kips/in.

Base metal shear strength of the L6 × 6 × 5/8:

Yielding: $\phi R_n = \phi(0.6F_y t) = 1.00(0.6)(36)(5/8) = 13.50$ kips/in.

Rupture: $\phi R_n = \phi(0.6F_u t) = 0.75(0.6)(58)(5/8) = 16.31$ kips/in.

The weld shear strength controls; $\phi R_n = 12.53$ kips/in.

Direct shear: $f_v = \dfrac{R_u}{2(6)} = \dfrac{1}{12}R_u$ kips/in

Tension: $M = 4R_u$

$$I = \frac{(6)^3}{12} \times 2 = 36.0 \text{ in.}^4, \qquad f_t = \frac{Mc}{I} = \frac{4R_u(3)}{36} = \frac{1}{3}R_u \text{ kips/in.}$$

$$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{\left(\frac{R_u}{12}\right)^2 + \left(\frac{R_u}{3}\right)^2} = 0.3436R_u \text{ kips/in.}$$

From $0.3436R_u = 12.53$, $\underline{R_u = 36.5 \text{ kips}}$

(b) ASD solution

Weld strength: $\frac{R_n}{\Omega} = 0.9279D = 0.9279(9) = 8.351$ kips/in.

Base metal shear strength of the W14 × 48:

$$\text{Yielding: } \frac{R_n}{\Omega} = \frac{0.6F_y t}{1.50} = \frac{0.6(50)(0.595)}{1.50} = 11.90 \text{ kips/in}$$

$$\text{Rupture: } \frac{R_n}{\Omega} = \frac{0.6F_u t}{2.00} = \frac{0.6(65)(0.595)}{2.00} = 11.60 \text{ kips/in.}$$

Base metal shear strength of the L6 × 6 × 5/8:

$$\text{Yielding: } \frac{R_n}{\Omega} = \frac{0.6F_y t}{1.50} = \frac{0.6(36)(5/8)}{1.50} = 9.0 \text{ kips/in}$$

$$\text{Rupture: } \frac{R_n}{\Omega} = \frac{0.6F_u t}{2.00} = \frac{0.6(58)(5/8)}{2.00} = 10.88 \text{ kips/in.}$$

The weld shear strength controls; $\frac{R_n}{\Omega} = 8.351$ kips/in.

Direct shear: $f_v = \frac{R_a}{2(6)} = \frac{1}{12}R_a$ kips/in

Tension: $M = 4R_a$

$$I = \frac{(6)^3}{12} \times 2 = 36.0 \text{ in.}^4, \quad f_t = \frac{Mc}{I} = \frac{4R_a(3)}{36} = \frac{1}{3}R_a \text{ kips/in.}$$

$$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{\left(\frac{R_a}{12}\right)^2 + \left(\frac{R_a}{3}\right)^2} = 0.3436R_a \text{ kips/in.}$$

From $0.3436R_a = 8.351$, $\underline{R_a = 24.3 \text{ kips}}$

8.5-4

(a) LRFD solution

$$P_u = 1.2D + 1.6L = 1.2(8) + 1.6(18) = 38.4 \text{ kips}$$

$$P_{ux} = 38.4\cos 70° = 13.13 \text{ kips}, \quad P_{uy} = 38.4\sin 70° = 36.08 \text{ kips}$$

Direct shear: $f_v = \frac{36.08}{2(12)} = 1.503$ kips/in.

Tension: $f_{t1} = \frac{13.13}{24} = 0.5471$ kips/in

$$M = 13.13(6) + 36.08(6) = 295.3 \text{ in.-kips}, \quad I = \frac{(12)^3}{12}(2) = 288.0 \text{ in.}^4$$

$$f_{t2} = \frac{Mc}{I} = \frac{295.3(6)}{288} = 6.152 \text{ kips/in.}$$

$$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{(1.503)^2 + (0.5471 + 6.152)^2} = 6.866 \text{ kips/in.}$$

$$D = \frac{6.866}{1.392} = 4.93 \text{ sixteenths. Try } w = \tfrac{5}{16} \text{ in.}$$

Check base metal shear (bracket plate controls)

$$f_v = \frac{36.08}{12} = 3.01 \text{ kips/in.}$$

Yielding: $\phi R_n = 0.6F_y t = 0.6(36)(3/8) = 8.1$ kips/in. > 3.01 kips/in. (OK)

Rupture: $\phi R_n = 0.45F_u t = 0.45(58)(3/8) = 9.79$ kips/in. > 3.01 kips/in. (OK)

Use a $\frac{5}{16}$-in. fillet weld.

(b) ASD solution

$P_a = D + L = 8 + 18 = 26$ kips

$P_{ax} = 26\cos 70° = 8.893$ kips, $P_{ay} = 26\sin 70° = 24.43$ kips

Direct shear: $f_v = \frac{24.43}{2(12)} = 1.018$ kips/in.

Tension: $f_{t1} = \frac{8.893}{24} = 0.3705$ kips/in

$M = 8.893(6) + 24.43(6) = 199.9$ in.-kips, $I = \frac{(12)^3}{12}(2) = 288.0 \text{ in.}^4$

$$f_{t2} = \frac{Mc}{I} = \frac{199.9(6)}{288} = 4.165 \text{ kips/in.}$$

$$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{(1.018)^2 + (0.3705 + 4.165)^2} = 4.648 \text{ kips/in.}$$

$$D = \frac{4.648}{0.9279} = 5.01 \text{ sixteenths. Try } w = \tfrac{3}{8} \text{ in.}$$

Check base metal shear (bracket plate controls)

$$f_v = \frac{24.43}{12} = 2.04 \text{ kips/in.}$$

Yielding: $R_n/\Omega = 0.4F_y t = 0.4(36)(3/8) = 5.4$ kips/in. > 2.04 kips/in. (OK)

Rupture: $R_n/\Omega = 0.3F_u t = 0.3(58)(3/8) = 6.53$ kips/in. > 2.04 kips/in. (OK)

Use a $\frac{3}{8}$-in. fillet weld.

8.5-5

(a) LRFD solution

Weld strength: $\phi R_n = 1.392D = 1.392(5) = 6.96$ kips/in.

Base metal shear strength (flange of WT controls):

Yielding: $\phi R_n = 0.6F_yt = 0.6(50)(0.855) = 25.65$ kips/in.

Rupture: $\phi R_n = 0.45F_ut = 0.45(65)(0.855) = 25.01$ kips/in.

The weld shear strength controls; $\phi R_n = 6.96$ kips/in.

Direct shear: $f_v = \dfrac{P_u}{2(16)} = 0.03125P_u$ kips/in

Tension: $M = 6P_u$

$$I = \frac{(16)^3}{12} \times 2 = 682.7 \text{ in.}^4, \qquad f_t = \frac{Mc}{I} = \frac{6P_u(8)}{682.7} = 0.07031P_u \text{ kips/in.}$$

$$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{(0.03125P_u)^2 + (0.07031P_u)^2} = 0.07694P_u \text{ kips/in.}$$

From $0.07694P_u = 6.96$, $\underline{P_u = 90.5 \text{ kips}}$

(b) ASD solution

Weld strength: $\dfrac{R_n}{\Omega} = 0.9279D = 0.9279(5) = 4.640$ kips/in.

Base metal shear strength (flange of WT controls):

Yielding: $\dfrac{R_n}{\Omega} = 0.4F_yt = 0.4(50)(0.855) = 17.1$ kips/in

Rupture: $\dfrac{R_n}{\Omega} = 0.3F_ut = 0.3(65)(0.855) = 16.67$ kips/in.

The weld shear strength controls; $\dfrac{R_n}{\Omega} = 4.640$ kips/in.

Direct shear: $f_v = \dfrac{P_a}{2(16)} = 0.03125P_a$ kips/in.

Tension: $M = 6P_a$

$$I = \frac{(16)^3}{12} \times 2 = 682.7 \text{ in.}^4, \qquad f_t = \frac{Mc}{I} = \frac{6P_a(8)}{682.7} = 0.07031P_a \text{ kips/in.}$$

$$f_r = \sqrt{f_v^2 + f_t^2} = \sqrt{(0.03125P_a)^2 + (0.07031P_a)^2} = 0.07694P_a \text{ kips/in.}$$

From $0.07694P_a = 4.640$, $\underline{P_a = 60.3 \text{ kips}}$

8.6-1

Nominal bolt shear strength: $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

For A325-N bolts,

$R_n = F_{nv}A_b = 48(0.6013) = 28.86$ kips/bolt

(a) LRFD solution

$\phi R_n = 0.75(28.86) = 21.65$ kips/bolt

For 8 bolts, $\phi R_n = 8(21.65) = 173$ kips

$$M_u = 1.2M_D + 1.6M_L = 1.2(45) + 1.6(135) = 270.0 \text{ ft-kips}$$

$$H = \frac{M_u}{d} = \frac{270(12)}{18.0} = 180 \text{ kips} > 173 \text{ kips} \qquad \text{(N.G.)}$$

Bolts do not have enough shear strength.

(b) ASD solution

$$\frac{R_n}{\Omega} = \frac{28.86}{2.00} = 14.43 \text{ kips/bolt}$$

For 8 bolts, $\frac{R_n}{\Omega} = 8(14.43) = 115$ kips

$$H = \frac{M_a}{d} = \frac{180(12)}{18.0} = 120 \text{ kips} > 115 \text{ kips} \qquad \text{(N.G.)}$$

Bolts do not have enough shear strength.

8.6-2

(a) Web plate:

Check bolt shear. $A_b = \pi d^2/4 = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

Shear capacity of one bolt is $\phi R_n = \phi F_{nv} A_b = 0.75(48)(0.6013) = 21.65$ kips/bolt

For 4 bolts, 4(21.65) = 86.6 kips

Check bearing on plate (plate is thinner than beam web, and F_u is smaller):

$$h = \frac{7}{8} + \frac{1}{16} = \frac{15}{16} \text{ in.}$$

For the hole nearest the edge, $L_e = 2.25$ in.

$$L_c = L_e - \frac{h}{2} = 2.25 - \frac{15/16}{2} = 1.781 \text{ in.}$$

$$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(1.781)(5/16)(58) = 29.05 \text{ kips}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(7/8)(5/16)(58) = 28.55 \text{ kips} < 29.05 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 28.55 \text{ kips/bolt}$$

For other bolts, $s = 3$ in.

$$L_c = s - h = 3 - \frac{15}{16} = 2.062 \text{ in.}$$

$$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(2.062)(5/16)(58) = 33.64 \text{ kips}$$

$$\phi(2.4dtF_u) = 28.55 \text{ kips} < 33.64 \text{ kips} \;\therefore \text{ use } \phi R_n = 28.55 \text{ kips/bolt}$$

For 4 bolts, $4(28.55) = 114$ kips

Check plate shear yielding strength:

$$\phi R_n = 1.00(0.60F_y A_g) = 1.00(0.6)(36)(13.5 \times 5/16) = 91.1 \text{ kips}$$

Check shear rupture strength. Use a hole diameter of $\frac{7}{8}+\frac{1}{8}=1$ in.

$$A_{nv}=(13.5-4\times1.0)\left(\frac{5}{16}\right)=2.969 \text{ in.}^2$$

$$\phi R_n=\phi(0.60F_uA_{nv})=0.75(0.60)(58)(2.969)=77.5 \text{ kips}$$

Check block shear. The shear areas are

$$A_{gv}=\frac{5}{16}(11.25)=3.516 \text{ in.}^2$$

$$A_{nv}=\frac{5}{16}\left[11.25-3.5\left(\frac{7}{8}+\frac{1}{8}\right)\right]=2.422 \text{ in.}^2$$

The tension area is $A_{nt}=\frac{5}{16}\left[2.25-0.5\left(\frac{7}{8}+\frac{1}{8}\right)\right]=0.5469 \text{ in.}^2$

$$R_n=0.6F_uA_{nv}+U_{bs}F_uA_{nt}$$

$$=0.6(58)(2.422)+1.0(58)(0.5469)=116.0 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv}+U_{bs}F_uA_{nt}=0.6(36)(3.516)+1.0(58)(0.5469)=107.7 \text{ kips}$$

The nominal block shear strength is therefore 107.7 kips. The design block shear strength is

$$\phi R_n=0.75(107.7)=80.8 \text{ kips}$$

Check connection of shear plate to column flange.

Weld strength = 1.392×4 sixteenths = 5.568 kips/in.

For two welds, use 2(5.568) = 11.14 kips/in.

Base metal (plate) shear strength:

Yielding: $\phi R_n=0.6F_yt=0.6(36)(5/16)=6.75$ kips/in.

Rupture: $\phi R_n=0.45F_ut=0.45(58)(5/16)=8.156$ kips/in.

Plate shear yielding controls: Use $\phi R_n=6.75$ kips/in. × 13.5 in. = 91.1 kips

Summary:

Bolt shear strength = 86.6 kips
Plate bearing strength = 114 kips
Plate shear yielding strength = 91.1 kips
Plate shear rupture strength = 77.5 kips
Plate block shear strength = 80.8 kips
Weld (base metal shear) strength = 91.1 kips.

Shear rupture controls: Shear design strength = 77.5 kips

(b) Top flange plate.

Tension on the gross section: $\phi_tP_n=0.90(36)(5/8\times7)=142$ kips

Tension on the net section: $A_n=(7-2\times1.0)(5/8)=3.125 \text{ in.}^2$

$\phi_t P_n = 0.75(58)(3.125) = 135.9$ kips

Check bolt shear. From part (a), $\phi R_n = 21.65$ kips/bolt

For 8 bolts, $8(21.65) = 173$ kips

Check bearing.

For the hole nearest the edge, $L_e = 2.25$ in.

$$L_c = L_e - \frac{h}{2} = 2.25 - \frac{15/16}{2} = 1.781 \text{ in.}$$

$$\phi R_n = \phi(1.2L_c tF_u) = 0.75(1.2)(1.781)(5/8)(58) = 58.11 \text{ kips}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(7/8)(5/8)(58) = 57.09 \text{ kips} < 58.11 \text{ kips}$$

$\therefore$ use $\phi R_n = 57.09$ kips/bolt

For other bolts, $s = 3$ in.

$$L_c = s - h = 3 - \frac{15}{16} = 2.062 \text{ in.}$$

$$\phi R_n = \phi(1.2L_c tF_u) = 0.75(1.2)(2.062)(5/8)(58) = 67.27 \text{ kips}$$

$\phi(2.4dtF_u) = 57.09$ kips < 67.27 kips $\therefore$ use $\phi R_n = 57.09$ kips/bolt

For 8 bolts, $8(57.09) = 457$ kips

Check block shear on the plate.

Shear areas: $A_{gv} = \frac{5}{8}(11.25) \times 2 = 14.06 \text{ in.}^2$

$A_{nv} = \frac{5}{8}[11.25 - 3.5(1.0)] \times 2 = 9.688 \text{ in.}^2$ (3.5 hole diameters each side)

Tension area: $A_{nt} = \frac{5}{8}[2.5 - 1(1.0)] = 0.9375 \text{ in.}^2$ (using 0.5+0.5 hole diameters)

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(9.688) + 1.0(58)(0.9375) = 391.5 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(14.06) + 1.0(58)(0.9375) = 358.1 \text{ kips}$$

The nominal block shear strength is therefore 358.1 kips. The design block shear strength is

$$\phi R_n = 0.75(358.1) = 269 \text{ kips}$$

Check block shear on the beam flange:

Shear areas: $A_{gv} = 0.565(11.25) \times 2 = 12.71 \text{ in.}^2$

$A_{nv} = 0.565[11.25 - 3.5(1.0)] \times 2 = 8.758 \text{ in.}^2$ (3.5 hole diameters each side)

Tension area: $A_{nt} = 0.565[2.25 - 0.5(1.0)](2) = 1.978 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(65)(8.758) + 1.0(65)(1.978) = 470.1 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(50)(12.71) + 1.0(65)(1.978) = 509.9 \text{ kips}$$

The nominal block shear strength is therefore 470.1 kips. The design block shear strength is

$$\phi R_n = 0.75(470.1) = 353 \text{ kips}$$

Check compression in the bottom plate. Assume that the plate acts as a fixed-end compression member between the end fastener and the weld. Use

$$KL = 0.65(2.25 + 0.5) = 1.788 \text{ in.}$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{7(5/8)^3/12}{7(5/8)}} = 0.1804 \text{ in.}, \qquad \frac{KL}{r} = \frac{1.788}{0.1804} = 9.911$$

From AISC J4.4, for compression elements with $KL/r < 25$, the nominal strength is

$$P_n = F_yA_g$$

$$\therefore\ \phi_cP_n = \phi_cF_yA_g = 0.90(36)(7 \times 5/8) = 142 \text{ kips}$$

Summary:

Tension on gross section: 142 kips
Tension on net section: 135.9 kips
Bolt shear: 173 kips
Bearing: 457 kips
Block shear on flange plate: 269 kips
Block shear on beam flange: 353 kips
Compression on bottom flange plate: 142 kips

Tension on net section controls. Use $H = 135.9$ kips and a moment arm equal to the beam depth (this is conservative).

$$\phi M_n = Hd = 135.9(16.1) = 2188 \text{ in.-kips} = 182 \text{ ft-kips}$$

Check beam for the effect of bolt holes in the tension flange. The gross area of one flange is

$$A_{fg} = t_fb_f = 0.565(7.04) = 3.978 \text{ in.}^2$$

The effective hole diameter is $d_h = \frac{7}{8} + \frac{1}{8} = 1$ in.

$$A_{fn} = A_{fg} - t_f\sum d_h = 3.978 - 0.565(2 \times 1) = 2.848 \text{ in.}^2$$

$$F_uA_{fn} = 65(2.848) = 185.1 \text{ kips}$$

$$Y_tF_yA_{fg} = 1.1(50)(3.978) = 218.8 \text{ kips}$$

Since $F_uA_{fn} < Y_tF_yA_{fg}$, the holes must be accounted for. From AISC Equation F13-1,

$$M_n = \frac{F_uA_{fn}}{A_{fg}}S_x = \frac{185.1}{3.978}(72.7) = 3383 \text{ in.-kips} = 281.9 \text{ ft-kips}$$

$$\phi_bM_n = 0.90(281.9) = 254 \text{ ft-kips} > 182 \text{ ft-kips} \qquad \underline{\phi_bM_n = 182 \text{ ft-kips}}$$

8.6-3

(a) Web plate: The nominal shear strength of one bolt is

$$R_n = F_{nv}A_b = 48(0.6013) = 28.86 \text{ kips}$$

and the allowable strength is

$$\frac{R_n}{\Omega} = \frac{28.86}{2.00} = 14.43 \text{ kips}$$

For 4 bolts, $R_n/\Omega = 4(14.43) = 57.7$ kips

Check bearing on plate (plate is thinner than beam web, and F_u is smaller):

$$h = d + \frac{1}{16} = \frac{7}{8} + \frac{1}{16} = \frac{15}{16} \text{ in.}$$ For the hole nearest the edge,

$$L_c = L_e - \frac{h}{2} = 2.25 - \frac{15/16}{2} = 1.781 \text{ in.}$$

$$\frac{R_n}{\Omega} = \frac{1.2L_ctF_u}{\Omega} = \frac{1.2(1.781)(5/16)(58)}{2.00} = 19.37 \text{ kips}$$

$$\frac{2.4dtF_u}{\Omega} = \frac{2.4(7/8)(5/16)(58)}{2.00} = 19.03 \text{ kips} < 19.37 \text{ kips}$$

$$\therefore \text{ use } \frac{R_n}{\Omega} = 19.03 \text{ kips/bolt}$$

For the other bolts, $s = 3$ in.

$$L_c = s - h = 3 - 15/16 = 2.063 \text{ in.}$$

$$\frac{R_n}{\Omega} = \frac{1.2L_ctF_u}{\Omega} = \frac{1.2(2.063)(5/16)(58)}{2.00} = 22.44 \text{ kips}$$

$$\frac{2.4dtF_u}{\Omega} = 19.03 \text{ kips} < 22.44 \text{ kips}$$

$$\therefore \text{ use } \frac{R_n}{\Omega} = 19.03 \text{ kips/bolt}$$

For the connection, $R_n/\Omega = 4(19.03) = 76.1$ kips

Shear yielding: from AISC Equation J4-3,

$$R_n = 0.60F_yA_g = 0.60(36)\left(\frac{5}{16} \times 13.5\right) = 91.13 \text{ kips}$$

$$\frac{R_n}{\Omega} = \frac{91.13}{1.50} = 60.8 \text{ kips}$$

For shear rupture, the net area is

$$A_{nv} = \left[13.5 - 4\left(\frac{7}{8} + \frac{1}{8}\right)\right](5/16) = 2.969 \text{ in.}^2$$

From AISC Equation J4-4,

$$R_n = 0.6F_uA_{nv} = 0.6(58)(2.969) = 103.3 \text{ kips}$$

$$\frac{R_n}{\Omega} = \frac{103.3}{2.00} = 51.7 \text{ kips}$$

Connection to column flange:

Shear strength of weld $= 0.9279D = 0.9279(4) = 3.712$ kips/in. For two welds, use $2(3.712) = 7.424$ kips/in.

Base metal shear strength:

yield strength: $\frac{R_n}{\Omega} = 0.4F_y t = 0.4(36)\left(\frac{5}{16}\right) = 4.5$ kips/in.

rupture strength: $\frac{R_n}{\Omega} = 0.3F_u t = 0.3(58)\left(\frac{5}{16}\right) = 5.438$ kips/in.

Base metal shear yield strength controls. Let

$$\frac{V_a}{13.5} = 4.5, \qquad V_a = 60.8 \text{ kips}$$

Check block shear. The shear areas are

$$A_{gv} = \frac{5}{16}(11.25) = 3.516 \text{ in.}^2$$

$$A_{nv} = \frac{5}{16}\left[11.25 - 3.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 2.422 \text{ in.}^2$$

The tension area is $A_{nt} = \frac{5}{16}\left[2.25 - 0.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 0.5469 \text{ in.}^2$

$$R_n = 0.6F_u A_{nv} + U_{bs} F_u A_{nt}$$

$$= 0.6(58)(2.422) + 1.0(58)(0.5469) = 116.0 \text{ kips}$$

with an upper limit of

$$0.6F_y A_{gv} + U_{bs} F_u A_{nt} = 0.6(36)(3.516) + 1.0(58)(0.5469) = 107.7 \text{ kips}$$

The nominal block shear strength is therefore 107.7 kips. The allowable strength is

$$\frac{R_n}{\Omega} = \frac{107.7}{2.00} = 53.9 \text{ kips}$$

Summary:

Bolt shear strength = 57.7 kips
Plate bearing strength = 76.1 kips
Plate shear yielding strength = 60.8 kips
Plate shear rupture strength = 51.7 kips
Weld (base metal) strength = 60.8 kips.
Plate block shear strength = 53.9 kips

Shear rupture controls: Available shear strength = $V_a = 51.7$ kips

(b) Flange plate:

Bolt shear strength from part (a) is

$\frac{R_n}{\Omega} = 14.43$ kips. For 8 bolts, $R_n/\Omega = 8(14.43) = 115.4$ kips

Check bearing. Bearing strength is proportional to t and F_u. For the plate, $tF_u = (5/8)(58) = 36.25$ kips. For the W16 × 45, $t_f F_u = 0.565(65) = 36.73$ kips. The plate bearing strength controls.

$h = d + \frac{1}{16} = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$ in. For the hole nearest the edge,

$$L_c = L_e - \frac{h}{2} = 2.25 - \frac{15/16}{2} = 1.781 \text{ in.}$$

$$\frac{R_n}{\Omega} = \frac{1.2L_c t F_u}{\Omega} = \frac{1.2(1.781)(5/8)(58)}{2.00} = 38.74 \text{ kips}$$

$$\frac{2.4dtF_u}{\Omega} = \frac{2.4(7/8)(5/8)(58)}{2.00} = 38.06 \text{ kips} < 38.74 \text{ kips}$$

$$\therefore \text{ use } \frac{R_n}{\Omega} = 38.06 \text{ kips/bolt}$$

For the other bolts, $s = 3$ in.

$$L_c = s - h = 3 - 15/16 = 2.063 \text{ in.}$$

$$\frac{R_n}{\Omega} = \frac{1.2L_c t F_u}{\Omega} = \frac{1.2(2.063)(5/8)(58)}{2.00} = 44.87 \text{ kips}$$

$$\frac{2.4dtF_u}{\Omega} = 38.06 \text{ kips} < 44.87 \text{ kips}$$

$$\therefore \text{ use } \frac{R_n}{\Omega} = 38.06 \text{ kips/bolt}$$

For 8 bolts, $R_n/\Omega = 8(38.06) = 304.5$ kips

Tension on the gross section: $\frac{R_n}{\Omega} = \frac{F_y A_g}{\Omega} = \frac{36\left(\frac{5}{8} \times 7\right)}{1.67} = 94.31$ kips

Tension on the net section: $A_n = \frac{5}{8}(7 - 2 \times 1.0) = 3.125 \text{ in.}^2$

$$\frac{R_n}{\Omega} = \frac{F_u A_n}{\Omega} = \frac{58(3.125)}{2.00} = 90.63 \text{ kips}$$

Check compression in the bottom plate. Assume that the plate acts as a fixed-end compression member between the end fastener and the weld. Use

$$KL = 0.65(2.25 + 0.5) = 1.788 \text{ in.}$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{7(5/8)^3/12}{7(5/8)}} = 0.1804 \text{ in.,} \qquad \frac{KL}{r} = \frac{1.788}{0.1804} = 9.911$$

From AISC J4.4, for compression elements with $KL/r < 25$, the nominal strength is

$$P_n = F_y A_g$$

$$\frac{P_n}{\Omega} = \frac{F_y A_g}{1.67} = \frac{36\left(\frac{5}{8} \times 7\right)}{1.67} = 94.31 \text{ kips}$$

Check block shear in the plate. The shear areas are

$$A_{gv} = \frac{5}{8}(11.25) \times 2 = 14.06 \text{ in.}^2$$

$$A_{nv} = \frac{5}{8}\left[11.25 - 3.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] \times 2 = 9.688 \text{ in.}^2$$

The tension area is $A_{nt} = \frac{5}{8}\left[2.5 - 1.0\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 0.9375 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(9.688) + 1.0(58)(0.9375) = 391.5 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(14.06) + 1.0(58)(0.9375) = 358.1 \text{ kips}$$

The nominal block shear strength is therefore 358.1 kips. The allowable strength is

$$\frac{R_n}{\Omega} = \frac{358.1}{2.00} = 179.1 \text{ kips}$$

Check block shear in the beam flange. The shear areas are

$$A_{gv} = 0.565(11.25) \times 2 = 12.71 \text{ in.}^2$$

$$A_{nv} = 0.565\left[11.25 - 3.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] \times 2 = 8.758 \text{ in.}^2$$

The tension area is $A_{nt} = 0.565\left[2.25 - 0.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] \times 2 = 1.978 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(65)(8.758) + 1.0(65)(1.978) = 470.1 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(50)(12.71) + 1.0(65)(1.978) = 509.9 \text{ kips}$$

The nominal block shear strength is therefore 470.1 kips. The allowable strength is

$$\frac{R_n}{\Omega} = \frac{470.1}{2.00} = 235.1 \text{ kips}$$

Summary:

Bolt shear: 115.4 kips
Bearing: 304.5 kips
Tension on gross section: 94.31 kips
Tension on net section: 90.63 kips
Compression on bottom flange plate: 94.31 kips
Block shear in flange plate: 179.1 kips
Block shear in beam flange: 235.1 kips

Tension on the net section controls. Use $H = 90.63$ kips and a moment arm equal to the beam depth (this is conservative).

$$M_a = Hd = 90.63(16.1) = 1459 \text{ in.-kips} = 122 \text{ ft-kips}$$

Check beam for the effect of bolt holes in the tension flange. The gross area of one flange is

$$A_{fg} = t_f b_f = 0.565(7.04) = 3.978 \text{ in.}^2$$

The effective hole diameter is $d_h = \frac{7}{8} + \frac{1}{8} = 1$ in.

$$A_{fn} = A_{fg} - t_f \sum d_h = 3.978 - 0.565(2 \times 1) = 2.848 \text{ in.}^2$$

$$F_u A_{fn} = 65(2.848) = 185.1 \text{ kips}$$

$$Y_t F_y A_{fg} = 1.1(50)(3.978) = 218.8 \text{ kips}$$

Since $F_u A_{fn} < Y_t F_y A_{fg}$, the holes must be accounted for. From AISC Equation F13-1,

$$M_n = \frac{F_u A_{fn}}{A_{fg}} S_x = \frac{185.1}{3.978}(72.7) = 3383 \text{ in.-kips} = 281.9 \text{ ft-kips}$$

$$\frac{M_n}{\Omega} = \frac{281.9}{1.67} = 169 \text{ ft-kips} > 122 \text{ ft-kips} \quad \therefore \text{ tension on the net section controls}$$

$$\underline{M_a = 122 \text{ ft-kips}}$$

8.6-4

LRFD solution

Reaction: $R_u = 1.2D + 1.6L = 1.2(8) + 1.6(21) = 43.2$ kips

Moment: $M_u = 1.2D + 1.6L = 1.2(42) + 1.6(104) = 216.8$ ft-kips

Web plate: neglect eccentricity.

Try $\frac{5}{8}$-in. diameter bolts. Assume that threads are in the plane of shear.

$$A_b = \pi d^2/4 = \pi(5/8)^2/4 = 0.3068 \text{ in.}^2$$

Shear capacity of one bolt is $\phi R_n = \phi F_{nv} A_b = 0.75(48)(0.3068) = 11.04$ kips/bolt

Number of bolts required is

$$\frac{43.2}{11.04} = 3.91 \text{ try 4 bolts}$$

Determine plate thickness required for bearing. Assume that

$$\phi R_n = \phi(2.4 d t F_u) = 0.75(2.4)(5/8)t(58) = 65.25t$$

Load resisted by each bolt $= \frac{43.2}{4} = 10.8$ kips. Let

$$\phi R_n = 10.8 = 65.25t \quad \Rightarrow \quad t = 0.166 \text{ in.} \qquad \text{Try } t = \tfrac{3}{16} \text{ in.}$$

Determine whether plate or beam web controls bearing. For the plate,

$$tF_u = \frac{3}{16}(58) = 10.9 \text{ kips/in.}$$

For the beam web, $t_w F_u = 0.300(65) = 19.5$ kips/in. > 10.9 kips/in. ∴ plate controls.

Check bearing strength assumption. $h = \frac{5}{8} + \frac{1}{16} = \frac{11}{16}$ in.

For the hole nearest the edge, min. $L_e = 1\frac{1}{8}$ in. Try $1\frac{1}{2}$ in.

$$L_c = L_e - \frac{h}{2} = 1.5 - \frac{11/16}{2} = 1.156 \text{ in.}$$

$$\phi R_n = \phi(1.2L_c tF_u) = 0.75(1.2)(1.156)(3/16)(58) = 11.31 \text{ kips}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(5/8)(3/16)(58) = 12.23 \text{ kips} > 11.31 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 11.31 \text{ kips/bolt}$$

For other bolts, min. $s = 2.667(5/8) = 1.667$ in. Use $s = 2\frac{1}{2}$ in.

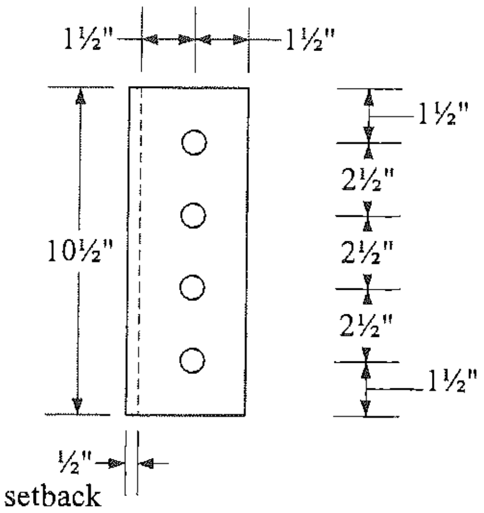
$$L_c = s - h = 2.5 - \frac{11}{16} = 1.813 \text{ in.}$$

$$\phi R_n = \phi(1.2L_c tF_u) = 0.75(1.2)(1.813)(3/16)(58) = 17.74 \text{ kips}$$

$$\phi(2.4dtF_u) = 12.23 \text{ kips} < 17.74 \text{ kips} \therefore \text{ use } \phi R_n = 12.23 \text{ kips/bolt}$$

Total bearing strength $= 11.31 + 3(12.23) = 48.0$ kips > 43.2 kips (OK)

Use four $\frac{5}{8}$-in. diameter A325 bolts .



Determine plate thickness required for shear. Shear yielding strength is

$$\phi R_n = 1.00(0.60F_y A_g)$$

Let $43.2 = 1.00(0.60)(36)(10.5t) \quad \Rightarrow \quad t = 0.191$ in. Try $t = \frac{1}{4}$ in.

Check shear rupture strength. Use hole diameter $= \frac{5}{8} + \frac{1}{8} = \frac{3}{4}$ in.

$$A_{nv} = (10.5 - 4 \times 3/4)\left(\frac{1}{4}\right) = 1.875 \text{ in.}^2$$

$$\phi R_n = \phi(0.60F_u A_{nv}) = 0.75(0.60)(58)(1.875) = 48.9 \text{ kips} > 43.2 \text{ kips} \qquad \text{(OK)}$$

Check block shear.

Shear areas: $A_{gv} = \frac{1}{4}(1.5 + 3 \times 2.5) = \frac{1}{4}(9) = 2.25 \text{ in.}^2$

$$A_{nv} = \frac{1}{4}[9 - 3.5(3/4)] = 1.594 \text{ in.}^2$$

Tension area: $A_{nt} = \frac{1}{4}[1.5 - 0.5(3/4)] = 0.2813 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(1.594) + 1.0(58)(0.2813) = 71.79 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(2.25) + 1.0(58)(0.2813) = 64.92 \text{ kips}$$

The nominal block shear strength is therefore 64.92 kips. The design block shear strength is

$\phi R_n = 0.75(64.92) = 48.7$ kips > 43.2 kips (OK)

Use a PL $\frac{1}{4} \times 3\frac{1}{2} \times 10\frac{1}{2}$ as shown above.

Connection of shear plate to column flange: Use E70 electrodes

Minimum weld size, based on the plate thickness, is $\frac{1}{8}$ inch. Try $w = \frac{1}{8}$ in.

Weld strength = $1.392 \times 2 = 2.784$ kips/in.

Base metal (plate) shear strength:

Yielding: $\phi R_n = 0.6F_yt = 0.6(36)(1/4) = 5.4$ kips/in. > 2.784 kips/in.

Rupture: $\phi R_n = 0.45F_ut = 0.45(58)(1/4) = 6.525$ kips/in. > 2.784 kips/in.

Total length required = $43.2/2.784 = 15.5$ in.

Use a continuous $\frac{1}{8}$ in. fillet weld, both sides of plate.

Flange plate: From $M = Hd$,

$$H = \frac{M}{d} = \frac{216.8(12)}{17.7} = 147.0 \text{ kips}$$

Try $\frac{7}{8}$ in. diameter bolts. Assume that threads are in the plane of shear.

$A_b = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$, $\phi R_n = 0.75(48)(0.6013) = 21.65$ kips/bolt

Number of bolts required for shear is

$\frac{147.0}{21.65} = 6.79$ try 8 bolts (4 pair)

Determine plate thickness required for bearing: $h = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$ in

Minimum $L_e = 1\frac{1}{2}$ in. (use). Minimum $s = 2.667(7/8) = 2.33$ in., use $2\frac{1}{2}$ in.

For the hole nearest the edge, $L_c = L_e - \frac{h}{2} = 1.5 - \frac{15/16}{2} = 1.031$ in.

$\phi R_n = \phi(1.2L_ctF_u) = 0.75(1.2)(1.031)t(58) = 53.82t$ kips

$\phi(2.4dtF_u) = 0.75(2.4)(7/8)t(58) = 91.35t$ kips > $53.82t$ kips

$\therefore$ use $\phi R_n = 53.82t$ kips/bolt

For other bolts, $L_c = s - h = 2.5 - \frac{15}{16} = 1.563$ in.

$$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(1.563)t(58) = 81.59t \text{ kips}$$

$$\phi(2.4dtF_u) = 91.35t \text{ kips} > 81.59t \text{ kips} \therefore \text{ use } \phi R_n = 81.59t \text{ kips/bolt}$$

Total bearing strength = $2(53.82t) + 6(81.59t) = 543.4t$

Let $543.4t = 147.0 \quad \Rightarrow \quad t = 0.271$ in. (required for bearing)

Design top flange plate as a tension connection element (AISC J4.2).

Tension on gross area:

$$\text{Required } A_g = \frac{H}{0.90F_y} = \frac{147.0}{0.9(36)} = 4.537 \text{ in.}^2$$

Tension on net area:

$$\text{Required } A_e = \frac{H}{0.75F_u} = \frac{147.0}{0.75(58)} = 3.379 \text{ in.}^2$$

Try a plate width of $w_g = 7$ in.

For gross area requirement, $t = \frac{A_g}{w_g} = \frac{4.537}{7} = 0.648$ in.

For net area requirement, hole diameter = $\frac{7}{8} + \frac{1}{8} = 1$ in.

$$t = \frac{A_n}{w_n} = \frac{A_n}{w_g - \Sigma d_{hole}} = \frac{3.379}{7 - 2(7/8 + 1/8)} = 0.676 \text{ in. (controls)}$$

Try a plate $\frac{3}{4} \times 7$

$$A_n = [7 - 2(1.0)](3/4) = 3.75 \text{ in.}^2$$

Check compression in the bottom plate. Assume that the plate acts as a fixed-end compression member between the end fastener and the weld. Use

$$KL = 0.65(1.5 + 0.5) = 1.3 \text{ in.}$$

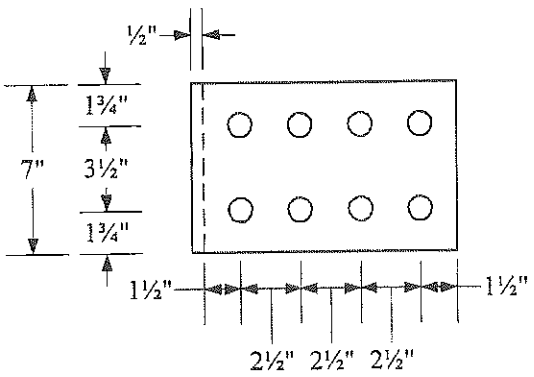
$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{7(5/8)^3/12}{7(5/8)}} = 0.1804 \text{ in.,} \qquad \frac{KL}{r} = \frac{1.3}{0.1804} = 7.21$$

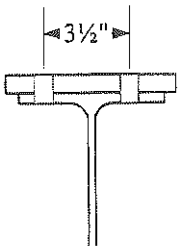
From AISC J4.4, for compression elements with $KL/r < 25$, the nominal strength is

$$P_n = F_y A_g$$

$$\therefore \phi P_n = \phi F_y A_g = 0.90(36)(7 \times 3/4) = 170 \text{ kips} > 147.0 \text{ kips} \qquad \text{(OK)}$$

Check block shear on the plate using the dimensions and bolt layout shown. (Use the "Workable Gage" from Part 1 of the Manual.)





Shear areas: $A_{gv} = \frac{3}{4}(9) \times 2 = 13.5 \text{ in.}^2$

$A_{nv} = \frac{3}{4}[9 - 3.5(1.0)] \times 2 = 8.25 \text{ in.}^2$ (3.5 hole diameters each side)

Tension area: $A_{nt} = \frac{3}{4}[3.5 - 1(1.0)] = 1.875 \text{ in.}^2$ (using 0.5 + 0.5 hole diameters)

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(8.25) + 1.0(58)(1.875) = 395.9 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(13.5) + 1.0(58)(1.875) = 400.4 \text{ kips}$$

The nominal block shear strength is therefore 395.9 kips. The design block shear strength is

$$\phi R_n = 0.75(395.9) = 297 \text{ kips} > 147.0 \text{ kips} \qquad \text{(OK)}$$

Check block shear strength of beam flange.

Transverse spacing = gage distance = 3.5 in.

Transverse edge distance = $\frac{6 - 3.5}{2} = 1.25$ in.

Longitudinal spacing and edge distance same as for plate.

Shear areas: $A_{gv} = 0.425(9) \times 2 = 7.65 \text{ in.}^2$

$A_{nv} = 0.425[9 - 3.5(1.0)] \times 2 = 4.675 \text{ in.}^2$

Tension area: $A_{nt} = 0.425[2.5 - 1(1.0)] = 0.6375 \text{ in.}^2$

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(4.675) + 1.0(58)(0.6375) = 199.7 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(7.65) + 1.0(58)(0.6375) = 202.2 \text{ kips}$$

The nominal block shear strength is therefore 199.7 kips. The design block shear strength is

$\phi R_n = 0.75(199.7) = 149.8 \text{ kips} > 147.0 \text{ kips}$ (OK)

Check beam for the effect of bolt holes in the tension flange. The gross area of one flange is

$$A_{fg} = t_f b_f = 0.425(6.00) = 2.55 \text{ in.}^2$$

The effective hole diameter is $d_h = \frac{7}{8} + \frac{1}{8} = 1$ in.

$$A_{fn} = A_{fg} - t_f \sum d_h = 2.55 - 0.425(2 \times 1) = 1.7 \text{ in.}^2$$

$$F_u A_{fn} = 65(1.7) = 110.5 \text{ kips}$$

$$Y_t F_y A_{fg} = 1.1(50)(2.55) = 140.3 \text{ kips}$$

Since $F_u A_{fn} < Y_t F_y A_{fg}$, the holes must be accounted for. From AISC Equation F13-1,

$$M_n = \frac{F_u A_{fn}}{A_{fg}} S_x = \frac{110.5}{2.55}(57.6) = 2496 \text{ in.-kips} = 208 \text{ ft-kips}$$

$$\phi_b M_n = 0.90(208) = 187 \text{ ft-kips} < 216.8 \text{ ft-kips} \quad \text{(N.G.)}$$

Try a smaller diameter bolt. Try ½-in. diameter bolts.

$$A_b = \pi d^2/4 = \pi(1/2)^2/4 = 0.1963 \text{ in.}^2$$

Nominal shearing strength $= \phi R_n = 0.75(48)(0.1963) = 7.067$ kips/bolt

Number of bolts required for shear is

$$\frac{147.0}{7.067} = 20.8 \text{ try 22 bolts (11 pair)}$$

Bearing and block shear will be satisfactory.

Check reduction in beam flange area: Use a hole diameter $= \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$ in.

$$A_{fn} = A_{fg} - t_f \sum d_h = 2.55 - 0.425(2 \times 5/8) = 2.019 \text{ in.}^2$$

$$F_u A_{fn} = 65(2.019) = 131.2 \text{ kips}$$

$$Y_t F_y A_{fg} = 1.1(50)(2.55) = 140.3 \text{ kips}$$

Since $F_u A_{fn} < Y_t F_y A_{fg}$, the holes must be accounted for. From AISC Equation F13-1,

$$M_n = \frac{F_u A_{fn}}{A_{fg}} S_x = \frac{131.2}{2.55}(57.6) = 2964 \text{ in.-kips} = 247 \text{ ft-kips}$$

$$\phi_b M_n = 0.90(247) = 222.3 \text{ ft-kips} > 216.8 \text{ ft-kips required} \quad \text{(OK)}$$

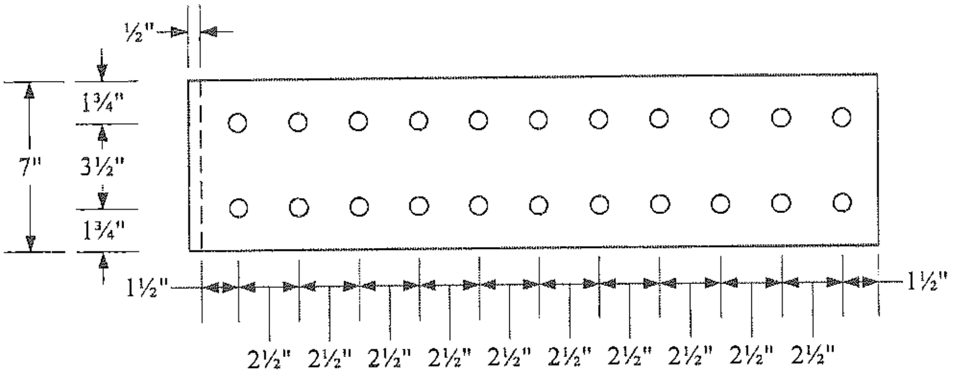
Plate length $= 10(2.5) + 1.5 + 1.5 + 0.5 = 28.5$ in.

Use a PL $\frac{3}{4} \times 7 \times 2' - 4\frac{1}{2}''$ with 22 bolts

Summary:

For the web plate, use a PL $\frac{1}{4} \times 3\frac{1}{2} \times 10\frac{1}{2}$ with four $\frac{5}{8}$-in. diameter A325 bolts as shown. Attach to column with a continuous $\frac{1}{8}$-in. fillet weld on both sides of the plate.

For each flange plate, use a PL $\frac{3}{4} \times 7 \times 2' - 4\frac{1}{2}''$ with twenty-two $\frac{1}{2}$-in. diameter A325 bolts as shown.



8.7-1

W16 × 45 beam, W10 × 45 column, A992 steel. Flange plates are 5/8 × 7.

(a) LRFD solution

$$M_u = 1.2(0.3 \times 118) + 1.6(0.7 \times 118) = 174.6 \text{ ft-kips}$$

The force in the flange plate can be conservatively taken as

$$H = \frac{M_u}{d_b} = \frac{174.6(12)}{16.1} = 130.1 \text{ kips} = P_{bf}$$

Local flange bending: from AISC Eq. J10-1,

$$\phi R_n = \phi(6.25 t_f^2 F_{yf}) = 0.90[6.25(0.620)^2(50)] = 108.1 \text{ kips} < 130.1 \text{ kips} \quad \text{(N.G.)}$$

Local web yielding:

$$\phi R_n = \phi[(5k + N)F_{yw} t_w] = 1.0[5(1.12) + 0.625)](50)(0.350)$$

$$= 108.9 \text{ kips} < 130.1 \text{ kips} \qquad \text{(N.G.)}$$

Web crippling: from AISC Eq. J10-4,

$$\phi R_n = \phi\left\{0.80 t_w^2\left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}}\right\}$$

$$= 0.75\left(0.80(0.350)^2\left[1 + 3\left(\frac{0.625}{10.1}\right)\left(\frac{0.350}{0.620}\right)^{1.5}\right]\sqrt{\frac{29000(50)(0.620)}{0.350}}\right)$$

$$= 127.1 \text{ kips} < 130.1 \text{ kips} \qquad \text{(N.G.)}$$

Local flange bending is the worst case. Required stiffener area is

$$A_{st} = \frac{P_{bf} - \phi R_{n\,\min}}{\phi_{st} F_{yst}} = \frac{130.1 - 108.1}{0.90(36)} = 0.679 \text{ in.}^2$$

Other requirements:

$$b \geq \frac{b_b}{3} - \frac{t_w}{2} = \frac{7}{3} - \frac{0.350}{2} = 2.16 \text{ in.}$$

$$b \leq \frac{b_{f\,col} - t_w}{2} = \frac{8.02 - 0.350}{2} = 3.84 \text{ in.}$$

$$t_{st} \geq \frac{t_b}{2} = \frac{0.625}{2} = 0.313 \text{ in.}$$

Try a plate $\frac{1}{4} \times 3$

$$A_{st} = \tfrac{1}{4}(3)(2) = 1.5 \text{ in.}^2 > 0.679 \text{ in.}^2 \qquad \text{(OK)}$$

$$\frac{b}{15} = \frac{3.5}{15} = 0.233 < 1/4 \qquad \text{(OK)}$$

Full-depth stiffeners are not required for this case. Use a depth of approximately

$$\frac{d}{2} = \frac{10.1}{2} = 5.05 \text{ in.}$$

Use 2 plates $\frac{1}{4} \times 3 \times 0'\text{-}5''$.
Clip inside corners to avoid flange-to-web fillets.

(b) ASD solution

The flange plate force can be conservatively taken as

$$P_{bf} = \frac{M_a}{d_b} = \frac{118(12)}{16.1} = 87.95 \text{ kips}$$

Check local flange bending with AISC Equation J10-1:

$$\frac{R_n}{\Omega} = \frac{6.25 t_f^2 F_{yf}}{\Omega}$$

$$= \frac{6.25(0.620)^2(50)}{1.67} = 71.93 \text{ kips} < 87.95 \text{ kips} \qquad \text{(N.G.)}$$

Check local web yielding with AISC Equation J10-2:

$$\frac{R_n}{\Omega} = \frac{(5k + N)F_{yw}t_w}{\Omega}$$

$$= \frac{[5(1.12) + 0.625](50)(0.350)}{1.50} = 72.63 \text{ kips} < 87.95 \text{ kips} \qquad \text{(N.G.)}$$

Check web crippling with AISC Equation J10-4:

$$\frac{R_n}{\Omega} = \frac{0.80 t_w^2 \left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{E F_{yw} t_f}{t_w}}}{\Omega}$$

$$= \frac{0.80(0.350)^2 \left[1 + 3\left(\frac{0.625}{10.1}\right)\left(\frac{0.350}{0.620}\right)^{1.5}\right]\sqrt{\frac{29000(50)0.620)}{0.350}}}{2.00}$$

$$= 84.71 \text{ kips} < 87.95 \text{ kips} \qquad \text{(N.G.)}$$

Stiffeners are required. The smallest strength is 71.93 kips, for the limit state of local flange bending. The required stiffener area is

$$A_{st} = \frac{P_{bf} - (R_n/\Omega)_{\min}}{F_{yst}/\Omega} = \frac{87.95 - 71.93}{36/1.67} = 0.743 \text{ in.}^2$$

Other requirements:

$$b \geq \frac{b_b}{3} - \frac{t_w}{2} = \frac{7}{3} - \frac{0.350}{2} = 2.16 \text{ in.}$$

$$b \leq \frac{8.02 - 0.350}{2} = 3.84 \text{ in.}$$

$$t_{st} \geq \frac{t_b}{2} = \frac{0.625}{2} = 0.313 \text{ in.}$$

Try a plate $\frac{1}{4} \times 3$

$$A_{st} = \tfrac{1}{4}(3)(2) = 1.5 \text{ in.}^2 > 0.743 \text{ in.}^2 \qquad \text{(OK)}$$

$$\frac{b}{15} = \frac{3.5}{15} = 0.233 < 1/4 \qquad \text{(OK)}$$

Full-depth stiffeners are not required for this case. Use a depth of approximately

$$\frac{d}{2} = \frac{10.1}{2} = 5.05 \text{ in.}$$

Use 2 plates $\frac{1}{4} \times 3 \times 0'\text{-}\ 5''$.

Clip inside corners to avoid flange-to-web fillets.

8.7-2

LRFD solution

Force developed in the beam flange plate is

$$H = F_y A = 36\left(\frac{11}{16} \times 9\right) = 222.8 \text{ kips} = P_{bf}$$

Local flange bending: from AISC Eq. J10-1,

$$\phi R_n = \phi 6.25 t_f^2 F_{yf} = 0.90[6.25(1.09)^2(50)] = 334 \text{ kips} > 222.8 \text{ kips} \qquad \text{(OK)}$$

Local web yielding: from AISC Eq. J10-2,

$$\phi R_n = \phi[(5k + N)F_{yw}t_w] = 1.0[(5 \times 1.69 + 11/16)(50)(0.680)]$$

$$= 311 \text{ kips} > 222.8 \text{ kips} \qquad \text{(OK)}$$

Web crippling: from AISC Eq. J10-4,

$$\phi R_n = \phi\left\{0.80t_w^2\left[1+3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}\right\}$$

$$= 0.75\left(0.80(0.680)^2\left[1+3\left(\frac{11/16}{14.8}\right)\left(\frac{0.680}{1.09}\right)^{1.5}\right]\sqrt{\frac{29{,}000(50)(1.09)}{0.680}}\right)$$

$= 452$ kips > 222.8 kips (OK) Stiffeners not required.

8.7-3

(a) Web plate: neglect eccentricity.

Try $\frac{3}{4}$-in. diameter A325 bearing-type bolts. Assume that threads are in the plane of shear.

$$A_b = \pi d^2/4 = \pi(3/4)^2/4 = 0.4418 \text{ in.}^2$$

Shear capacity of one bolt is $\phi R_n = \phi F_{nv} A_b = 0.75(48)(0.4418) = 15.9$ kips/bolt

Number of bolts required is

$$\frac{45}{15.9} = 2.83 \text{ try 4 bolts}$$

Determine plate thickness required for bearing. Assume that

$$\phi R_n = \phi(2.4dtF_u) = 0.75(2.4)(3/4)t(58) = 78.3t$$

Load resisted by each bolt $= \frac{45}{4} = 11.25$ kips. Let

$$\phi R_n = 11.25 = 78.3t \quad \Rightarrow \quad t = 0.144 \text{ in.} \qquad \text{Try } t = \tfrac{3}{16} \text{ in.}$$

Determine whether plate or beam web controls bearing. For the plate,

$$tF_u = \frac{3}{16}(58) = 10.9 \text{ kips/in.}$$

For the beam web, $t_w F_u = 0.300(65) = 19.5$ kips/in. > 10.9 kips/in. $\therefore$ plate controls.

Check bearing strength assumption. $h = \frac{3}{4} + \frac{1}{16} = \frac{13}{16}$ in.

For the hole nearest the edge, min. $L_e = 1\frac{1}{4}$ in. Try $1\frac{1}{2}$ in.

$$L_c = L_e - \frac{h}{2} = 1.5 - \frac{13/16}{2} = 1.094 \text{ in.}$$

$$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(1.094)(3/16)(58) = 10.71 \text{ kips}$$

$$\phi(2.4dtF_u) = 0.75(2.4)(3/4)(3/16)(58) = 14.68 \text{ kips} > 10.71 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 10.71 \text{ kips/bolt}$$

For other bolts, min. $s = 2.667(3/4) = 2.0$ in. Try $s = 2\frac{1}{2}$ in.

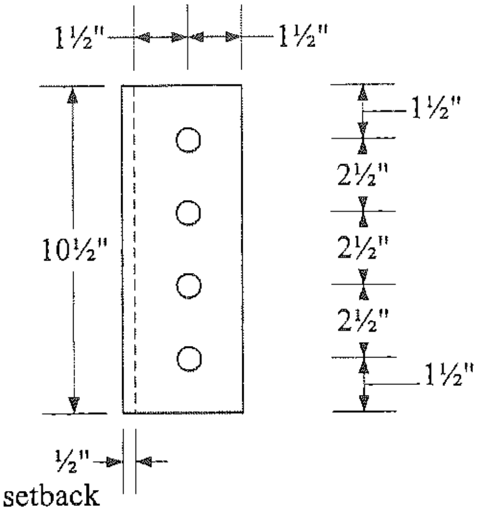
$$L_c = s - h = 2.5 - \frac{13}{16} = 1.688 \text{ in.}$$

$$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(1.688)(3/16)(58) = 16.52 \text{ kips}$$

$\phi(2.4dtF_u) = 14.68$ kips < 16.52 kips $\therefore$ use $\phi R_n = 14.68$ kips/bolt

Total bearing strength = $10.71 + 3(14.68) = 54.8$ kips > 45 kips (OK)

Use four $\frac{3}{4}$-in. diameter A325 bolts.



Determine plate thickness required for shear. Shear yielding strength is

$$\phi R_n = 1.00(0.60F_yA_g)$$

Let $45 = 1.00(0.60)(36)(10.5t) \quad \Rightarrow \quad t = 0.198$ in. Try $t = \frac{1}{4}$ in.

Check shear rupture strength. Use hole diameter = $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in.

$$A_{nv} = (10.5 - 4 \times 7/8)\left(\frac{1}{4}\right) = 1.75 \text{ in.}^2$$

$$\phi R_n = \phi(0.6A_{nv}F_u) = 0.75(0.6)(1.75)(58) = 45.7 \text{ kips} > 45 \text{ kips} \quad \text{(OK)}$$

Check block shear.

Shear areas: $A_{gv} = \frac{1}{4}(1.5 + 3 \times 2.5) = \frac{1}{4}(9) = 2.25$ in.^{2}

$$A_{nv} = \frac{1}{4}[9 - 3.5(7/8)] = 1.484 \text{ in.}^2$$

Tension area: $A_{nt} = \frac{1}{4}[1.5 - 0.5(7/8)] = 0.2656$ in.^{2}

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt}$$

$$= 0.6(58)(1.484) + 1.0(58)(0.2656) = 67.1 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(2.25) + 1.0(58)(0.2656) = 64.0 \text{ kips}$$

The nominal block shear strength is therefore 64.0 kips. The design block shear strength is

$$\phi R_n = 0.75(64.0) = 48.0 \text{ kips} > 45 \text{ kips (OK)}$$

Use a plate $\frac{1}{4} \times 3\frac{1}{2} \times 10\frac{1}{2}$ as shown above.

Connection of shear plate to column flange: Use E70 electrodes.

Minimum weld size, based on the shear plate thickness of $\frac{1}{4}$ inch, is $\frac{1}{8}$ inch. Try $w = \frac{1}{8}$ in.

Weld strength = $1.392 \times 2 = 2.784$ kips/in.

Base metal (plate) shear strength:

Yielding: $\phi R_n = 0.6F_y t = 0.6(36)(1/4) = 5.4$ kips/in. > 2.784 kips/in.

Rupture: $\phi R_n = 0.45F_u t = 0.45(58)(1/4) = 6.525$ kips/in. > 2.784 kips/in.

Weld strength controls: Total length required = $45/2.784 = 16.16$ in.

Use a continuous $\frac{1}{8}$-in. fillet weld on each side of plate.

Check for column stiffener requirements. The force developed in the beam flange is

$$H = \frac{M_u}{d_b - t_b} = \frac{220(12)}{17.7 - 0.425} = 152.8 \text{ kips} = P_{bf}$$

Local flange bending: from AISC Eq. J10-1,

$$\phi R_n = \phi 6.25 t_f^2 F_{yf} = 0.90[6.25(0.660)^2(50)] = 122.5 \text{ kips} < 152.8 \text{ kips} \quad \text{(N.G.)}$$

Local web yielding: from AISC Eq. J10-2,

$$\phi R_n = \phi[(5k + N)F_{yw}t_w] = 1.00[(5 \times 1.25 + 0.425)(50)(0.370)]$$

$$= 123.5 \text{ kips} < 152.8 \text{ kips} \qquad \text{(N.G.)}$$

Web crippling: from AISC Eq. J10-4,

$$\phi R_n = \phi\left\{0.80t_w^2\left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}\right\}$$

$$= 0.75\left(0.80(0.370)^2\left[1 + 3\left(\frac{0.425}{13.9}\right)\left(\frac{0.370}{0.660}\right)^{1.5}\right]\sqrt{\frac{29,000(50)(0.660)}{0.370}}\right)$$

$$= 137.2 \text{ kips} < 152.8 \text{ kips} \qquad \text{(N.G.)}$$

Local flange bending is the worst case. Required stiffener area is

$$A_{st} = \frac{P_{bf} - \phi R_{n\,\text{min}}}{\phi_{st}F_{yst}} = \frac{152.8 - 122.5}{0.90(36)} = 0.935 \text{ in.}^2$$

Other requirements:

$$b \geq \frac{b_b}{3} - \frac{t_w}{2} = \frac{6.00}{3} - \frac{0.370}{2} = 1.82 \text{ in.}$$

$$b \leq \frac{b_{fcol} - t_w}{2} = \frac{8.06 - 0.370}{2} = 3.85 \text{ in.}$$

$$t_{st} \geq \frac{t_b}{2} = \frac{0.425}{2} = 0.213 \text{ in.}$$

Try a plate $\frac{1}{4} \times 3$

$$A_{st} = \frac{1}{4}(3)(2) = 1.5 \text{ in.}^2 > 0.935 \text{ in.}^2 \qquad \text{(OK)}$$

Check for $t_{st} > b/15$:

$$\frac{b}{15} = \frac{3}{15} = 0.2 \text{ in.} < \frac{1}{4} \text{ in.} \qquad \text{(OK)}$$

Full-depth stiffeners are not required for this case. Use a depth of approximately

$$\frac{d}{2} = \frac{13.9}{2} = 6.95 \text{ in.}$$

Use 2 plates $\frac{1}{4} \times 3 \times$ 0′- 7”.

Clip inside corners $\frac{5}{8}$ in. to avoid flange-to-web fillets.

(b) Check panel zone reinforcement requirements.

$$P = \frac{(M_1 + M_2)}{d_m} - V = \frac{M_u}{d_b - t_b} - V_u = \frac{220(12)}{17.7 - 0.425} - 0 = 152.8 \text{ kips}$$

$P_u = 0.6P_y > 0.4P_y$, so use AISC Equation J10-10.

$$R_n = 0.60F_y d_c t_w\left(1.4 - \frac{P_r}{P_c}\right) = 0.60F_y d_c t_w\left(1.4 - \frac{P_u}{P_y}\right)$$

$$= 0.60(50)(13.9)(0.370)(1.4 - 0.6) = 123.4 \text{ kips}$$

$$\phi R_n = 0.90(123.4) = 111.1 \text{ kips} < 152.8 \text{ kips (N.G.)}$$

Alternative 1: use a web doubler plate.

Use AISC Equation J10-10 to find the required doubler plate thickness. Multiplying both sides by ϕ and solving for t_w gives

$$t_w = \frac{\phi R_n}{\phi(0.60F_y d_c)(1 - P_u/P_y)}$$

Substituting the plate thickness t_d for t_w and using the yield stress of the doubler plate, we get

$$t_d = \frac{\phi R_n}{\phi(0.60F_y d_c)(1 - P_u/P_y)} = \frac{152.8 - 111.1}{0.90(0.60)(36)(13.9)(1 - 0.6)} = 0.386 \text{ in.}$$

where $152.8 - 111.1$ is the extra strength, in kips, to be furnished by the doubler plate.

Use a 7/16-inch doubler plate.

(The welds will not be designed. For an explanation, see Example 8.12 in the textbook.)

Alternative 2: use a diagonal stiffener. With this alternative, use full-depth horizontal stiffeners.

The shear force to be resisted by the web reinforcement is $152.8 - 111.1 = 41.7$ kips. If this force is taken as the horizontal component of an axial compressive force P in the stiffener,

$$P\cos\theta = 41.7 \text{ kips}$$

where

$$\theta = \tan^{-1}\left(\frac{d_b}{d_c}\right) = \tan^{-1}\left(\frac{17.7}{13.9}\right) = 51.86°$$

$$P = \frac{41.7}{\cos(51.86°)} = 67.52 \text{ kips}$$

Since the stiffener is continuously connected along its length, we will treat it as a compression member whose effective length *KL* is zero. From AISC J4.4, for compression elements with *KL*/*r* < 25, the nominal strength is

$$P_n = F_y A_g$$

$$\therefore\ \phi P_n = \phi F_y A_g = 0.90(36)A_{st}$$

Equating this strength to the required strength, we obtain the required area of stiffener:

$$0.90(36)A_{st} = 67.52 \qquad \Rightarrow \qquad A_{st} = 2.08 \text{ in.}^2$$

Try 2 stiffeners, $3 \times \frac{3}{8}$, one on each side of the web.

$$A_{st}\text{ provided} = 2\left(3 \times \frac{3}{8}\right) = 2.25 \text{ in.}^2 > 2.08 \text{ in.}^2 \text{ required (OK)}$$

Check for $t_{st} > b/15$:

$$\frac{b}{15} = \frac{3}{15} = 0.2 \text{ in.} < \frac{3}{8} \text{ in.} \qquad \text{(OK)}$$

Design the welds. The approximate length of each diagonal stiffener is

$$L_{st} = \frac{d_c}{\cos\theta} = \frac{13.9}{\cos(51.86°)} = 22.51 \text{ in.}$$

If welds are used on both sides of the stiffeners, the available length for welding is

$$L = 22.51(4) = 90.04 \text{ in.}$$

The weld size, in sixteenths of an inch, required for strength is

$$D = \frac{P}{1.392L} = \frac{67.52}{1.392(90.04)} = 0.539 \text{ sixteenths}$$

Use the minimum size of $\frac{3}{16}$ inch (AISC Table J2.4).

Because of the small size required for strength, use intermittent welds. From AISC J2.2b,

Minimum length = $4w = 4\left(\frac{3}{16}\right) = 0.75$ in., but not less than 1.5 in. (1.5 in. controls)

For a group of four welds, the capacity is

$$4(1.392DL) = 4(1.392)(3)(1.5) = 25.06 \text{ kips}$$

Required weld capacity per inch = $\frac{P}{L_{st}} = \frac{67.52}{22.51} = 3.00$ kips/in.

Base metal shear strength of column web:

Yielding: $\phi R_n = 0.6F_y t_w \times 2 = 0.6(50)(0.370) \times 2 = 22.2$ kips/in. > 3.00 kips/in.

Rupture: $\phi R_n = 0.45F_u t_w \times 2 = 0.45(65)(0.370) \times 2 = 21.7$ kips/in. > 3.00 kips/in.

Base metal shear strength of stiffener:

Yielding: $\phi R_n = 0.6F_y t_{st} \times 2 = 0.6(36)(3/8) \times 2 = 16.2$ kips/in. > 3.00 kips/in.

Rupture: $\phi R_n = 0.45F_u t_{st} \times 2 = 0.45(58)(3/8) \times 2 = 19.6$ kips/in. > 3.00 kips/in.

Required spacing of welds = $\dfrac{25.06}{3} = 8.35$ in.

Use $\frac{3}{16}$-in. × $1\frac{1}{2}$-in. intermittent fillet welds spaced at 8 inches on center, on each side of each diagonal stiffener.

8.8-1

(a) LRFD solution

Factored shear and moment:

$V_u = 1.2(0.25 \times 27) + 1.6(0.75 \times 27) = 40.5$ kips

$M_u = 1.2(0.25 \times 67) + 1.6(0.75 \times 67) = 100.5$ ft-kips

Is required moment strength at least 60% of available moment strength?

$\phi_b M_n = \phi_b M_p = 125$ ft-kips, $\quad 0.60(125) = 75.0$ ft-kips

Required moment strength = $M_u = 100.5$ ft-kips > 75.0 ft-kips (OK)

Flange force = $\dfrac{M_u}{d - t_f} = \dfrac{100.5(12)}{13.7 - 0.335} = 90.24$ kips

Tensile load per bolt = $90.24/4 = 22.6$ kips

Tensile strength: $A_b = \pi(0.75)^2/4 = 0.4418$ in.^{2}

$\phi R_n = \phi F_{nt} A_b = 0.75(90)(0.4418) = 29.82$ kips/bolt > 22.6 kips/bolt (OK)

Shear load per bolt = $40.5/4 = 10.1$ kips (compression side bolts)

Shear strength:

$\phi R_n = \phi F_{nv} A_b = 0.75(48)(0.4418) = 15.9$ kips/bolt > 10.1 kips/bolt (OK)

Bearing strength: Assume that the upper limit of $2.4dtF_u$ controls and that column flange thickness controls ($t_f = 0.435$ in.).

$\phi R_n = \phi(2.4dtF_u) = 0.75(2.4)(3/4)(0.435)(65)$

$= 38.2$ kips/bolt > 10.1 kips/bolt (OK) Bolts are adequate

(b) ASD solution

Is required moment strength at least 60% of available moment strength?

$$\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 82.8 \text{ ft-kips}, \qquad 0.60(82.8) = 49.7 \text{ ft-kips}$$

Required moment strength = $M_a = 67$ ft-kips > 49.7 ft-kips (OK)

$$\text{Flange force} = \frac{M_a}{d - t_f} = \frac{67(12)}{13.7 - 0.335} = 60.16 \text{ kips}$$

Tensile load per bolt = 60.16/4 = 15.0 kips

Tensile strength: $A_b = \pi(0.75)^2/4 = 0.4418 \text{ in.}^2$

$$\frac{R_n}{\Omega} = \frac{F_{nt}A_b}{\Omega} = \frac{90(0.4418)}{2.00} = 19.9 \text{ kips/bolt} > 15.0 \text{ kips/bolt} \quad \text{(OK)}$$

Shear load per bolt = 27/4 = 6.75 kips (compression side bolts)

Shear strength:

$$\frac{R_n}{\Omega} = \frac{F_{nv}A_b}{\Omega} = \frac{48(0.4418)}{2.00} = 10.6 \text{ kips/bolt} > 6.75 \text{ kips/bolt} \quad \text{(OK)}$$

Bearing strength: Assume that the upper limit of $2.4dtF_u$ controls and that column flange thickness controls ($t_f = 0.435$ in.).

$$\frac{R_n}{\Omega} = \frac{2.4dtF_u}{\Omega} = \frac{2.4(3/4)(0.435)(65)}{2.00}$$

$$= 25.5 \text{ kips/bolt} > 6.75 \text{ kips/bolt} \quad \text{(OK)}$$

Bolts are adequate

8.8-2

LRFD solution

Factored shear and moment:

$$V_u = 1.2(0.25 \times 53) + 1.6(0.75 \times 53) = 79.5 \text{ kips}$$

$$M_u = 1.2(0.25 \times 134) + 1.6(0.75 \times 134) = 201.0 \text{ ft-kips}$$

Is required moment strength at least 60% of available moment strength?

$$\phi_b M_n = \phi_b M_p = 249 \text{ ft-kips}, \qquad 0.60(249) = 149.4 \text{ ft-kips}$$

Required moment strength = $M_u = 201.0$ ft-kips > 149.6 ft-kips (OK)

$$\text{Flange force} = \frac{M_u}{d - t_f} = \frac{201.0(12)}{17.7 - 0.425} = 139.6 \text{ kips}$$

Tensile load per bolt = 139.6/4 = 34.9 kips

Tensile strength: $A_b = \pi(7/8)^2/4 = 0.6013 \text{ in.}^2$

$$\phi R_n = \phi F_{nt}A_b = 0.75(90)(0.6013) = 40.59 \text{ kips/bolt} > 34.9 \text{ kips/bolt} \quad \text{(OK)}$$

Shear load per bolt = 79.5/4 = 19.9 kips (compression side bolts)

Shear strength:

$$\phi R_n = \phi F_{nv} A_b = 0.75(48)(0.6013) = 21.65 \text{ kips/bolt} > 19.9 \text{ kips/bolt} \qquad \text{(OK)}$$

Bearing strength: Assume that the upper limit of $2.4dtF_u$ controls and that column flange thickness controls ($t_f = 0.560$ in.).

$$\phi R_n = \phi(2.4dtF_u) = 0.75(2.4)(7/8)(0.560)(65)$$

$$= 57.3 \text{ kips/bolt} > 19.9 \text{ kips/bolt} \qquad \text{(OK)}$$

Bolts are adequate

8.8-3

LRFD solution

From the Z_x tables, for a W18 × 40,

$$M_u = \phi_b M_{px} = 294 \text{ ft-kips}$$

$$V_u = \phi_v V_n = 169 \text{ kips}$$

From the dimensions and properties tables

$d = 17.9$ in., $t_w = 0.315$ in., $b_{fb} = 6.02$ in., $t_{fb} = 0.525$ in., workable gage = 3.50 in.

For the bolt pitch, try $p_{fo} = p_{fi} = 2$ in.

For the gage distance, use the workable gage $g = 3.50$ in.

Required bolt diameter:

$$h_0 = d - \frac{t_{fb}}{2} + p_{fo} = 17.9 - \frac{0.525}{2} + 2 = 19.64 \text{ in.}$$

$$h_1 = d - \frac{t_{fb}}{2} - t_{fb} - p_{fi} = 17.9 - \frac{0.525}{2} - 0.525 - 2 = 15.11 \text{ in.}$$

$$d_{b\text{Req'd}} = \sqrt{\frac{2M_u}{\pi \phi F_t (h_0 + h_1)}} = \sqrt{\frac{2(294 \times 12)}{\pi(0.75)(90)(19.64 + 15.11)}} = 0.979 \text{ in.}$$

Try $d_b = 1$ inch.

Moment strength based on bolt strength:

$$P_t = F_t A_b = 90\pi(1)^2/4 = 70.69 \text{ kips/bolt}$$

$$M_n = 2P_t(h_0 + h_1) = 2(70.69)(19.64 + 15.11) = 4913 \text{ in.-kips}$$

$$\phi M_n = 0.75(4913) = 3685 \text{ in-kips} = 307.1 \text{ ft-kips} > \phi_b M_{px} = 294 \text{ ft-kips}$$

$$\therefore \text{ use } \phi M_n = \phi_b M_{px} = 294 \text{ ft-kips} = 3528 \text{ in.-kips}$$

Determine end-plate width. From AISC Table J3.4,

$$\text{minimum } L_e = 1\tfrac{3}{4} \text{ in.}$$

The minimum plate width is $g + 2L_e = 3.50 + 2(1.75) = 7.0$ in.

but no less than the beam flange width of 6.02 in.

Maximum *effective* end-plate width = $b_{fb} + 1 = 6.02 + 1 = 7.02$ in.

Try $b_p = 7$ in. Compute the required plate thickness:

$$s = \frac{1}{2}\sqrt{b_p g} = \frac{1}{2}\sqrt{7(3.5)} = 2.475 \text{ in.} > p_{fi} \therefore \text{ use the original value of } p_{fi} = 2.0 \text{ in.}$$

$$Y_p = \frac{b_p}{2}\left[h_1\left(\frac{1}{p_{fi}} + \frac{1}{s}\right) + h_0\left(\frac{1}{p_{fo}}\right) - \frac{1}{2}\right] + \frac{2}{g}[h_1(p_{fi} + s)]$$

$$= \frac{7}{2}\left[15.11\left(\frac{1}{2} + \frac{1}{2.475}\right) + 19.64\left(\frac{1}{2}\right) - \frac{1}{2}\right] + \frac{2}{3.5}[15.11(2 + 2.475)]$$

$$= 119.1$$

$$\text{Required } t_p = \sqrt{\frac{1.11\phi M_n}{\phi_b F_y Y_p}} = \sqrt{\frac{1.11(3528)}{0.9(36)(119.1)}} = 1.01 \text{ in.}$$

Try $t_p = 1\frac{1}{4}$ inch.

Beam flange force:

$$F_{fu} = \frac{M_u}{d - t_{fb}} = \frac{294 \times 12}{17.9 - 0.525} = 203.1 \text{ kips}$$

$$\frac{F_{fu}}{2} = \frac{203.1}{2} = 101.6 \text{ kips}$$

The shear yield strength of the end plate is

$$\phi(0.6)F_y t_p b_p = 0.90(0.6)(36)(1.25)(7) = 170.1 \text{ kips} > 101.6 \text{ kips} \qquad \text{(OK)}$$

Shear rupture strength of end plate:

$$A_n = t_p[b_p - 2(d_b + 1/8)] = (1.25)\left[7 - 2\left(1 + \frac{1}{8}\right)\right] = 5.938 \text{ in.}^2$$

$$\phi(0.6)F_u A_n = 0.75(0.6)(58)(5.938) = 155.0 \text{ kips} > 101.6 \text{ kips} \qquad \text{(OK)}$$

Check bolt shear. The compression side bolts must be capable of resisting the entire vertical shear.

$$A_b = \frac{\pi d_b^2}{4} = \frac{\pi(1)^2}{4} = 0.7854 \text{ in.}^2$$

$$\phi R_n = \phi F_{nv} A_b = 0.75(48)(0.7854) = 28.27 \text{ kips/bolt}$$

For 4 bolts, $\phi R_n = 4 \times 28.27 = 113.1$ kips

$$V_u = 169 \text{ kips} > 113.1 \text{ kips} \qquad \text{(N.G.)}$$

Try $d_b = 1\frac{1}{4}$ inch.

$$A_b = \frac{\pi d_b^2}{4} = \frac{\pi(1.25)^2}{4} = 1.227 \text{ in.}^2$$

$$\phi R_n = \phi F_{nv} A_b = 0.75(48)(1.227) = 44.17 \text{ kips/bolt}$$

For 4 bolts, $\phi R_n = 4 \times 44.17 = 177$ kips > 169 kips (OK)

Recompute the plate dimensions. For 1¼-inch diameter bolts, minimum $L_e = 2\frac{1}{4}$ in.

The minimum plate width is

$$g + 2L_e = 3.50 + 2(2.25) = 8.0 \text{ in.}$$

Maximum *effective* end-plate width = $b_{fb} + 1 = 6.02 + 1 = 7.02$ in.

Try $b_p = 8$ in., with an effective width of 7.02 inches. Compute the required plate thickness:

$$s = \frac{1}{2}\sqrt{b_p g} = \frac{1}{2}\sqrt{7.02(3.5)} = 2.478 \text{ in.} > p_{fi} \therefore \text{ use the original value of } p_{fi} = 2.0 \text{ in.}$$

$$Y_p = \frac{b_p}{2}\left[h_1\left(\frac{1}{p_{fi}} + \frac{1}{s}\right) + h_0\left(\frac{1}{p_{fo}}\right) - \frac{1}{2}\right] + \frac{2}{g}[h_1(p_{fi} + s)]$$

$$= \frac{7.02}{2}\left[15.11\left(\frac{1}{2} + \frac{1}{2.478}\right) + 19.64\left(\frac{1}{2}\right) - \frac{1}{2}\right] + \frac{2}{3.5}[15.11(2 + 2.478)]$$

$$= 119.3$$

$$\text{Required } t_p = \sqrt{\frac{1.11\phi M_n}{\phi_b F_y Y_p}} = \sqrt{\frac{1.11(3528)}{0.9(36)(119.3)}} = 1.01 \text{ in.}$$

Try $t_p = 1\frac{1}{4}$ inch.

Because the bolt size has increased, check the shear rupture strength of the end plate:

$$A_n = t_p[b_p - 2(d_b + 1/8)] = (1.25)[7.02 - 2(1.25 + 1/8)] = 5.338 \text{ in.}^2$$

$$\phi(0.6)F_u A_n = 0.75(0.6)(58)(5.338) = 139 \text{ kips} > 101.6 \text{ kips} \qquad \text{(OK)}$$

Check bearing in the plate at the compression side bolts.

$$h = d + \frac{1}{16} = 1\frac{1}{4} + \frac{1}{16} = 1.313 \text{ in.}$$

For the outer bolts,

$$L_c = p_{fo} + t_{fb} + p_{fi} - h = 2 + 0.525 + 2 - 1.313 = 3.212 \text{ in.}$$

$$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(3.212)(1.25)(58) = 209.6 \text{ kips}$$

The upper limit is

$$\phi(2.4dtF_u) = 0.75(2.4)(1.25)(1.25)(58) = 163.1 \text{ kips} < 209.6 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 163.1 \text{ kips/bolt}$$

Since the inner bolts are not near an edge or adjacent bolts, the outer bolts control. The total bearing strength is

$$4 \times 163.1 = 652 \text{ kips} > V_u = 169 \text{ kips} \qquad \text{(OK)}$$

Check bearing in the column flange. Use $L_c = 3.212$ in.

$$\phi R_n = \phi(1.2L_c t F_u) = 0.75(1.2)(3.212)(0.525)(65) = 98.65 \text{ kips}$$

The upper limit is

$$\phi(2.4dtF_u) = 0.75(2.4)(1.25)(0.525)(65) = 76.78 \text{ kips} < 98.65 \text{ kips}$$

$\therefore$ use $\phi R_n = 76.78$ kips/bolt

The total bearing strength is

$$4 \times 76.78 = 307 \text{ kips} > V_u = 169 \text{ kips} \qquad \text{(OK)}$$

The plate length, using detailing dimensions and the notation of Figure 8.51 in the textbook, is

$$d + 2p_{fo} + 2d_e = 17\tfrac{7}{8} + 2(2) + 2\left(2\tfrac{1}{4}\right) = 26\tfrac{3}{8} \text{ in.}$$

Use a PL 1¼ × 8 × $2' - 2\tfrac{3}{8}''$ and four 1¼-inch diameter A325 fully-tightened bolts at each flange.

Beam flange-to-plate weld design: The flange force is

$$F_{fu} = 203.1 \text{ kips}$$

AISC Design Guide 4 recommends that the minimum design flange force should be 60% of the flange yield strength:

$$\text{Min. } F_{fu} = 0.6F_y(b_{fb}t_{fb}) = 0.6(50)(6.02)(0.525) = 94.8 \text{ kips} < 203.1 \text{ kips}$$

Therefore, use the actual flange force of 203.1 kips. The flange weld length is

$$b_{fb} + (b_{fb} - t_w) = 6.02 + (6.02 - 0.315) = 11.73 \text{ in.}$$

The weld strength is

$$\phi R_n = 1.392D \times 11.73 \times 1.5$$

where D is the weld size in sixteenths of an inch, and the factor of 1.5 accounts for the direction of the load on the weld. If we equate the weld strength to the flange force,

$$1.392D \times 11.73 \times 1.5 = 203.1, \qquad D = 8.29 \text{ sixteenths}$$

From AISC Table J2.4, the minimum weld size is ¼ in. (based on the thickness of the flange, which is the thinner connected part).

Use a 9/16-inch fillet weld at each flange.

Beam web-to-plate weld design: To develop the yield stress in the web near the tension bolts, let

$$1.392D \times 2 = 0.6F_y t_w$$

for two welds, one on each side of the web. The required weld size is

$$D = \frac{0.6F_y t_w}{1.392(2)} = \frac{0.6(50)(0.315)}{1.392(2)} = 3.39 \text{ sixteenths}$$

Use a ¼-inch fillet weld on each side of the web in the tension region.

The applied shear of $V_u = 169$ kips must be resisted by welding a length of web equal to the smaller of the following two lengths:

1. from mid-depth to the compression flange:

$$L = \frac{d}{2} - t_{fb} = \frac{17.9}{2} - 0.525 = 8.425 \text{ in.}$$

2. from the inner row of tension bolts plus $2d_b$ to the compression flange:

$$L = d - 2t_{fb} - p_{fi} - 2d_b = 17.9 - 2(0.525) - 2.0 - 2(1.25) = 12.35 \text{ in.} > 8.425 \text{ in.}$$

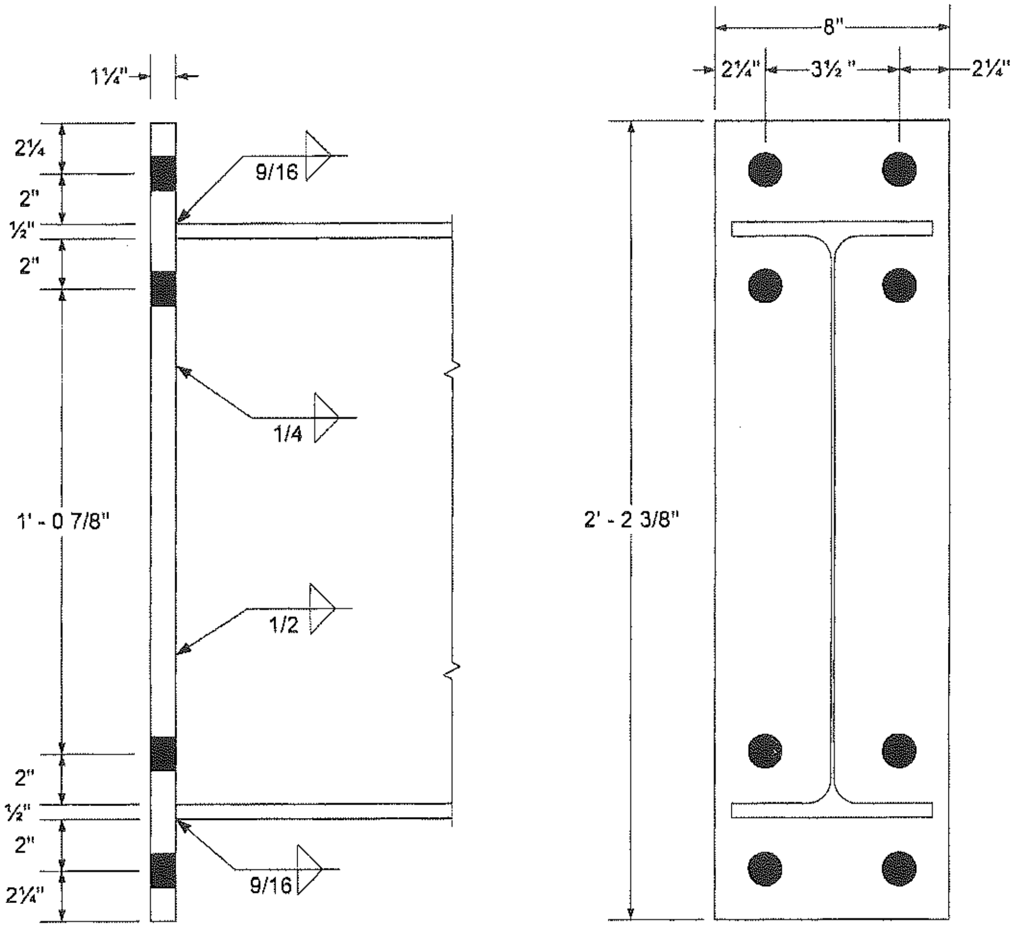
Equating the weld strength to the required shear strength, we get

$$1.392D \times 8.425 \times 2 = 169, \qquad D = 7.21 \text{ sixteenths } (w = ½ \text{ in.})$$

From AISC Table J2.4, the minimum weld size is 3/16 in.

Use a ½-inch fillet weld on each side of the web between mid-depth and the compression flange.

The design is summarized in the figure below.



Not to scale

8.8-4

LRFD solution

Factored shear and moment:

$$V_u = 1.2(13) + 1.6(34) = 70.0 \text{ kips}$$

$$M_u = 1.2(20) + 1.6(48) = 100.8 \text{ ft-kips}$$

From the dimensions and properties tables

$d = 12.3$ in., $t_w = 0.260$ in., $b_{fb} = 6.52$ in., $t_{fb} = 0.440$ in., workable gage = 3.50 in.

For the bolt pitch, try $p_{fo} = p_{fi} = 2$ in.

For the gage distance, use the workable gage $g = 3.50$ in.

Required bolt diameter:

$$h_0 = d - \frac{t_{fb}}{2} + p_{fo} = 12.3 - \frac{0.440}{2} + 2 = 14.08 \text{ in.}$$

$$h_1 = d - \frac{t_{fb}}{2} - t_{fb} - p_{fi} = 12.3 - \frac{0.440}{2} - 0.440 - 2 = 9.64 \text{ in.}$$

$$d_{b\,\text{Req'd}} = \sqrt{\frac{2M_u}{\pi\phi F_t(h_0 + h_1)}} = \sqrt{\frac{2(100.8 \times 12)}{\pi(0.75)(90)(14.08 + 9.64)}} = 0.694 \text{ in.}$$

Try $d_b = \frac{3}{4}$ inch.

Moment strength based on bolt strength:

$$P_t = F_t A_b = 90\pi(3/4)^2/4 = 39.76 \text{ kips/bolt}$$

$$M_n = 2P_t(h_0 + h_1) = 2(39.76)(14.08 + 9.64) = 1886 \text{ in.-kips}$$

$$\phi M_n = 0.75(1886) = 1415 \text{ in-kips} = 117.9 \text{ ft-kips}$$

Determine end-plate width. From AISC Table J3.4,

minimum $L_e = 1\frac{1}{4}$ in.

The minimum plate width is $g + 2L_e = 3.50 + 2(1.25) = 6.0$ in.

but no less than the beam flange width of 6.52 in.

Maximum *effective* end-plate width = $b_{fb} + 1 = 6.52 + 1 = 7.52$ in.

Try $b_p = 8$ in., with an effective $b_p = 7.52$ in. Compute the required plate thickness:

$$s = \frac{1}{2}\sqrt{b_p g} = \frac{1}{2}\sqrt{7.52(3.5)} = 2.565 \text{ in.} > p_{fi} \;\therefore\; \text{use the original value of } p_{fi} = 2.0 \text{ in.}$$

$$Y_p = \frac{b_p}{2}\left[h_1\left(\frac{1}{p_{fi}} + \frac{1}{s}\right) + h_0\left(\frac{1}{p_{fo}}\right) - \frac{1}{2}\right] + \frac{2}{g}[h_1(p_{fi} + s)]$$

$$= \frac{7.52}{2}\left[9.64\left(\frac{1}{2} + \frac{1}{2.565}\right) + 14.08\left(\frac{1}{2}\right) - \frac{1}{2}\right] + \frac{2}{3.5}[9.64(2 + 2.565)]$$

$$= 81.99$$

$$\text{Required } t_p = \sqrt{\frac{1.11\phi M_n}{\phi_b F_y Y_p}} = \sqrt{\frac{1.11(1415)}{0.90(36)(81.99)}} = 0.769 \text{ in.}$$

Try $t_p = \frac{7}{8}$ inch.

Beam flange force:

$$F_{fu} = \frac{M_u}{d - t_{fb}} = \frac{100.8 \times 12}{12.3 - 0.440} = 102.0 \text{ kips}$$

$$\frac{F_{fu}}{2} = \frac{102.0}{2} = 51 \text{ kips}$$

The shear yield strength of the end plate is

$$\phi(0.6)F_y t_p b_p = 0.90(0.6)(36)(7/8)(7.52) = 128 \text{ kips} > 51 \text{ kips} \qquad \text{(OK)}$$

Shear rupture strength of end plate:

$$A_n = t_p[b_p - 2(d_b + 1/8)] = \frac{7}{8}\left[7.52 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\right] = 5.049 \text{ in.}^2$$

$$\phi(0.6)F_u A_n = 0.75(0.6)(58)(5.049) = 132 \text{ kips} > 51 \text{ kips} \qquad \text{(OK)}$$

Check bolt shear. The compression side bolts must be capable of resisting the entire vertical shear.

$$A_b = \frac{\pi d_b^2}{4} = \frac{\pi(3/4)^2}{4} = 0.4418 \text{ in.}^2$$

$$\phi R_n = \phi F_{nv} A_b = 0.75(48)(0.4418) = 15.9 \text{ kips/bolt}$$

For 4 bolts, $\phi R_n = 4 \times 15.9 = 63.6$ kips

$$V_u = 70 \text{ kips} > 63.6 \text{ kips} \qquad \text{(N.G.)}$$

Try $d_b = \frac{7}{8}$ inch.

$$A_b = \frac{\pi d_b^2}{4} = \frac{\pi(7/8)^2}{4} = 0.6013 \text{ in.}^2$$

$$\phi R_n = \phi F_{nv} A_b = 0.75(48)(0.6013) = 21.65 \text{ kips/bolt}$$

For 4 bolts, $\phi R_n = 4 \times 21.65 = 86.6$ kips > 70 kips (OK)

Recompute the plate dimensions. For 7/8-inch diameter bolts,

minimum $L_e = 1\frac{1}{2}$ in.

The minimum plate width is

$$g + 2L_e = 3.50 + 2(1.5) = 6.5 \text{ in.}$$

Maximum *effective* end-plate width = $b_{fb} + 1 = 6.52 + 1 = 7.52$ in.

Try $b_p = 8$ in., with an effective width of 7.52 inches. From previous calculations,

$$Y_p = 125.0$$

Moment strength based on bolt strength:

$P_t = F_t A_b = 90(0.6013) = 54.12$ kips/bolt

$M_n = 2P_t(h_0 + h_1) = 2(54.12)(14.08 + 9.64) = 2567$ in.-kips

$\phi M_n = 0.75(2567) = 1925$ in-kips = 160.4 ft-kips

$$\text{Required } t_p = \sqrt{\frac{1.11\phi M_n}{\phi_b F_y Y_p}} = \sqrt{\frac{1.11(1925)}{0.9(36)(125.0)}} = 0.726 \text{ in.}$$

Try t_p = ¾ inch.

The shear yield strength of the end plate is

$$\phi(0.6)F_y t_p b_p = 0.90(0.6)(36)(3/4)(7.52) = 110 \text{ kips} > \frac{F_{fu}}{2} = 51 \text{ kips} \qquad \text{(OK)}$$

Shear rupture strength of end plate:

$$A_n = t_p[b_p - 2(d_b + 1/8)] = \frac{3}{4}\left[7.52 - 2\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 4.14 \text{ in.}^2$$

$$\phi(0.6)F_u A_n = 0.75(0.6)(58)(4.14) = 108 \text{ kips} > 51 \text{ kips} \qquad \text{(OK)}$$

Check bearing in the plate at the compression side bolts.

$$h = d + \frac{1}{16} = \frac{7}{8} + \frac{1}{16} = \frac{15}{16} \text{ in.}$$

For the outer bolts,

$$L_c = p_{fo} + t_{fb} + p_{fi} - h = 2 + 0.440 + 2 - 15/16 = 3.503 \text{ in.}$$

$$\phi R_n = \phi(1.2 L_c t F_u) = 0.75(1.2)(3.503)(3/4)(58) = 137.1 \text{ kips}$$

The upper limit is

$$\phi(2.4 d t F_u) = 0.75(2.4)(7/8)(3/4)(58) = 68.51 \text{ kips} < 137.1 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 68.51 \text{ kips/bolt}$$

Since the inner bolts are not near an edge or adjacent bolts, the outer bolts control. The total bearing strength is

$$4 \times 68.51 = 274 \text{ kips} > V_u = 70 \text{ kips} \qquad \text{(OK)}$$

Check bearing in the column flange. Use $L_c = 3.503$ in.

$$\phi R_n = \phi(1.2 L_c t F_u) = 0.75(1.2)(3.503)(0.680)(65) = 139.3 \text{ kips}$$

The upper limit is

$$\phi(2.4 d t F_u) = 0.75(2.4)(7/8)(0.680)(65) = 69.62 \text{ kips} < 139.3 \text{ kips}$$

$$\therefore \text{ use } \phi R_n = 69.62 \text{ kips/bolt}$$

The total bearing strength is

$$4 \times 69.62 = 279 \text{ kips} > V_u = 70 \text{ kips} \qquad \text{(OK)}$$

The plate length, using detailing dimensions and the notation of Figure 8.51 in the textbook, is

$$d + 2p_{fo} + 2d_e = 12\tfrac{3}{8} + 2(2) + 2\left(1\tfrac{1}{2}\right) = 19\tfrac{3}{8} \text{ in.}$$

Use a PL $\frac{3}{4} \times 8 \times 1' - 7\frac{3}{8}''$ and four $\frac{7}{8}$-inch diameter A325 fully-tightened bolts at each flange.

Beam flange-to-plate weld design: The flange force is

$$F_{fu} = 102.0 \text{ kips}$$

AISC Design Guide 4 recommends that the minimum design flange force should be 60% of the flange yield strength:

$$\text{Min. } F_{fu} = 0.6F_y(b_{fb}t_{fb}) = 0.6(50)(6.52)(0.440) = 86.06 \text{ kips} < 102.0 \text{ kips}$$

Therefore, use the actual flange force of 102.0 kips. The flange weld length is

$$b_{fb} + (b_{fb} - t_w) = 6.52 + (6.52 - 0.260) = 12.78 \text{ in.}$$

The weld strength is

$$\phi R_n = 1.392D \times 12.78 \times 1.5$$

where D is the weld size in sixteenths of an inch, and the factor of 1.5 accounts for the direction of the load on the weld. If we equate the weld strength to the flange force,

$$1.392D \times 12.78 \times 1.5 = 102.0, \qquad D = 3.82 \text{ sixteenths}$$

From AISC Table J2.4, the minimum weld size is 3/16 in. (based on the thickness of the flange, which is the thinner connected part).

Use a ¼-inch fillet weld at each flange.

Beam web-to-plate weld design: To develop the yield stress in the web near the tension bolts, let

$$1.392D \times 2 = 0.6F_y t_w$$

for two welds, one on each side of the web. The required weld size is

$$D = \frac{0.6F_y t_w}{1.392(2)} = \frac{0.6(50)(0.260)}{1.392(2)} = 2.80 \text{ sixteenths}$$

Minimum size = 3/16 in., based on web thickness.

Use a 3/16-inch fillet weld on each side of the web in the tension region.

The applied shear of $V_u = 70$ kips must be resisted by welding a length of web equal to the smaller of the following two lengths:

1. from mid-depth to the compression flange:

$$L = \frac{d}{2} - t_{fb} = \frac{12.3}{2} - 0.440 = 5.71 \text{ in.}$$

2. from the inner row of tension bolts plus $2d_b$ to the compression flange:

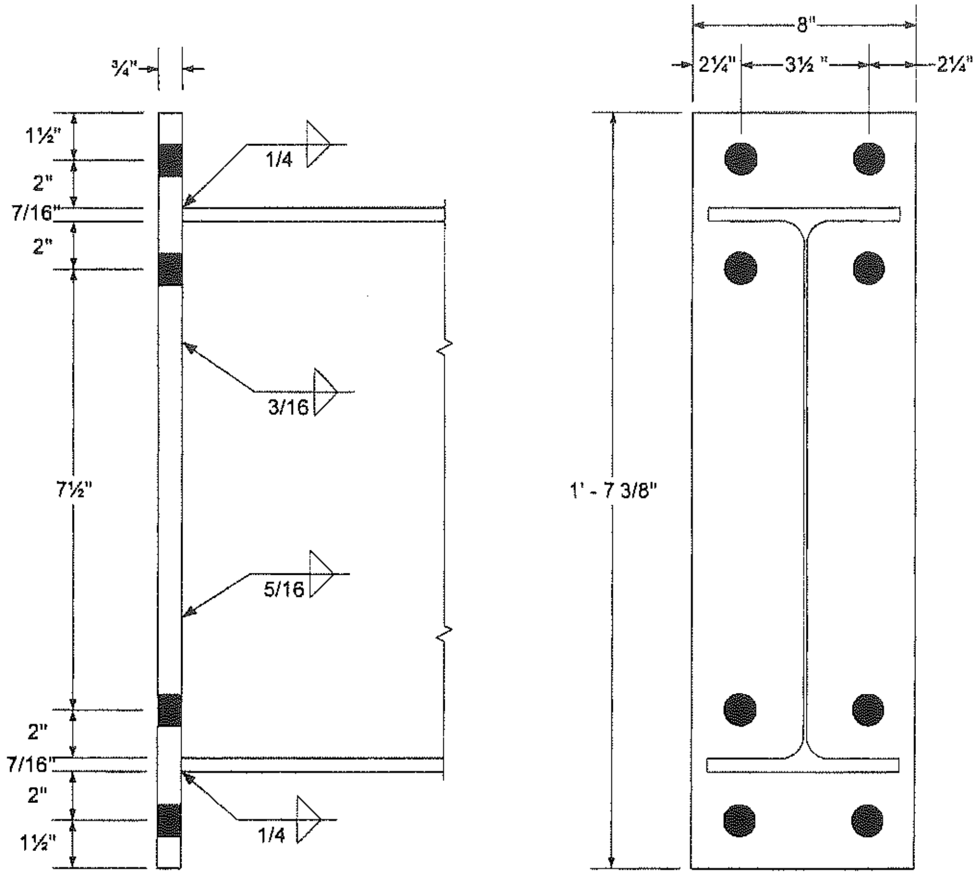
$$L = d - 2t_{fb} - p_{fi} - 2d_b = 12.3 - 2(0.440) - 2.0 - 2(7/8) = 7.67 \text{ in.} > 5.71 \text{ in.}$$

Use $L = 5.71$ inches. Equating the weld strength to the required shear strength, we get

$$1.392D \times 5.71 \times 2 = 70, \qquad D = 4.40 \text{ sixteenths}$$

Use a 5/16-inch fillet weld on each side of the web between mid-depth and the compression flange.

The design is summarized in the figure below.



Not to scale

CHAPTER 9 - COMPOSITE CONSTRUCTION

9.1-1

(a) $E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}, \qquad n = \dfrac{E_s}{E_c} = \dfrac{29{,}000}{3492} = 8.3$ use 8

$$\frac{b}{n} = \frac{81}{8} = 10.13 \text{ in.}$$

For a W18 × 40, $A_s = 11.8 \text{ in.}^2$, $d = 17.9 \text{ in.}$, $I_x = 612 \text{ in.}^4$

Component	A	y	Ay	$\bar{I}$	d	$\bar{I}+Ad^2$
Slab	40.52	2	81.04	54.03	2.469	301.0
W	11.8	12.95	152.8	612	8.481	1461
Sum	52.32		233.8			1762

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{233.8}{52.32} = 4.469 \text{ in.}, \qquad \underline{I_{tr} = 1762 \text{ in.}^4}$$

(b) Top of steel:

$$f_s = \frac{My}{I_{tr}} = \frac{(290 \times 12)(4.469 - 4)}{1762} = \underline{0.926 \text{ ksi (compression)}}$$

Bottom of steel:

$$f_s = \frac{My}{I_{tr}} = \frac{(290 \times 12)(4 + 17.9 - 4.469)}{1762} = \underline{34.4 \text{ ksi (tension)}}$$

Top of slab:

$$f_s = \frac{M\bar{y}}{nI_{tr}} = \frac{(290 \times 12)(4.469)}{8(1762)} = \underline{1.10 \text{ ksi (compression)}}$$

9.1-3

(a) $E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}, \qquad n = \dfrac{E_s}{E_c} = \dfrac{29{,}000}{3492} = 8.3$ use 8

$$\frac{b}{n} = \frac{78}{8} = 9.75 \text{ in.}$$

For a W24 × 55, $A_s = 16.2 \text{ in.}^2$, $d = 23.6 \text{ in.}$, $I_x = 1350 \text{ in.}^4$

Component	A	y	Ay	$\bar{I}$	d	$\bar{I}+Ad^2$
Slab	43.88	2.25	98.73	74.04	3.789	704.0
W	16.2	16.3	264.1	1350	10.26	3055
Sum	60.08		362.8			3759

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{362.8}{60.08} = 6.039 \text{ in.}, \qquad \underline{I_{tr} = 3759 \text{ in.}^4}$$

(b) Top of steel:

$$f_s = \frac{My}{I_{tr}} = \frac{(450 \times 12)(6.039 - 4.5)}{3759} = \underline{2.21 \text{ ksi (compression)}}$$

Bottom of steel:

$$f_s = \frac{My}{I_{tr}} = \frac{(450 \times 12)(4.5 + 23.6 - 6.039)}{3759} = \underline{31.7 \text{ ksi (tension)}}$$

Top of slab:

$$f_s = \frac{M\bar{y}}{nI_{tr}} = \frac{(450 \times 12)(6.039)}{8(3759)} = \underline{1.08 \text{ ksi (compression)}}$$

9.1-5

Determine location of plastic neutral axis:

$$A_sF_y = 16.7(50) = 835.0 \text{ kips}, \quad 0.85f_c'bt = 0.85(4)(75)(5) = 1275 \text{ kips}$$

Since 835 kips < 1275 kips, PNA is in the slab and $C = 835$ kips.

From $C = T$,

$$0.85f_c'ab = A_sF_y$$

$0.85(4)a(75) = 835$, Solution is: $\{a = 3.275\}$ in.

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{21.1}{2} + 5 - \frac{3.275}{2} = 13.91 \text{ in.}$$

$M_n = Ty = 835(13.91) = 11,610$ in.-kips = 968 ft-kips

$\underline{M_n = 968 \text{ ft-kips}}$

9.2-1

Loads applied before the concrete cures:

$$\text{slab weight} = \left(\frac{4}{12}\right)(150) = 50 \text{ psf}, \qquad 50(7.5) = 375 \text{ lb/ft}$$

$$w_D = 375 + 22 = 397 \text{ lb/ft}, \qquad w_L = 20(7.5) = 150 \text{ lb/ft}$$

Loads applied after the concrete cures:

$$w_D = 375 + 5 = 380 \text{ lb/ft}, \qquad w_L = (60 + 10)(7.5) = 525 \text{ lb/ft}$$

Strength of the composite section:

$$A_sF_y = 6.49(50) = 324.5 \text{ kips}, \qquad 0.85f_c'A_c = 0.85(4)(4 \times 90) = 1224 \text{ kips}$$

Use $C = 324.5$ kips.

$$a = \frac{C}{0.85f_c'b} = \frac{324.5}{0.85(4)(90)} = 1.060 \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{13.7}{2} + 4 - \frac{1.060}{2} = 10.32 \text{ in.}$$

$$M_n = Cy = 324.5(10.32) = 3349 \text{ in.-kips} = 279.1 \text{ ft-kips}$$

(a) LRFD solution

Before the concrete cures:

From the Z_x table, $\phi_b M_n = \phi_b M_p = 125$ ft-kips

$$w_u = 1.2w_D + 1.6w_L = 1.2(397) + 1.6(150) = 716.4 \text{ lb/ft}$$

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(0.7164)(30)^2 = 80.6 \text{ ft-kips} < 125 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures:

$$\phi_b M_n = 0.90(279.1) = 251 \text{ ft-kips}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(380) + 1.6(525) = 1296 \text{ lb/ft}$$

$$M_u = \frac{1}{8}(1.296)(30)^2 = 146 \text{ ft-kips} < 251 \text{ ft-kips} \qquad \text{(OK)}$$

Flexural strength is adequate.

(b) ASD solution

Before the concrete cures,

From the Z_x table, $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 82.8$ ft-kips

$$w_a = w_D + w_L = 397 + 150 = 547 \text{ lb/ft}$$

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(0.547)(30)^2 = 61.5 \text{ ft-kips} < 82.8 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures,

$$\frac{M_n}{\Omega_b} = \frac{279.1}{1.67} = 167 \text{ ft-kips}$$

$$w_a = w_D + w_L = 380 + 525 = 905 \text{ lb/ft}$$

$$M_a = \frac{1}{8}(0.905)(30)^2 = 102 \text{ ft-kips} < 167 \text{ ft-kips} \qquad \text{(OK)}$$

Flexural strength is adequate.

9.3-1

Loads applied before the concrete cures:

$$\text{slab weight} = \left(\frac{4}{12}\right)(150) = 50 \text{ psf}, \qquad 50(7) = 350 \text{ lb/ft}$$

$$w_D = 350 + 16 = 366 \text{ lb/ft}, \qquad w_L = 20(7) = 140 \text{ lb/ft}$$

Loads applied after the concrete cures:

$$w_D = 350 \text{ lb/ft}, \qquad w_L = (125 + 15)(7) = 980 \text{ lb/ft}$$

Strength of the composite section:

Effective flange width = $(25 \times 12)/4 = 75$ in. or $7(12) = 84$ in., use $b = 75$ in.

$$A_sF_y = 4.71(50) = 235.5 \text{ kips}, \qquad 0.85f_c'A_c = 0.85(4)(4 \times 75) = 1020 \text{ kips}$$

Use $C = 235.5$ kips.

$$a = \frac{C}{0.85f_c'b} = \frac{235.5}{0.85(4)(75)} = 0.9235 \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{12.0}{2} + 4 - \frac{0.9235}{2} = 9.538 \text{ in.}$$

$$M_n = Cy = 235.5(9.538) = 2246 \text{ in.-kips} = 187.2 \text{ ft-kips}$$

(a) LRFD solution

Before the concrete cures:

From the Z_x table, $\phi_bM_n = \phi_bM_p = 75.4$ ft-kips

$$w_u = 1.2w_D + 1.6w_L = 1.2(366) + 1.6(140) = 663.2 \text{ lb/ft}$$

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(0.6632)(25)^2 = 51.8 \text{ ft-kips} < 75.4 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures:

$$\phi_bM_n = 0.90(187.2) = 168 \text{ ft-kips}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(350) + 1.6(980) = 1988 \text{ lb/ft}$$

$$M_u = \frac{1}{8}(1.988)(25)^2 = 155 \text{ ft-kips} < 168 \text{ ft-kips} \qquad \text{(OK)}$$

Shear: $\phi_vV_n = 79.1$ kips

$$V_u = \frac{w_uL}{2} = \frac{1.988(25)}{2} = 24.9 \text{ kips} < 79.1 \text{ kips} \qquad \text{(OK)}$$

Beam is satisfactory

(b) ASD solution

Before the concrete cures:

From the Z_x table, $\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 50.1$ ft-kips

$$w_a = w_D + w_L = 366 + 140 = 506 \text{ lb/ft}$$

$$M_a = \frac{1}{8}w_aL^2 = \frac{1}{8}(0.506)(25)^2 = 39.5 \text{ ft-kips} < 50.1 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures,

$$\frac{M_n}{\Omega_b} = \frac{187.2}{1.67} = 112 \text{ ft-kips}$$

$$w_a = w_D + w_L = 350 + 980 = 1330 \text{ lb/ft}$$

$$M_a = \frac{1}{8}(1.330)(25)^2 = 104 \text{ ft-kips} < 112 \text{ ft-kips} \qquad \text{(OK)}$$

Shear: $\dfrac{V_n}{\Omega_v} = 52.8$ kips

$$V_a = \frac{w_a L}{2} = \frac{1.330(25)}{2} = 16.6 \text{ kips} < 52.8 \text{ kips} \qquad \text{(OK)}$$

Beam is satisfactory

9.4-1

Loads applied before the concrete cures:

$$\text{slab weight} = \left(\frac{6}{12}\right)(150) = 75 \text{ psf}, \qquad 75(9) = 675 \text{ lb/ft}$$

$$w_D = 675 + 57 = 732 \text{ lb/ft}, \qquad w_L = 20(9) = 180 \text{ lb/ft}$$

Loads applied after the concrete cures:

$$w_D = 732 \text{ lb/ft}, \qquad w_L = 250(9) = 2250 \text{ lb/ft}$$

Strength of the composite section:

Effective flange width = $(40 \times 12)/4 = 120$ in. or $9(12) = 108$ in., use $b = 108$ in.

$$A_s F_y = 16.7(50) = 835.0 \text{ kips}, \qquad 0.85 f_c' A_c = 0.85(4)(6 \times 108) = 2203 \text{ kips}$$

Use $C = 835$ kips.

$$a = \frac{C}{0.85 f_c' b} = \frac{835}{0.85(4)(108)} = 2.274 \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{21.1}{2} + 6 - \frac{2.274}{2} = 15.41 \text{ in.}$$

$$M_n = Cy = 835(15.41) = 12,870 \text{ in.-kips} = 1073 \text{ ft-kips}$$

(a) Before the concrete cures:

From the Z_x table, $\phi_b M_n = \phi_b M_p = 484$ ft-kips

$$w_u = 1.2 w_D + 1.6 w_L = 1.2(732) + 1.6(180) = 1166 \text{ lb/ft}$$

$$M_u = \frac{1}{8} w_u L^2 = \frac{1}{8}(1.166)(40)^2 = 233 \text{ ft-kips} < 484 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures:

$$\phi_b M_n = 0.90(1073) = 966 \text{ ft-kips}$$

$$w_u = 1.2 w_D + 1.6 w_L = 1.2(732) + 1.6(2250) = 4478 \text{ lb/ft}$$

$$M_u = \frac{1}{8}(4.478)(40)^2 = 896 \text{ ft-kips} < 966 \text{ ft-kips} \qquad \text{(OK)}$$

Shear: $\phi_v V_n = 256$ kips

$$V_u = \frac{w_u L}{2} = \frac{4.478(40)}{2} = 89.6 \text{ kips} < 256 \text{ kips} \qquad \text{(OK)}$$

Beam is satisfactory

(b) Before the concrete cures:

From the Z_x table, $\dfrac{M_n}{\Omega_b} = \dfrac{M_p}{\Omega_b} = 322$ ft-kips

$$w_a = w_D + w_L = 732 + 180 = 912 \text{ lb/ft}$$

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(0.912)(40)^2 = 182 \text{ ft-kips} < 322 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures,

$$\frac{M_n}{\Omega_b} = \frac{1073}{1.67} = 643 \text{ ft-kips}$$

$$w_a = w_D + w_L = 732 + 2250 = 2982 \text{ lb/ft}$$

$$M_a = \frac{1}{8}(2.982)(40)^2 = 596 \text{ ft-kips} < 643 \text{ ft-kips} \qquad \text{(OK)}$$

Shear: $\dfrac{V_n}{\Omega_v} = 171$ kips

$$V_a = \frac{w_a L}{2} = \frac{2.982(40)}{2} = 59.6 \text{ kips} < 171 \text{ kips} \qquad \text{(OK)}$$

Beam is satisfactory

(c) $A_{sc} = \dfrac{\pi(3/4)^2}{4} = 0.4418 \text{ in.}^2$, $\qquad E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492$ ksi

$$Q_n = 0.5A_{sc}\sqrt{f_c' E_c} \le R_g R_p A_{sc} F_u$$

$$= 0.5(0.4418)\sqrt{4(3492)} = 26.11 \text{ kips}$$

$$R_g R_p A_{sc} F_u = 1.0(0.6)(0.4418)(65)$$

$$= 17.23 \text{ kips} < 26.11 \text{ kips} \qquad \therefore \text{ use } Q_n = 17.23 \text{ kips}$$

$$N_1 = \frac{V'}{Q_n} = \frac{835}{17.23} = 48.5, \text{ round up to 49.}$$

total number = 2(49) = 98

Use 98 studs

9.4-3

$$A_s F_y = 11.8(50) = 590.0 \text{ kips}, \qquad 0.85 f_c' A_c = 0.85(4)(4 \times 81) = 1102 \text{ kips}$$

Use $C = V' = 590$ kips.

$$A_{sc} = \frac{\pi(3/4)^2}{4} = 0.4418 \text{ in.}^2, \qquad E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

$$Q_n = 0.5A_{sc}\sqrt{f_c'E_c} \le R_gR_pA_{sc}F_u$$

$$= 0.5(0.4418)\sqrt{4(3492)} = 26.11 \text{ kips}$$

$$R_gR_pA_{sc}F_u = 1.0(0.6)(0.4418)(65)$$

$$= 17.23 \text{ kips} < 26.11 \text{ kips} \qquad \therefore \text{ use } Q_n = 17.23 \text{ kips}$$

$$N_1 = \frac{V'}{Q_n} = \frac{590}{17.23} = 34.2, \text{ round up to 35.}$$

total number = 2(35) = 70 Use 70 studs

9.6-1

(a) Before concrete cures:

$$\text{Slab: } \frac{4}{12}(150)(7.5) = 375.0 \text{ lb/ft}$$

$$w_D = 375 + 22 = 397 \text{ lb/ft}, \qquad w_{const} = 20(7.5) = 150.0 \text{ lb/ft}, \qquad I_s = 199 \text{ in.}^4$$

$$\Delta_D = \frac{5w_DL^4}{384EI_s} = \frac{5(0.397/12)(30 \times 12)^4}{384(29,000)(199)} = 1.254 \text{ in.}$$

$$\Delta_{const} = \frac{5w_{const}L^4}{384EI_s} = \frac{5(0.150/12)(30 \times 12)^4}{384(29,000)(199)} = 0.4737 \text{ in.}$$

$$\Delta = \Delta_D + \Delta_{const} = 1.254 + 0.4737 = 1.728 \text{ in.} \qquad \Delta = 1.73 \text{ in.}$$

(b) After concrete has cured:

Transformed moment of inertia:

$$E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}, \qquad n = \frac{E_s}{E_c} = \frac{29,000}{3492} = 8.3 \quad \text{use 8}$$

$$\frac{b}{n} = \frac{90}{8} = 11.25 \text{ in.}$$

For a W14 × 22, $A_s = 6.49 \text{ in.}^2$, $d = 13.7$ in., $I_x = 199 \text{ in.}^4$

Component	A	y	Ay	$\bar{I}$	d	$\bar{I} + Ad^2$
Slab	4.500E+01	2.000E+00	9.000E+01	6.000E+01	1.115E+00	1.159E+02
W	6.490E+00	1.085E+01	7.042E+01	1.990E+02	7.735E+00	5.873E+02
Sum	5.149E+01		1.604E+02			7.032E+02

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{160.4}{51.49} = 3.115 \text{ in.}, \qquad I_{tr} = 703.2 \text{ in.}^4$$

Use $I_{eff} = 0.75I_{tr} = 0.75(703.2) = 527.4 \text{ in.}^4$

$$w_L = 60(7.5) = 450.0 \text{ lb/ft}$$

$$w_{part} = 10(7.5) = 75.0 \text{ lb/ft}$$

$$w_{ceil} = 5(7.5) = 37.5 \text{ lb/ft}$$

$$\Delta = \Delta_D + \Delta_L + \Delta_{part} + \Delta_{ceil} = \Delta_D + \frac{5(w_L + w_{part} + w_{ceil})L^4}{384EI_{eff}}$$

$$= 1.254 + \frac{5[(0.450 + 0.075 + 0.0375)/12](30 \times 12)^4}{384(29{,}000)(527.4)}$$

$$= 1.254 + 0.6703 = 1.924 \text{ in.} \qquad \underline{\Delta = 1.92 \text{ in.}}$$

9.6-3

(a) From Problem 9.3-1, a W12 × 16 is used, with $t = 4$ in., $s = 7$ ft, $L = 25$ ft, $q_{const} = 20$ psf, $q_{part} = 15$ psf, $q_L = 125$ psf, A992 steel and 4 ksi concrete.

Before concrete cures:

$$\text{Slab: } \frac{4}{12}(150)(7) = 350 \text{ lb/ft}$$

$$w_D = 350 + 16 = 366 \text{ lb/ft}, \qquad w_{const} = 20(7) = 140 \text{ lb/ft}, \qquad I_s = 103 \text{ in.}^4$$

$$\Delta_D = \frac{5w_DL^4}{384EI_s} = \frac{5(0.366/12)(25 \times 12)^4}{384(29{,}000)(103)} = 1.077 \text{ in.}$$

$$\Delta_{const} = \frac{5w_{const}L^4}{384EI_s} = \frac{5(0.140/12)(25 \times 12)^4}{384(29{,}000)(103)} = 0.4119 \text{ in.}$$

$$\Delta = \Delta_D + \Delta_{const} = 1.077 + 0.4119 = 1.489 \text{ in.} \qquad \underline{\Delta = 1.49 \text{ in.}}$$

After concrete has cured:

$$w_{part} = 15(7) = 105 \text{ lb/ft}, \qquad w_L = 125(7) = 875 \text{ lb/ft}$$

Transformed moment of inertia:

$$E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}, \qquad n = \frac{E_s}{E_c} = \frac{29{,}000}{3492} = 8.3 \quad \text{use } 8$$

Effective flange width = (25 × 12)/4 = 75 in. or 7(12) = 84 in., use $b = 75$ in.

$$\frac{b}{n} = \frac{75}{8} = 9.375 \text{ in.}$$

For a W12 × 16, $A_s = 4.71$ in.^{2}, $d = 12.0$ in., $I_x = 103$ in.^{4}

Component	A	y	Ay	$\bar{I}$	d	$\bar{I} + Ad^2$
Slab	3.750E+01	2.000E+00	7.500E+01	5.000E+01	8.930E-01	7.990E+01
W	4.710E+00	1.000E+01	4.710E+01	1.030E+02	7.107E+00	3.409E+02
Sum	4.221E+01		1.221E+02			4.208E+02

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{122.1}{42.21} = 2.893 \text{ in.}, \qquad I_{tr} = 420.8 \text{ in.}^4$$

Use $I_{eff} = 0.75 I_{tr} = 0.75(420.8) = 315.6 \text{ in.}^4$

$$\Delta_{part} = \frac{5w_{part}L^4}{384EI_{eff}} = \frac{5(0.105/12)(25 \times 12)^4}{384(29,000)(315.6)} = 0.1008 \text{ in.}$$

$$\Delta_L = \frac{5w_L L^4}{384EI_{eff}} = \frac{5(0.875/12)(25 \times 12)^4}{384(29,000)(315.6)} = 0.8403 \text{ in.}$$

$$\Delta = \Delta_D + \Delta_{part} + \Delta_L = 1.077 + 0.1008 + 0.8403 = 2.018 \text{ in.} \qquad \underline{\Delta = 2.02 \text{ in.}}$$

(b) Maximum permissible $\Delta_L = \frac{L}{360} = \frac{25 \times 12}{360} = 0.8333 \text{ in.} < 0.8403 \text{ in.}$ (N.G.)

Use LRFD and select another shape.

$$\text{Required } I_{eff} = \frac{5w_L L^4}{384E\Delta_{\max}} = \frac{5(0.875/12)(25 \times 12)^4}{384(29,000)(0.8333)} = 318.2 \text{ in.}^4$$

Required $I_{tr} = I_{eff}/0.75 = 318.2/0.75 = 424.3 \text{ in.}^4$

Try a W12 × 19.

Loads applied before the concrete cures:

$$\text{slab weight} = \left(\frac{4}{12}\right)(150) = 50 \text{ psf}, \qquad 50(7) = 350 \text{ lb/ft}$$

$$w_D = 350 + 19 = 369 \text{ lb/ft}, \qquad w_{const} = 20(7) = 140 \text{ lb/ft}$$

Loads applied after the concrete cures:

$$w_D = 369 \text{ lb/ft}, \qquad w_L = (125 + 15)(7) = 980 \text{ lb/ft}$$

Check strength before the concrete cures:

From the Z_x table, $\phi_b M_n = \phi_b M_p = 92.6$ ft-kips

$$w_u = 1.2w_D + 1.6w_L = 1.2(369) + 1.6(140) = 666.8 \text{ lb/ft}$$

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(0.6668)(25)^2 = 52.1 \text{ ft-kips} < 92.6 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures: Compute the strength of the composite section.

$$A_s F_y = 5.57(50) = 278.5 \text{ kips}, \qquad 0.85f_c'A_c = 0.85(4)(4 \times 75) = 1020 \text{ kips}$$

Use $C = 278.5$ kips.

$$a = \frac{C}{0.85f_c'b} = \frac{278.5}{0.85(4)(75)} = 1.092 \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{12.2}{2} + 4 - \frac{1.092}{2} = 9.554 \text{ in.}$$

$$M_n = Cy = 278.5(9.554) = 2661 \text{ in.-kips} = 221.8 \text{ ft-kips}$$

$$\phi_b M_n = 0.90(221.8) = 200 \text{ ft-kips}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(369) + 1.6(980) = 2011 \text{ lb/ft}$$

$$M_u = \frac{1}{8}(2.011)(25)^2 = 157 \text{ ft-kips} < 200 \text{ ft-kips} \qquad \text{(OK)}$$

Shear: $\phi_v V_n = 85.7$ kips

$$V_u = \frac{w_u L}{2} = \frac{2.011(25)}{2} = 25.1 \text{ kips} < 85.7 \text{ kips} \qquad \text{(OK)}$$

Compute deflections. Transformed moment of inertia:

Component	A	y	Ay	$\bar{I}$	d	$\bar{I} + Ad^2$
Slab	3.750E+01	2.000E+00	7.500E+01	5.000E+01	1.049E+00	9.127E+01
W	5.570E+00	1.010E+01	5.626E+01	1.300E+02	7.051E+00	4.069E+02
Sum	4.307E+01		1.313E+02			4.982E+02

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{131.3}{43.07} = 3.049 \text{ in.,}$$

$$I_{tr} = 498.2 \text{ in.}^4 > 424.3 \text{ in.}^4 \text{ required} \qquad \text{(OK)}$$

$$I_{eff} = 0.75I_{tr} = 0.75(498.2) = 373.7 \text{ in.}^4$$

$$\Delta_D = \frac{5w_D L^4}{384EI_s} = \frac{5(0.369/12)(25 \times 12)^4}{384(29{,}000)(130)} = 0.8603 \text{ in.}$$

$$\Delta_{part} = \frac{5w_{part} L^4}{384EI_{eff}} = \frac{5(0.105/12)(25 \times 12)^4}{384(29{,}000)(373.7)} = 0.08516 \text{ in.}$$

$$\Delta_L = \frac{5w_L L^4}{384EI_{eff}} = \frac{5(0.875/12)(25 \times 12)^4}{384(29{,}000)(373.7)} = 0.7096 \text{ in.}$$

$$\Delta = \Delta_D + \Delta_{part} + \Delta_L = 0.8603 + 0.08516 + 0.7096 = 1.66 \text{ in.}$$

Use a W12 × 19

9.6-5

(a) From Problem 9.4-2, a W14 × 22 is used, with $t = 4$ in., $s = 8$ ft, $L = 27$ ft, $q_{const} = 20$ psf, $q_{part} = 20$ psf, $q_L = 120$ psf, A992 steel and 4 ksi concrete.

Before concrete cures:

$$\text{Slab: } \frac{4}{12}(150)(8) = 400 \text{ lb/ft}$$

$$w_D = 400 + 22 = 422 \text{ lb/ft}, \qquad w_{const} = 20(8) = 160 \text{ lb/ft}, \qquad I_s = 199 \text{ in.}^4$$

$$\Delta_D = \frac{5w_D L^4}{384EI_s} = \frac{5(0.422/12)(27 \times 12)^4}{384(29{,}000)(199)} = 0.8744 \text{ in.}$$

$$\Delta_{const} = \frac{5w_{const} L^4}{384EI_s} = \frac{5(0.160/12)(27 \times 12)^4}{384(29{,}000)(199)} = 0.3315 \text{ in.}$$

$\Delta = \Delta_D + \Delta_{const} = 0.8744 + 0.3315 = 1.206$ in. $\quad \underline{\Delta = 1.21 \text{ in.}}$

After concrete has cured:

$w_{part} = 20(8) = 160$ lb/ft, $\quad w_L = 120(8) = 960$ lb/ft

Transformed moment of inertia:

$$E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}, \qquad n = \frac{E_s}{E_c} = \frac{29,000}{3492} = 8.3 \quad \text{use } 8$$

Effective flange width = (27 × 12)/4 = 81 in. or 8(12) = 96 in., use $b = 81$ in.

$$\frac{b}{n} = \frac{81}{8} = 10.13 \text{ in.}$$

For a W14 × 22, $A_s = 6.49$ in.^{2}, $d = 13.7$ in., $I_x = 199$ in.^{4}

Component	A	y	Ay	$\bar{I}$	d	$\bar{I} + Ad^2$
Slab	4.050E+01	2.000E+00	8.100E+01	5.400E+01	1.222E+00	1.145E+02
W	6.490E+00	1.085E+01	7.042E+01	1.990E+02	7.628E+00	5.766E+02
Sum	4.699E+01		1.514E+02			6.911E+02

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{151.4}{46.99} = 3.222 \text{ in.}, \qquad I_{tr} = 691.1 \text{ in.}^4$$

Use $I_{eff} = 0.75 I_{tr} = 0.75(691.1) = 518.3$ in.^{4}

$$\Delta_{part} = \frac{5 w_{part} L^4}{384 E I_{eff}} = \frac{5(0.160/12)(27 \times 12)^4}{384(29,000)(518.3)} = 0.1273 \text{ in.}$$

$$\Delta_L = \frac{5 w_L L^4}{384 E I_{eff}} = \frac{5(0.960/12)(27 \times 12)^4}{384(29,000)(518.3)} = 0.7637 \text{ in.}$$

$\Delta = \Delta_D + \Delta_{part} + \Delta_L = 0.8744 + 0.1273 + 0.7637 = 1.765$ in. $\quad \underline{\Delta = 1.77 \text{ in.}}$

(b) Maximum permissible $\Delta_L = \dfrac{L}{360} = \dfrac{27 \times 12}{360} = 0.9000 \text{ in.} > 0.7637 \text{ in.}$ (OK)

No redesign necessary

9.7-1

(a) Transformed moment of inertia:

$$E_c = w_c^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}, \qquad n = \frac{E_s}{E_c} = \frac{29,000}{3492} = 8.31 \quad \text{use } 8$$

Effective flange width = (30 × 12)/4 = 90 in. or 6(12) = 72 in. (controls).

$$\frac{b}{n} = \frac{72}{8} = 9 \text{ in.}$$

For area and moment of inertia computations for the slab, use a depth of $4.5 - 2 = 2.5$ in.

For a W18 × 35, $A_s = 10.3$ in.^{2}, $d = 17.7$ in., $I_x = 510$ in.^{4}

Component	A	y	Ay	$\bar{I}$	d	$\bar{I}+Ad^2$
Slab	2.250E+01	1.250E+00	2.813E+01	1.172E+01	3.799E+00	3.364E+02
W	1.030E+01	1.335E+01	1.375E+02	5.100E+02	8.301E+00	1.220E+03
Sum	3.280E+01		1.656E+02			1.556E+03

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{165.6}{32.80} = 5.049 \text{ in.} \qquad \underline{I_{tr} = 1556 \text{ in.}^4}$$

Use $I_{eff} = 0.75 I_{tr} = 0.75(1556) = 1167$ in.^{4}

$$\Delta_L = \frac{5 w_L L^4}{384 E I_{tr}} = \frac{5(1.0/12)(30 \times 12)^4}{384(29{,}000)(1167)} = 0.5385 \text{ in.} \qquad \underline{\Delta_L = 0.539 \text{ in.}}$$

(b) $A_s F_y = 10.3(50) = 515.0$ kips, $0.85 f_c{}' bt = 0.85(4)(72)(4.5-2) = 612.0$ kips

Use $C = 515$ kips.

$$0.85 f_c{}' ab = A_s F_y$$

$0.85(4)a(72) = 515$, Solution is: $\{a = 2.104\}$ in.

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{17.7}{2} + 4.5 - \frac{2.104}{2} = 12.30 \text{ in.}$$

$M_n = Ty = 515(12.30) = 6335$ in.-kips $= 528$ ft-kips

$$\underline{M_n = 528 \text{ ft-kips}}$$

9.7-3

Shear connectors:

Maximum diameter $= 2.5 t_f = 2.5(0.420) = 1.05$ in. $> 3/4$ in. (OK)

Maximum diameter = 3/4 in. with formed steel deck. (OK)

$$A_{sc} = \frac{\pi(3/4)^2}{4} = 0.4418 \text{ in.}^2, \qquad E_c = w_c^{1.5}\sqrt{f_c{}'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

$$Q_n = 0.5 A_{sc}\sqrt{f_c E_c} \le R_g R_p A_{sc} F_u$$

$$= 0.5(0.4418)\sqrt{4(3492)} = 26.11 \text{ kips}$$

$$R_g R_p A_{sc} F_u = 1.0(0.6)(0.4418)(65)$$

$$= 17.23 \text{ kips} < 26.11 \text{ kips} \qquad \therefore \text{ use } Q_n = 17.23 \text{ kips}$$

$$N_1 = \text{total no. of studs} \div 2 = \frac{(30 \times 12)/(2 \times 6)}{2} = 15$$

$$\sum Q_n = Q_n \times N_1 = 17.23(15) = 258.5 \text{ kips}$$

$$A_s F_y = 7.69(50) = 384.5 \text{ kips}$$

$$0.85 f_c{}' bt = 0.85(4)(66)(4.5 - 1.5) = 673.2 \text{ kips}$$

Since $\sum Q_n$ is the smallest of the three possibilities, $C = 258.5$ kips, there is partial composite action, and the PNA is in the steel section. If the PNA were at the bottom of the compression flange,

$$P_{yf} = b_f t_f F_y = 5.03(0.420)(50) = 105.6 \text{ kips}$$

The net force to be transferred between steel and concrete would be

$$T - C_s = T - P_{yf} = (A_s F_y - P_{yf}) - P_{yf} = A_s F_y - 2P_{yf}$$
$$= 384.5 - 2(105.6) = 173.3 \text{ kips} < 258.5 \text{ kips}$$

∴ PNA is within the steel flange.

$$V' = T - C_s = (A_s F_y - b_f t' F_y) - b_f t' F_y = A_s F_y - 2b_f t' F_y$$

or $258.5 = 384.5 - 2[5.03t'(50)]$, Solution is: $\{t' = 0.2505\}$

$$C_s = b_f t' F_y = 5.03(0.2505)(50) = 63.0 \text{ kips}$$

Compute $\bar{y}$, the distance from the top of the steel to the centroid of the area below the PNA.

Component	A	y	Ay
W16 x 26	7.69	6.950	53.45
Flange segment	-1.260	0.1253	-0.1578
Sum	6.430		53.29

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{53.29}{6.430} = 8.288 \text{ in.}$$

Location of concrete compressive force:

$$a = \frac{C}{0.85 f'_c b} = \frac{258.5}{0.85(4)(66)} = 1.152$$

Moment arm for concrete compressive force is

$$\bar{y} + t - \frac{a}{2} = 8.288 + 4.5 - \frac{1.152}{2} = 12.21 \text{ in.}$$

Moment arm for compressive force in the steel is

$$\bar{y} - \frac{t'}{2} = 8.288 - \frac{0.2505}{2} = 8.163 \text{ in.}$$

$$M_n = \sum M_T = C(12.21) + C_s(8.163)$$
$$= 258.5(12.21) + 63.0(8.163) = 3671 \text{ in.-kips} = 305.9 \text{ ft-kips}$$

$$\underline{M_n = 306 \text{ ft-kips}}$$

9.8-1

From the solution to problem 9.7-3, for ¾-in. studs and $f_c' = 4$ ksi, $Q_n = 17.23$ kips

$$N_1 = \frac{30 \times 12}{12(2)} = 15.0$$

$$\sum Q_n = N_1 Q_n = 15(17.23) = 258.5 \text{ kips}$$

$$A_s F_y = 7.69(50) = 384.5 \text{ kips}$$

$$0.85 f_c' bt = 0.85(4)(66)(4.5 - 1.5) = 673.2 \text{ kips}$$

The smallest of these three controls; $\therefore C = 258.5$ kips (this is denoted as $\sum Q_n$ in the tables)

$$a = \frac{C}{0.85 f_c' b} = \frac{258.5}{0.85(4)(66)} = 1.152 \text{ in.}$$

$$Y2 = t - \frac{a}{2} = 4.5 - \frac{1.152}{2} = 3.924 \text{ in.}$$

Since we are seeking a nominal strength, we can use either the LRFD value or the ASD value from the table. We will use the LRFD value.

Interpolate for ϕM_n. First, interpolate vertically (create an intermediate row), then horizontally.

ΣQ_n	$Y2$		
	3.500E+00	**3.924E+00**	4.000E+00
3.850E+02	3.020E+02		3.160E+02
2.585E+02	**2.662E+02**	**2.760E+02**	**2.778E+02**
3.320E+02	2.870E+02		3.000E+02

$$M_n = \frac{\phi M_n}{\phi} = \frac{276.0}{0.90} = 306.7 \text{ ft-kips}$$

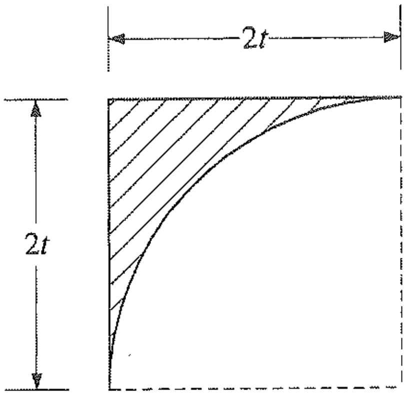
$\underline{\phi M_n = 307 \text{ ft-kips}}$

9.10-1

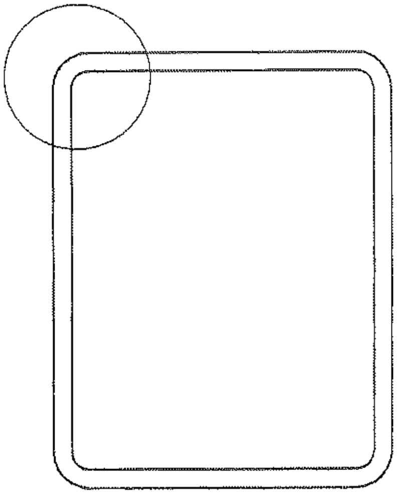
The following dimensions and properties from Part 1 of the Manual will be needed: $A_s = 10.4$ in.^{2}, design wall thickness = 0.349 in., and $I_y = 80.4$ in.^{4}

$$E_c = w^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

The area of concrete A_c can be estimated as $bd - A_s$, or more accurately as follows:



Detail



The corner radius is given in Part 1 of the Manual as $2t$. The area of an quarter-circle spandrel with a radius r is

$$r^2 - \frac{1}{4}\pi r^2 = (2t)^2 - \frac{1}{4}\pi(2t)^2 = t^2(4-\pi) = (0.349)^2(4-\pi) = 0.1046\text{ in.}^2$$

and since there are four of these segments to be deducted,

$$A_c = bd - A_s - 4(0.1046) = 7(9) - 10.4 - 4(0.1046) = 52.18\text{ in.}^2$$

For computing the moment of inertia of the concrete, the moment of inertia of the spandrel about an axis parallel to the 9-inch side through the point of tangency is

$$I = \left(\frac{1}{3} - \frac{\pi}{16}\right)r^4 = \left(\frac{1}{3} - \frac{\pi}{16}\right)(2t)^4 = \left(\frac{16}{3} - \pi\right)t^4$$

$$= \left(\frac{16}{3} - \pi\right)(0.349)^4 = 3.252 \times 10^{-2}\text{ in.}^4$$

The distance to the centroid of the spandrel from this axis is

$$\bar{x} = \frac{2r}{3(4-\pi)} = \frac{2(2 \times 0.349)}{3(4-\pi)} = 0.5421\text{ in.}$$

From the parallel axis theorem, the moment of inertia of the spandrel about a centroidal axis parallel to the 9-inch side is

$$\bar{I} = I - A\bar{x}^2 = 0.03252 - 0.1046(0.5421)^2 = 1.781 \times 10^{-3}$$

Use the parallel-axis theorem and the following table to obtain the moment of inertia of the concrete:

Segment	A	Ibar	d	Ibar +Ad2
Outer rectangle	6.300E+01	2.573E+02	0.000E+00	2.573E+02
Steel shape	-1.040E+01	-8.040E+01	0.000E+00	-8.040E+01
spandrel	-1.046E-01	-1.781E-03	3.344E+00	-1.171E+00
spandrel	-1.046E-01	-1.781E-03	3.344E+00	-1.171E+00
spandrel	-1.046E-01	-1.781E-03	3.344E+00	-1.171E+00
spandrel	-1.046E-01	-1.781E-03	3.344E+00	-1.171E+00
			Sum	1.722E+02

$$I_c = 172.2 \text{ in.}^4$$

Note that a slightly unconservative approximate solution can be obtained by using .

$$I_c = \frac{hb^3}{12} - I_s = \frac{9(7)^3}{12} - 80.4 = 177 \text{ in.}^4$$

From AISC Equation I2-13,

$$P_o = A_sF_y + A_{sr}F_{yr} + C_2A_cf'_c = 10.4(46) + 0 + 0.85(52.18)(4) = 655.8 \text{ kips}$$

From AISC Equation I2-15,

$$C_3 = 0.6 + 2\left(\frac{A_s}{A_c + A_s}\right) \leq 0.9$$

$$= 0.6 + 2\left(\frac{10.4}{52.18 + 10.4}\right) = 0.9324 > 0.9 \quad \therefore \text{ use } C_3 = 0.9$$

From AISC Equation I2-14,

$$(EI)_{eff} = E_sI_s + E_sI_{sr} + C_3E_cI_c$$

$$= 29{,}000(80.4) + 0 + 0.9(3492)(172.2) = 2.873 \times 10^6 \text{ kip-in.}^2$$

From AISC Equation I2-5,

$$P_e = \frac{\pi^2(EI)_{eff}}{(KL)^2} = \frac{\pi^2(2.873 \times 10^6)}{(0.65 \times 20 \times 12)^2} = 1165 \text{ kips}$$

Then

$$0.44P_o = 0.44(655.8) = 288.6 \text{ kips}$$

$$P_e = 1165 \text{ kips} > 288.6 \text{ kips} \quad \therefore \text{ use AISC Equation I2-2:}$$

$$P_n = P_o\left[0.658^{\left(\frac{P_o}{P_e}\right)}\right] = 655.8\left[0.658^{\left(\frac{655.8}{1165}\right)}\right] = 518.1 \text{ kips}$$

$$\underline{P_n = 518 \text{ kips}}$$

From the tables, with $KL = 0.65 \times 20 = 13.0$ ft, $\phi_cP_n = 388$ kips.

$$P_n = \phi_cP_n/\phi = 388/0.75 = 517 \text{ kips}$$

10.4-1

Check web width-thickness ratio:

$$\lambda = \frac{h}{t_w} = \frac{45}{3/8} = 120, \qquad \lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{29,000}{50}} = 90.55$$

$$\lambda_r = 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29000}{50}} = 137.3$$

Since $\lambda_p < \lambda < \lambda_r$, web is noncompact. AISC Section F4 applies, but Section F5 may be used. Compute the section modulus:

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h+t_f}{2}\right)^2 = \frac{1}{12}(3/8)(45)^3 + 2(10)\left(\frac{45+1}{2}\right)^2 = 13,430 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2+t_f)} = \frac{13,430}{(45/2+1)} = 571.5 \text{ in.}^3$$

From AISC Equation F5-10, the tension flange strength based on yielding is

$$M_n = F_y S_{xt} = 50(571.5) = 2.858 \times 10^4 \text{ in.-kips} = 2382 \text{ ft-kips}$$

The compression flange strength is given by AISC Equation F5-7:

$$M_n = R_{pg} F_{cr} S_{xc}$$

where the critical stress F_{cr} is based on either flange local buckling or yielding. For flange local buckling, the relevant slenderness parameters are

$$\lambda = \frac{b_f}{2t_f} = \frac{10}{2(1.0)} = 5, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29000}{50}} = 9.152$$

Since $\lambda < \lambda_p$, there is no flange local buckling. The compression flange strength is therefore based on yielding, and $F_{cr} = F_y = 50$ ksi.

To compute the bending strength reduction factor R_{pg}, the value of a_w will be needed:

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{45(3/8)}{10(1.0)} = 1.688 < 10$$

From AISC Equation F5-6,

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \le 1.0$$

$$= 1 - \frac{1.688}{1200 + 300(1.688)}\left(120 - 5.7\sqrt{\frac{29000}{50}}\right) = 1.017$$

Since 1.017 > 1.0, use $R_{pg} = 1.0$

$$M_n = R_{pg} F_{cr} S_{xc} = 1.0(50)(571.5) = 2.858 \times 10^4 \text{ in.-kips} = 2382 \text{ ft-kips}$$

Because the girder has continuous lateral support, lateral-torsional buckling does not have to be checked

$$\underline{M_n = 2380 \text{ ft-kips}}$$

10.4-3

Check web width-thickness ratio:

$$\lambda = \frac{h}{t_w} = \frac{60}{3/8} = 160, \qquad \lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{29,000}{50}} = 90.55$$

$$\lambda_r = 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29000}{50}} = 137.3$$

Since $\lambda > \lambda_r$, web is slender and AISC Section F5 applies. Compute the section modulus:

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h+t_f}{2}\right)^2 = \frac{1}{12}(3/8)(60)^3 + 2\left(\frac{7}{8}\times 12\right)\left(\frac{60+7/8}{2}\right)^2$$

$$= 2.621 \times 10^4 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2+t_f)} = \frac{26210}{(60/2+7/8)} = 848.9 \text{ in.}^3$$

From AISC Equation F5-10, the tension flange strength based on yielding is

$$M_n = F_y S_{xt} = 50(848.9) = 4.245 \times 10^4 \text{ in.-kips} = 3538 \text{ ft-kips}$$

The compression flange strength is given by AISC Equation F5-7:

$$M_n = R_{pg} F_{cr} S_{xc}$$

where the critical stress F_{cr} is based on either flange local buckling or yielding. For flange local buckling, the relevant slenderness parameters are

$$\lambda = \frac{b_f}{2t_f} = \frac{12}{2(7/8)} = 6.857, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29000}{50}} = 9.152$$

Since $\lambda < \lambda_p$, there is no flange local buckling. The compression flange strength is therefore based on yielding, and $F_{cr} = F_y = 50$ ksi.

To compute the bending strength reduction factor R_{pg}, the value of a_w will be needed:

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}} = \frac{60(3/8)}{12(7/8)} = 2.143 < 10$$

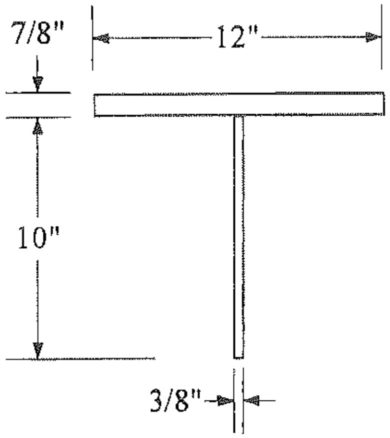
From AISC Equation F5-6,

$$R_{pg} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \le 1.0$$

$$= 1 - \frac{2.143}{1200 + 300(2.143)}\left(160 - 5.7\sqrt{\frac{29000}{50}}\right) = 0.9736$$

$$M_n = R_{pg} F_{cr} S_{xc} = 0.9736(50)(848.9) = 4.132 \times 10^4 \text{ in.-kips} = 3443 \text{ ft-kips}$$

Check lateral-torsional buckling.



(not to scale)

$$\frac{h}{6} = \frac{60}{6} = 10 \text{ in.}, \qquad I = \frac{1}{12}(10)(3/8)^3 + \frac{1}{12}(7/8)(12)^3 = 126.0 \text{ in.}^4$$

$$A = 10(3/8) + 12(7/8) = 14.25 \text{ in.}^2, \qquad r_t = \sqrt{\frac{I}{A}} = \sqrt{\frac{126}{14.25}} = 2.974 \text{ in.}$$

$$L_b = 40/2 = 20 \text{ ft}$$

$$L_p = 1.1 r_t\sqrt{\frac{E}{F_y}} = 1.1(2.974)\sqrt{\frac{29000}{50}} = 78.79 \text{ in.} = 6.566 \text{ ft}$$

$$L_r = \pi r_t\sqrt{\frac{E}{0.7F_y}} = \pi(2.974)\sqrt{\frac{29000}{0.7(50)}} = 268.9 \text{ in.} = 22.40 \text{ ft}$$

Since $L_p < L_b < L_r$, the girder is subject to inelastic lateral-torsional buckling. From AISC Equation F5-3,

$$F_{cr} = C_b\left[F_y - 0.3F_y\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le F_y$$

$$= 1.30\left[50 - (0.3 \times 50)\left(\frac{20 - 6.566}{22.40 - 6.566}\right)\right] = 48.46 \text{ ksi} \le 50 \text{ ksi}$$

where $C_b = 1.30$ is from Figure 5.15 in the textbook. LTB has the lowest critical stress and controls.

$$M_n = R_{pg}F_{cr}S_{xc} = 0.9736(48.46)(848.9) = 4.005 \times 10^4 \text{ in.-kips} = 3338 \text{ ft-kips}$$

$$\underline{M_n = 3340 \text{ ft-kips}}$$

10.4-5

Check web width-thickness ratio:

$$\lambda = \frac{h}{t_w} = \frac{78}{1/2} = 156, \qquad \lambda_p = 3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{29,000}{50}} = 90.55$$

$$\lambda_r = 5.70\sqrt{\frac{E}{F_y}} = 5.70\sqrt{\frac{29000}{50}} = 137.3$$

Since $\lambda > \lambda_r$, web is slender, and AISC Section F5 applies. Compute the section modulus:

$$I_x = \frac{1}{12}t_w h^3 + 2A_f\left(\frac{h+t_f}{2}\right)^2 = \frac{1}{12}(1/2)(78)^3 + 2(2.5\times 22)\left(\frac{78+2.5}{2}\right)^2$$

$$= 1.980\times 10^5 \text{ in.}^4$$

$$S_x = \frac{I_x}{c} = \frac{I_x}{(h/2+t_f)} = \frac{1.980\times 10^5}{(78/2+2.5)} = 4771 \text{ in.}^3$$

From AISC Equation F5-10, the tension flange strength based on yielding is

$$M_n = F_y S_{xt} = 50(4771) = 2.386\times 10^5 \text{ in.-kips} = 19,880 \text{ ft-kips}$$

The compression flange strength is given by AISC Equation F5-7:

$$M_n = R_{pg}F_{cr}S_{xc}$$

where the critical stress F_{cr} is based on either flange local buckling or yielding. For flange local buckling, the relevant slenderness parameters are

$$\lambda = \frac{b_f}{2t_f} = \frac{22}{2(2.5)} = 4.4, \qquad \lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29000}{50}} = 9.152$$

Since $\lambda < \lambda_p$, there is no flange local buckling. The compression flange strength is therefore based on yielding, and $F_{cr} = F_y = 50$ ksi.

To compute the bending strength reduction factor R_{pg}, the value of a_w will be needed:

$$a_w = \frac{h_c t_w}{b_{fc}t_{fc}} = \frac{78(1/2)}{22(2.5)} = 0.7091 < 10$$

From AISC Equation F5-6,

$$R_{pg} = 1 - \frac{a_w}{1200+300a_w}\left(\frac{h_c}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right) \leq 1.0$$

$$= 1 - \frac{0.7091}{1200+300(0.7091)}\left(156 - 5.7\sqrt{\frac{29000}{50}}\right) = 0.9906$$

$$M_n = R_{pg}F_{cr}S_{xc} = 0.9906(50)(4771) = 2.363\times 10^5 \text{ in.-kips} = 19{,}690 \text{ ft-kips}$$

Because the girder has continuous lateral support, lateral-torsional buckling does not have to be checked. Compression flange buckling controls, and $M_n = 19,690$ ft-kips.

(a) LRFD solution

$$\phi M_n = 0.90(19690) = 17,700 \text{ ft-kips}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(1.0) + 1.6(2) = 4.4 \text{ kips/ft}$$

$$P_u = 1.6P_L = 1.6(475) = 760.0 \text{ kips}$$

$$M_u = \frac{1}{8}w_uL^2 + \frac{P_uL}{4} = \frac{1}{8}(4.4)(80)^2 + \frac{760(80)}{4}$$

$= 18{,}700$ ft-kips $> 17{,}700$ ft-kips (N.G.) Flexural strength is not adequate

(b) ASD solution

$$\frac{M_n}{\Omega_b} = \frac{19690}{1.67} = 11{,}800 \text{ ft-kips}$$

$$w_a = w_D + w_L = 1.0 + 2.0 = 3 \text{ kips/ft}$$

$$P_a = P_L = 475 \text{ kips}$$

$$M_a = \frac{1}{8}w_aL^2 + \frac{P_aL}{4} = \frac{1}{8}(3)(80)^2 + \frac{475(80)}{4}$$

$= 11{,}900$ ft-kips $> 11{,}800$ ft-kips (N.G.) Flexural strength is not adequate

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10.5-1

(a) $\frac{h}{t_w} = \frac{70}{1/2} = 140, \qquad \frac{a}{h} = \frac{70}{70} = 1 < 3$

$$\left(\frac{260}{h/t_w}\right)^2 = \left(\frac{260}{140}\right)^2 = 3.45 > \frac{a}{h}$$

$$k_v = 5 + \frac{5}{(a/h)^2} = 5 + \frac{5}{(1.0)^2} = 10$$

$$1.10\sqrt{\frac{k_vE}{F_y}} = 1.10\sqrt{\frac{10(29000)}{50}} = 83.77$$

$$1.37\sqrt{\frac{k_vE}{F_y}} = 1.37\sqrt{\frac{10(29000)}{50}} = 104.3$$

Since $\frac{h}{t_w} > 104.3, \qquad C_v = \frac{1.51Ek_v}{(h/t_w)^2F_y} = \frac{1.51(29000)(10)}{(140)^2(50)} = 0.4468$

Tension field action cannot be used in an end panel:

$$V_n = 0.6F_yA_wC_v = 0.6(50)(0.5 \times 70)(0.4468) = 469.1 \text{ kips}$$

$V_n = 469$ kips

(b) $\frac{a}{h} = \frac{200}{70} = 2.857 < \left(\frac{260}{h/t_w}\right)^2$ and < 3

$$k_v = 5 + \frac{5}{(a/h)^2} = 5 + \frac{5}{(2.857)^2} = 5.613$$

$$1.10\sqrt{\frac{k_vE}{F_y}} = 1.10\sqrt{\frac{5.613(29000)}{50}} = 62.76$$

Since $\frac{h}{t_w} > 62.76$, tension field action can be used, and

$$V_n = 0.6F_yA_w\left(C_v + \frac{1-C_v}{1.15\sqrt{1+(a/h)^2}}\right).$$

Compute C_v. $\quad 1.37\sqrt{\frac{k_vE}{F_y}} = 1.37\sqrt{\frac{5.613(29000)}{50}} = 78.17$

Since $\frac{h}{t_w} > 78.11$, $\quad C_v = \frac{1.51Ek_v}{(h/t_w)^2F_y} = \frac{1.51(29000)(5.613)}{(140)^2(50)} = 0.2508$

$$V_n = 0.6(50)(0.5\times 70)\left(0.2508 + \frac{1-0.2508}{1.15\sqrt{1+(2.857)^2}}\right) = 489.3 \text{ kips}$$

$\underline{V_n = 489 \text{ kips}}$

(c) If no intermediate stiffeners are used, $\frac{a}{h} > 3$, and $k_v = 5$ (no tension field permitted)

$$1.10\sqrt{\frac{k_vE}{F_y}} = 1.10\sqrt{\frac{5(29{,}000)}{50}} = 59.24$$

Since $\frac{h}{t_w} > 59.24$, $V_n = 0.6F_yA_wC_v$. Compute C_v.

$$1.37\sqrt{\frac{k_vE}{F_y}} = 1.37\sqrt{\frac{5(29{,}000)}{50}} = 73.78$$

Since $\frac{h}{t_w} > 73.78$, $\quad C_v = \frac{1.51Ek_v}{(h/t_w)^2F_y} = \frac{1.51(29000)(5)}{(140)^2(50)} = 0.2234$

$$V_n = 0.6F_yA_wC_v = 0.6(50)(0.5\times 70)(0.2234) = 234.6 \text{ kips}$$

$\underline{V_n = 235 \text{ kips}}$

10.5-3

Before developing the LRFD and ASD solutions, compute the nominal shear strength of each panel.

$$\frac{h}{t_w} = \frac{66}{5/16} = 211.2$$

End panel: $\quad a = 6(12) + 2 = 74$ in., $\quad \frac{a}{h} = \frac{74}{66} = 1.121 < 3$

$$\left(\frac{260}{h/t_w}\right)^2 = \left(\frac{260}{211.2}\right)^2 = 1.516 > \frac{a}{h}$$

$$k_v = 5 + \frac{5}{(a/h)^2} = 5 + \frac{5}{(1.121)^2} = 8.979$$

$$1.10\sqrt{\frac{k_vE}{F_y}} = 1.10\sqrt{\frac{8.979(29000)}{50}} = 79.38$$

$$1.37\sqrt{\frac{k_vE}{F_y}} = 1.37\sqrt{\frac{8.979(29000)}{50}} = 98.87$$

Since $\frac{h}{t_w} > 98.87$, $\quad C_v = \frac{1.51Ek_v}{(h/t_w)^2F_y} = \frac{1.51(29000)(8.979)}{(211.2)^2(50)} = 0.1763$

Tension field action cannot be used in an end panel:

$$V_n = 0.6F_yA_wC_v = 0.6(50)(5/16 \times 66)(0.1763) = 109.1 \text{ kips}$$

Second panel:

$$a = 12(12) + 9 - 74 = 79 \text{ in.}$$

$$\frac{a}{h} = \frac{79}{66} = 1.197 < \left(\frac{260}{h/t_w}\right)^2 \text{ and} < 3$$

$$k_v = 5 + \frac{5}{(a/h)^2} = 5 + \frac{5}{(1.197)^2} = 8.490$$

$$1.10\sqrt{\frac{k_vE}{F_y}} = 1.10\sqrt{\frac{8.490(29000)}{50}} = 77.19$$

Since $\frac{h}{t_w} > 77.19$, tension field action can be used, and

$$V_n = 0.6F_yA_w\left(C_v + \frac{1 - C_v}{1.15\sqrt{1 + (a/h)^2}}\right)$$

Compute C_v. $\quad 1.37\sqrt{\frac{k_vE}{F_y}} = 1.37\sqrt{\frac{8.490(29000)}{50}} = 96.14$

Since $\frac{h}{t_w} > 96.14$, $\quad C_v = \frac{1.51Ek_v}{(h/t_w)^2F_y} = \frac{1.51(29000)(8.490)}{(211.2)^2(50)} = 0.1667$

$$V_n = 0.6(50)(5/16 \times 66)\left(0.1667 + \frac{1 - 0.1667}{1.15\sqrt{1 + (1.197)^2}}\right) = 390.6 \text{ kips}$$

Middle panel:

$$a = 55(12) - 2(12 \times 12 + 9) = 354 \text{ in.,}$$

$$\frac{a}{h} = \frac{354}{66} = 5.364 > 3 \quad \therefore k_v = 5 \text{ and tension-field action cannot be used.}$$

$$1.10\sqrt{\frac{k_vE}{F_y}} = 1.10\sqrt{\frac{5(29000)}{50}} = 59.24$$

$$1.37\sqrt{\frac{k_vE}{F_y}} = 1.37\sqrt{\frac{5(29000)}{50}} = 73.78$$

Compute C_v.

Since $\frac{h}{t_w} > 73.78$, $\quad C_v = \frac{1.51Ek_v}{(h/t_w)^2F_y} = \frac{1.51(29000)(5)}{(211.2)^2(50)} = 9.817 \times 10^{-2}$

$$V_n = 0.6F_yA_wC_v = 0.6(50)(5/16 \times 66)(0.09817) = 60.74 \text{ kips}$$

(a) LRFD solution

End panel: Design strength $= \phi_vV_n = 0.90(109.1) = 98.2$ kips

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.225) + 1.6(2.0) = 3.470 \text{ kips/ft}$$

Left reaction $= V_L = \frac{3.470(55)}{2} = 95.43 \text{ kips} < 98.2 \text{ kips}$ (OK)

Second panel: Design strength $= \phi_vV_n = 0.90(390.6) = 352$ kips

V_u at beginning of panel $= V_L - w_ux = 95.43 - 3.470(6.167)$

$= 74.0 \text{ kips} < 352 \text{ kips}$ (OK)

Middle panel: Design strength $= \phi_vV_n = 0.90(60.74) = 54.7$ kips

V_u at beginning of panel $= V_L - w_ux = 95.43 - 3.470(12.75)$

$= 51.2 \text{ kips} < 54.7 \text{ kips}$ (OK)

Girder has enough shear strength

(b) ASD solution

End panel: Allowable strength $= \frac{V_n}{\Omega_v} = \frac{109.1}{1.67} = 65.3$ kips

$$w_a = w_D + w_L = 0.225 + 2.0 = 2.225 \text{ kips/ft}$$

Left reaction $= V_L = \frac{2.225(55)}{2} = 61.19 \text{ kips} < 65.3 \text{ kips}$ (OK)

Second panel: Allowable strength $= \frac{V_n}{\Omega_v} = \frac{390.6}{1.67} = 234$ kips

V_a at beginning of panel $= V_L - w_ax = 61.19 - 2.225(6.167)$

$= 47.5 \text{ kips} < 234 \text{ kips}$ (OK)

Middle panel: Allowable strength $= \frac{V_n}{\Omega_v} = \frac{60.74}{1.67} = 36.4$ kips

V_a at beginning of panel $= V_L - w_ax = 61.19 - 2.225(12.75)$

$= 32.8 \text{ kips} < 36.4 \text{ kips}$ (OK)

Girder has enough shear strength

10.6-1

Bearing strength: $A_{pb} = (6 - 0.5)(1/2) \times 2 = 5.5 \text{ in.}^2$

$$R_n = 1.8F_yA_{pb} = 1.8(36)(5.5) = 356.4 \text{ kips}$$

Compressive strength: Use a length of web equal to

$$25t_w = 25(5/16) = 7.813 \text{ in.}$$

Compute the radius of gyration about an axis along the middle of the web:

$$I = \frac{1}{12}(7.813)(5/16)^3 + 2\left[\frac{1}{12}(1/2)(6)^3 + 6(1/2)(3 + 5/32)^2\right] = 77.79 \text{ in.}^4$$

$$A = 7.813(5/16) + 2(6)(1/2) = 8.442 \text{ in.}^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{77.79}{8.442}} = 3.036 \text{ in.}$$

The slenderness ratio is $\frac{KL}{r} = \frac{Kh}{r} = \frac{0.75(56)}{3.036} = 13.83 < 25$

$\therefore P_n = F_yA_g = 36(8.442) = 303.9 \text{ kips}$

(a) LRFD solution

Bearing strength $= \phi R_n = 0.75(356.4) = 267 \text{ kips}$

Compressive strength $= \phi P_n = 0.90(303.9) = 274 \text{ kips}$

Bearing controls: Maximum factored concentrated load = 267 kips

(b) ASD solution

Bearing strength $= \frac{R_n}{\Omega} = \frac{356.4}{2.00} = 178 \text{ kips}$

Compressive strength $= \frac{P_n}{\Omega} = \frac{303.9}{1.67} = 182 \text{ kips}$

Bearing controls: Maximum service concentrated load = 178 kips